

Interactive Disentanglement: Learning Concepts by Interacting with their Prototype Representations

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Abstract

Learning visual concepts from raw images without strong supervision is a challenging task. In this work, we show the advantages of prototype representations for understanding and revising the latent space of neural concept learners. For this purpose, we introduce interactive Concept Swapping Networks (iCSNs), a novel framework for learning concept-grounded representations via weak supervision and implicit prototype representations. iCSNs learn to bind conceptual information to specific prototype slots by swapping the latent representations of paired images. This semantically grounded and discrete latent space facilitates human understanding and human-machine interaction. We support this claim by conducting experiments on our novel data set “Elementary Concept Reasoning” (ECR), focusing on visual concepts shared by geometric objects.¹

1. Introduction

Learning an adequate representation of concepts from raw data without strong supervision is a challenging task. However, it remains important for research in areas of knowledge discovery where sufficient prior knowledge is missing, and the goal is to attain novel understandings. With better representations and architectural components of machine learning models, this appears to become more and more achievable [73]. However, if remained unchecked this bears the danger of learning incorrect concepts or even confounding features [18, 70]. A further difficult aspect of concept learning, regardless of the level of supervision, is its dynamic and subjective nature. One downstream task might require more fine-grained concepts than others, but also when encountering evidence on novel concepts (e.g. in an online learning setting), the knowledge and hierarchy of concepts should be constantly re-approved, discussed, and

¹Code available at: <https://github.com/ml-research/XICConceptLearning>

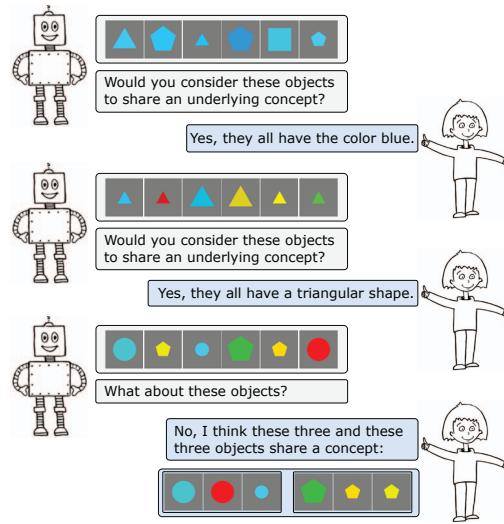


Figure 1. A trained model (left) queries the human user (right) if the concepts that it has extracted from the data coincides with the knowledge of the user. Subsequently, the model can receive revisions from the user.

possibly updated. It thus remains desirable that the representations learned by such concept learners to be human-understandable and revisable.

An evident approach to teaching concept information to a machine learning model is to train it in a supervised fashion through symbolic representations, e.g., one-hot encoding vectors and corresponding raw input [44, 84]. However, this requires extensive prior knowledge of relevant concepts and seems impractical given the subjective and dynamic nature of concept learning.

Another branch of research focuses on learning disentangled latent distribution models [27, 31, 80]. Although initially focused on unsupervised approaches, many recent studies have shifted away from unsupervised learning and show promising results with weak supervision [41, 49, 51, 58, 74, 81]. An often implicit assumption of dis-

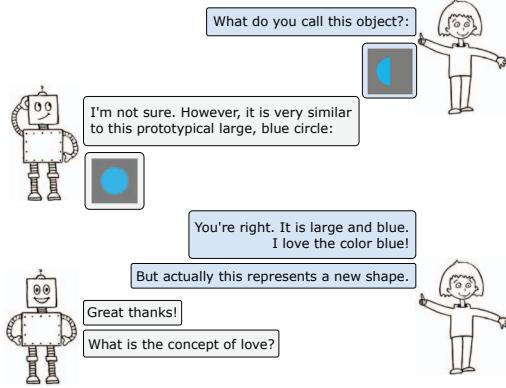


Figure 2. Human-machine interaction for learning about novel concepts. The user queries an object and guides the machine’s prototype suggestion if necessary.

entanglement research is that the learned latent representations should correspond to human-interpretable factors. Many state-of-the-art variational [51, 58] and generative adversarial [13, 46, 56, 59, 60] approaches, however, learn continuous latent representations, making these difficult for a human to understand without additional techniques for interpreting the latent space [68].

Due to the intricate nature of concept learning and inspired by findings on concept prototypes in the fields of psychology and cognitive science, we investigate the advantages of prototype representations in learning human-understandable and revisable concept representations for neural concept learners. To this end, we introduce the novel framework of Interactive Concept Swapping Network (iCSN) that learns to implicitly bind semantic concepts to latent prototype representations through weak supervision. This binding is enforced via a discretized distance estimation and swapping of shared concept representations between paired data samples. Among other things, iCSNs allow for querying and revising its learned concepts *cf.* Fig. 1, and integrating knowledge about unseen concepts *cf.* Fig. 2.

Explicitly focusing on learning object-centric visual concepts, we develop Elementary Concept Reasoning (ECR), a novel data set containing images of 2D geometrical objects and perform multiple experiments, emphasizing the advantages of our approach. To summarize, our work highlights the advantages of prototype representations for (i) learning a consistent and human-understandable latent space through weak supervision, (ii) revising concept representations via human interactions, and (iii) updating these in an online learning fashion.

2. Related Work

Concept learning. Many previous concept learning approaches focus on predicting selected high-level concepts

for improving additional downstream tasks [3, 44, 84]. Several studies highlight the benefits of concept-based machine learning for explainability [1, 25, 54, 83] and human interactions [77]. To communicate the concepts to a human user, some approaches include first-order logic formulas [16], causal relationships [82], user defined concepts [42], prediction of intermediate dataset-labels [3, 54], and one-hot encoded bottlenecks [44]. All of these approaches, however, focus on supervised concept learning.

Concept representations in psychology. The term concept is rooted in psychology where it can be defined as “*the label of a set of things that have something in common*” [2], though different notions do exist [23]. Most common approaches to represent concepts are exemplars and prototypes. Where the former approach assumes that one or multiple typical examples of a concept are maintained in memory, the latter only assumes an average representation over several observed examples [39, 61, 71, 75, 78]. The lines between exemplar and prototype representation become more blurred in the field of cognitive psychology, and their contribution to concept representations is still an open problem, with recent work hinting at the use of both representations [6, 24]. Nonetheless, there remains evidence of the use and importance of prototypes in the human memory system [17, 30, 38]. Inspiration sparked by such findings gave rise to Prototype Learning Systems [86].

Neural prototype learning. Recent approaches to artificial prototype learning systems focus on neural networks with prototype vectors as internal latent representations. These vectors can be converted into explainable visualizations via decoding approaches [45] or used for finding the most similar training example [11]. In both works, a class prediction is made based on the similarity of an encoded input to the model’s prototypes via a simple distance metric. Lastly, especially in the context of few-shot learning, prototypes show advantageous properties [37, 62, 64, 76].

Disentanglement. The field of disentanglement research is also closely related to our work. Here the goal is to extract independent underlying factors that are responsible for generating the data [4]. Recently, through the work of Locatello *et al.* [49] much of disentanglement research has shifted from unsupervised learning to the weakly supervised setting. Shu *et al.* [74] show that supervision via match pairing for a known subset of factors gives guarantees for disentanglement via their defined calculus of consistency and restrictiveness. Literature also extends to group-based disentanglement, allowing for grouping of the identified generative factors [5, 33, 34, 36, 85, 87]. However, the interpretation of the latent representations from these approaches remains an open question [68].

Explanatory interactive learning (XIL). The notion of human interactions on a model’s latent concept representations, *e.g.*, to correct confounding behaviour, is closely re-

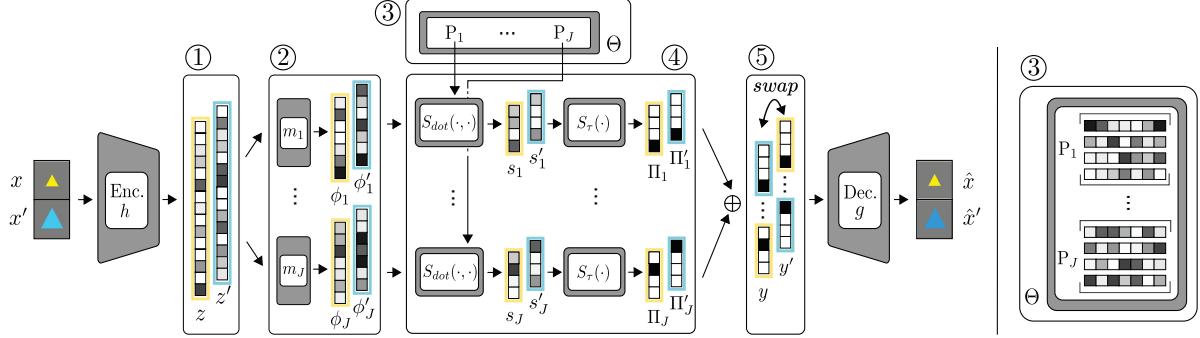


Figure 3. Interactive Concept Swapping Network. An iCSN is based on a deterministic autoencoder structure providing an initially entangled latent encoding (1). Several read-out encoders (2) extract relevant information from this latent space and compare their extracted concept encodings to a set of prototype slots (3) via a weighted, softmax-based dot product (4). This leads to a discretized code that indicates the most similar prototype slot of each concept encoding. iCSNs are trained via a simple reconstruction loss, weak supervision via match pairing and a swapping approach that swaps (5) the latent concept representations for shared concepts, enforcing the binding of semantic information to specific prototype representations.

lated to the field of XIL [69, 70, 72, 77, 79]. Specifically, XIL incorporates the human user into the training loop by allowing for them to interact via a model’s explanations. Rather than interacting via post-hoc explanations of previous XIL approaches, we focus on interacting directly with the latent representations of a model. These, nonetheless, possess a connection to the model’s explanations in our setup. Even though we take a more direct approach to revising a model’s internal representations, similar feedback methods as in XIL are applicable for our work.

3. Interactive Concept Swapping Networks

In this section, we explain the basic architectural components of an Interactive Concept Swapping Network (iCSN) before introducing the training procedure and how to interact with the implicit prototype representations of these networks. For an overview, see Fig. 3.

Prototype-based concept architecture. Assume an input $x_i \in X$, with $X := [x_1, \dots, x_N] \in \mathbb{R}^{N \times D}$. For the sake of simplicity, we remove the sample index i from further notations below and denote with $x \in \mathbb{R}^D$ an entire image. However, in our framework, x can also be a latent representation of a subregion of the image. This subregion can be implicitly or explicitly extracted from the image by a pre-processing step, *e.g.* via segmentation algorithms [8, 26, 29, 66] or compositional generative scene models [7, 19–21, 28, 48, 52]. Additionally, we assume each x to contain several attributes such as color, shape and size. Specifically, we refer to the realizations of these attributes, *e.g.*, a “blue color” or “triangular shape” as a *basic concept*. In contrast, we refer to “color” as a category concept or, as often called in the field of cognitive and psychological sciences, *superordinate concept* [22]. Each image x therefore has the ground truth basic concepts $c := [c_1, \dots, c_J]$ with

J denoting the total number of superordinate concepts. We make the necessary assumption that x can only contain one basic concept realization per superordinate concept. For simplicity, we furthermore assume that each superordinate concept contains the same number of basic concepts K which might vary in practice as we are going to show in our experiments.

Assuming an encoder-decoder structure, we define an input encoder $h(\cdot)$ that receives the image x and encodes it into a latent representation $z \in \mathbb{R}^Z$ by $h(x) = z$. Rather than reconstructing directly from z , as done by many autoencoder-based approaches, an iCSN first applies several read-out encoders $m_j(\cdot)$ to the latent representation z resulting in $m_j(z) = \phi_j \in \mathbb{R}^Q$ with $j \in [1, \dots, J]$. We refer to the encoding ϕ_j as a *concept encoding*. The goal of each read-out encoder is to extract the relevant information from the entangled latent space z that corresponds to a superordinate concept, *e.g.* color. We discuss how we enforce this extraction of concept specific information below.

One central component of the iCSN is a set of codebooks each containing multiple prototype slots. We define this set as $\Theta := [P_1, \dots, P_J]$, with a single codebook as $P_j \in \mathbb{R}^{Q \times K}$. Each codebook contains an ordered set of trainable, randomly initialized prototype slots $p_j \in \mathbb{R}^Q$, i.e., $P_j := [p_j^1, \dots, p_j^K]$.

To enforce the assignment of each concept encoding ϕ_j to one prototype slot of P_j , we define a similarity score $S_{dot}(\cdot, \cdot)$ as a softmax over the dot product between its two inputs. This way we obtain the similarity between a concept encoding, ϕ_j , and specific prototype slot, p_j^k with:

$$s_j^k = S_{dot}(\phi_j, p_j^k) = \frac{\exp(\phi_j \cdot p_j^k / \sqrt{Q})}{\sum_{k=1}^K \exp(\phi_j \cdot p_j^k / \sqrt{Q})} \quad (1)$$

The resulting similarity vector $s_j \in \mathbb{R}^K$ contains the sim-

ilarity score for each prototype slot of category j with the concept encoding ϕ_j . To enforce further discretization and the binding of concepts to individual prototype slots, we introduce a second function $S_\tau(\cdot)$ to apply a weighted softmax function to the similarity scores:

$$\Pi_j^k = S_\tau(s_j^k) = \frac{\exp(s_j^k/\tau)}{\sum_{k=1}^K \exp(s_j^k/\tau)}, \quad (2)$$

with $\Pi_j \in \mathbb{R}^K$ and weight parameter $\tau \in \mathbb{R}^+$. In our experiments we step-wise decrease τ to gradually enforce the binding of information. In the extreme case of $\tau \rightarrow 0$, Π_j resembles a one-hot vector (multi-label one-hot vector in the case of $J > 1$), indicating which prototype slot of category j the concept encoding ϕ_j is most similar to.

Finally, we concatenate the weighted similarity scores of each category into a single vector to receive the final prototype distance codes $y := [\Pi_1, \dots, \Pi_J] \in [0, 1]^{J \cdot K}$ which we pass to the decoder $g(\cdot)$ to reconstruct the image: $g(y) = \hat{x} \in \mathbb{R}^D$.

Concept swapping and weak supervision. Prior to training, i.e., after initialization, there is no semantic knowledge bound to the prototype slots yet. Each prototype carries just as little meaning as the other. The semantic knowledge found in converged iCSNs, however, is indirectly learned via a weakly-supervised training procedure and by employing a simple swapping trick.

We adopt the match pairing approach of Shu *et al.* [74], a practical weakly-supervised training procedure to overcome the issues of unsupervised disentanglement [50]. In this approach, a pair of images (x, x') is observed that shares values for a known subset of underlying factors of variation within the data, *e.g.* color, while the total number of shared factors can vary between 1 and $J - 1$. In this way, a model can use the additional information of the pairing to constrain and guide the learning of its latent representations.

Previous works on weakly-supervised training, specifically of VAEs, reverted to applying a product [5] or an average [35] of the encoder distributions of x and x' at the shared factor IDs. Locatello *et al.* [51], extended these works to a setting with an even weaker form of supervision but carries fewer disentanglement guarantees. In comparison to these works, an iCSN uses a simple swapping trick between paired representations, similar to Caron *et al.* [9]. Specifically, with v being the shared factor ID between the image pairs (x, x') the corresponding similarity scores (Π_v, Π'_v) are swapped between the final corresponding prototype codes, resulting in:

$$y := [\Pi_1, \dots, \Pi'_v, \dots, \Pi_J], \quad y' := [\Pi'_1, \dots, \Pi_v, \dots, \Pi'_J].$$

This swapping procedure has the intuitive semantic that it forces an iCSN to extract information from the first image that it can use to represent properties of the category v of the second image.

Pseudo-code of iCSNs can be found in the Supplementary Materials.

Training objective. iCSNs are finally trained with a single pixel-wise reconstruction loss per paired image over batches of size N :

$$L = \frac{1}{2N} \sum_{i=1}^N (x_i - \hat{x}_i)^2 + (x'_i - \hat{x}'_i)^2 \quad (3)$$

This simple loss term stands in contrast to several previous works on prototype learning, which enforce semantic binding via an additional consistency loss [45, 57, 64]. By including the semantic binding implicitly into the network architecture, we eliminate the need for additional hyper-parameters and a more complex optimization process over multiple objectives.

Interacting with iCSNs. The goal of iCSNs, especially in comparison to VAEs, is not necessarily to be a generative latent-variable model that learns the underlying data distribution, but to learn prototypical concept representations that are human-understandable and interactable. The autoencoder structure is thus a means to an end rather than a necessity. However, instead of discarding the decoder after convergence, an iCSN can present an input sample’s closest prototypical reconstruction of each concept. Thus, by querying these prototypical reconstructions at test time, a human user can confirm whether the predicted concepts make sense and possibly detect undesired model behavior. By defining a threshold on the test time reconstruction error, an iCSN can give a heuristic indication of its certainty in recognizing concepts in novel samples.

Due to the discrete and semantically bound latent code y , a human user can easily interact with iCSNs by treating y as a multi-label one-hot encoding. Specifically, a human user can revise and add knowledge via additional loss terms to the full extent of Stammer *et al.* [77]. For example, via logical statements such as $\forall \text{img. } \Rightarrow \neg \text{hasconcept}(\text{img}, p_1^1)$ or $\forall \text{img. } \text{isin}(\text{img}, \text{imgset}) \Rightarrow \text{hasconcept}(\text{img}, p_2^1)$, a user can formulate logical constraints which read as “Never detect the concept represented by the prototype p_1^1 .” and “For every image in this set of images you should be detecting the concept represented by prototype p_2^1 .”, respectively. The set of incorrectly represented images can be curated by the user interactively.

Lastly, the modularity of iCSNs has additional advantages or interactive online learning, *e.g.*, when the model is provided with data samples that contain novel concepts or when a factor that is present in the data is initially deemed unimportant but considered important retrospective to the initial learning phase. The approach for interaction in both cases depends on the hierarchy of the concept to be learned, namely if it is a basic concept or a superordinate concept. In the case of a novel basic concept, the approach is straightforward. Assuming human user satisfaction with the previous concept representations of an iCSN, and that J , the total

number of prototype slots per codebook, was set to be overestimated, a user can simply give the feedback to represent a novel basic concept via one of the unused prototype slots of the relevant category.

In case a novel superordinate concept should be learned, one can apply a simple trick during the initial training phase by adding an additional read-out encoder $\tilde{m}_{J+1}(z) = \tilde{\phi}_{J+1} \in \mathbb{R}^L$. In contrast to the other read-out encoders, this one does not map to the space of the prototype slots where it would consecutively be discretized via $S_{dot}(\cdot, \cdot)$ and $S_{\tau}(\cdot)$. Instead, $\tilde{\phi}_{J+1}$ remains a continuous representation that is directly concatenated to the final latent codes $y := [\Pi_0, \Pi_1, \dots, \Pi_J, \tilde{\phi}_{J+1}]$. In this way, $\tilde{m}_{J+1}(\cdot)$ can learn to incorporate all information that should not be discretized via the usual procedures. Ultimately, the initial latent space z of an iCSN can be trained to represent the full data distribution, even though only specific concepts should be discretized from this space. To include concepts that were initially considered irrelevant, one can just expand J which means to add a new read-out encoder $m_{J+1}(z) = \phi_{J+1} \in \mathbb{R}^Q$ and codebook P_{J+1} to the iCSN. Then, m_{J+1} learns to bind novel basic concepts from the “novel” superordinate concept to P_{J+1} which only requires novel data pairs exemplifying the previously unimportant concept.

Additional remarks. To summarize: the gradient of Eq. (3) provides the learning signal for the entire network, including the initial encoder, read-out encoders, prototype slots and decoder. Furthermore, by decreasing the softmax temperature τ in a step-wise fashion, one can enforce the binding of information to specific prototypes such that a specific concept is mapped to an individual prototype slot. Through this discretization process, the decoder learns to produce reconstructions that correspond to the prototypical representations present in the data. For example, given a pair of images of blue objects that vary in their shade of blue, an iCSN would learn to map the color information of these objects to the same prototype slot, thus learning the prototypical blue of both shades. This discretization step is a key difference to the various GAN, and VAE approaches with Gaussian distributions, which try to learn a continuous latent space of the underlying factors.

4. Elementary Concept Reasoning Data Set

Recent studies show the benefits of object-centric learning for performing complex downstream tasks [52, 77, 84]. Thus, rather than learning concepts of an entire image, *e.g.* as Kim *et al.* [43], we introduce a novel benchmark data set, Elementary Concept Reasoning (ECR), which explicitly focuses on object-centric visual concept learning.

ECR consists of RGB images ($64 \times 64 \times 3$) of 2D geometric objects on a constant colored background. Objects can vary in shape (circle, triangle, square, and pentagon), size (large and small), and color (red, green, blue, yellow).

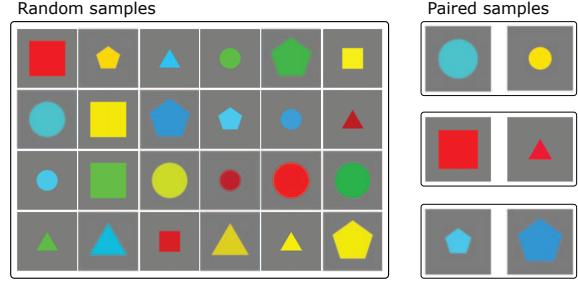


Figure 4. Samples of the Elementary Concept Reasoning data set. Each sample image (left) depicts a centered 2D object with three different properties: color, shape, and size. Images are paired such that the objects share between one and two concepts (right).

We add uniform jitter to each color, resulting in various color shades. Each image contains a single object which is fixed to the center of the image. Furthermore, ECR contains image pairs following the match pairing setup of Shu *et al.* [74]. We pair images such that the objects in the individual images share at least one, but at most $J - 1$ common properties. ECR contains a training set size of 5000 image pairs and 2000 images for validation.

Figure 4 shows example images of ECR. Random samples are presented on the left, exemplifying shape, size, and color combinations. Example image pairs are presented on the right. An important feature of ECR is that although various shades of colors exist, they all map to four discrete colors. Notice, *e.g.*, the color difference of the two paired blue objects. Even though both objects present different shades of blue, their state of being paired indicates that they share the same distinct shape (pentagon) and color (blue) concept.

5. Results

In this section, we demonstrate the advantages of prototype-based representations via Interactive Concept Swapping Networks. We begin our analysis by investigating the sparsity and semantics of iCSNs’ latent representations. Next, we show that the model can communicate the extracted concepts to a human user due to its discretized latent space. Subsequently, we simulate human user interactions via simple feedback rules, which are sufficient to revise an iCSNs’ latent concept space. Lastly, we show that novel concepts can easily be added into the concept space of iCSNs via simple human interactions.

Experimental details. For our experiments, we compare the iCSN to several baselines including the unsupervisedly-trained β -VAE [31] and Ada-VAE by Locatello *et al.* [51], using the arithmetic mean of the encoder distributions as in [35]. For a fair comparison with iCSNs which are trained via the shared match pairing of [74] and the Ada-VAE, which was originally introduced as a weaker form of supervision, we also trained the Ada-VAE with known shared

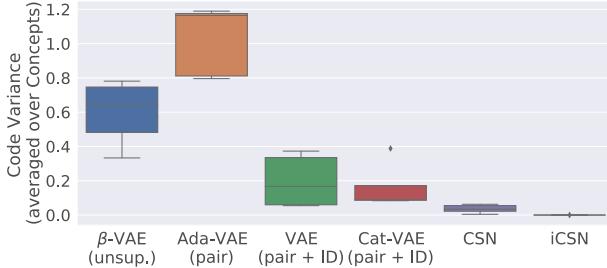


Figure 5. Average latent code variance given the ground truth concept labels for different model types and training settings. The compared models: unsupervised trained β -VAE, Ada-VAE with paired images, VAE and categorical VAE with paired images and known shared factor IDs, the novel CSN and iCSN with additional interactions on the learned concept space. Note that a lower variance is desirable.

factor IDs. This baseline essentially resembles a β -VAE with an averaging of encoder distributions between pairs of images at the known shared factor IDs. It is denoted as VAE in the results below. Lastly, we compare to a discretizing VAE approach which uses a categorical distribution via the Gumbel-softmax trick [40, 53] (Cat-VAE). Cat-VAE is trained the same way as the VAE, i.e., via share pairing and averaging over encoder distributions.

We train the iCSN with a simple reconstruction loss as in Eq. (3) and decreasing softmax temperature. We present the results of two iCSN configurations. The vanilla setting, denoted as *Concept Swapping Network* (CSN), corresponds to an iCSN prior to human interactions with the correct number of superordinate concepts ($J = 3$) and an over-estimated number K of prototype slots per superordinate concept ($K = 6$). Finally, *iCSN* denotes a CSN after the initial training phase with additional user interactions.

The number of latent variables for β -VAE, Ada-VAE, and VAE was set to the ground truth number of superordinate concepts. All Cat-VAE runs were performed with three categorical distributions each with $k = 6$ events.

All configurations were trained with five random seed initializations, and the results present the mean and standard deviation of these runs. All presented results were obtained from a held-out validation set. Further details can be found in the Supplementary Materials.

Reduced code variance. Human understandability of and interactions with a model’s latent space strongly benefit from consistency in a model’s concept representations. In other words, the representation of the color blue should be specific to this concept and the latent representation for the blue color of one object should be very similar to that of a second blue object. If this is not the case, it remains difficult for a human user to identify and interact with these learned concepts.

Motivated by this intuition, we first investigate the vari-

	DT	LR
β -VAE (unsup.)	88.98 ± 11.53	73.86 ± 18.07
Ada-VAE (pair)	63.39 ± 11.32	74.07 ± 6.40
VAE (pair + ID)	97.28 ± 5.32	97.88 ± 0.73
Cat-VAE (pair + ID)	74.79 ± 19.45	91.17 ± 13.13
CSN	99.92 ± 0.05	99.87 ± 0.07
iCSN	100.0 ± 0.00	100.0 ± 0.00
Ablation		
Cat-VAE w. swap.	60.85 ± 7.29	65.71 ± 10.66
iCSN w. avg.	68.84 ± 39.4	68.81 ± 39.4

Table 1. Linear probing via decision tree (DT) and logistic regression (LR). (top) Probing on the latent codes of iCSN models and various baselines. (bottom) Ablation study via probing on the latent codes of Cat-VAE with encoder distribution swapping and iCSN concept encoding averaging. All classification accuracies were computed on a held out test set.

ance of the latent representations given the ground truth multi-label information of each validation image. For this, we compute the latent code variance over all validation images of each concept. In mathematical notation this corresponds to:

$$\frac{1}{K \cdot J} \sum_{j=1}^{J=3} \sum_{k=1}^K \text{Var}(\{\bar{z}_j\}_{l==k}) \quad (4)$$

with $\text{Var}(\cdot)$ denoting the variance, \bar{z}_j denoting a place holder for the latent representation of a corresponding model (the discretized prototype distance code y for iCSNs, the distribution means for VAEs with Gaussian distributions and the event probabilities for Cat-VAE). $\{\cdot\}_{l==k}$ denotes the set of latent representations from images for which the ground truth basic concept of category j corresponds to k .

The resulting code variances over all models can be seen in Fig. 5. Note that a low code variance is desirable and indicates how well a concept is mapped to a distinct representation. The results in Fig. 5 suggest that the variance of the latent space from CSNs is much lower, showing more consistent concept representations. However, a reduced latent code variance is not a sufficient criterion for concept consistency and human understandability. For example, a model that learns to map all concepts to a single representation has zero latent code variance but also no representational power. Therefore, we turn to probing the latent concept space via linear models next.

Probing the latent space. Similar to works of the self-supervision community [10, 12, 14, 15, 63], we investigate the latent code of each model via linear probing. For this, the latent codes of each model on a held-out data set are inferred, as in the previous experiment. Next, ground truth labels are obtained by converting each multi-label ground truth vector, c , of this data set to a 32-dimensional one-hot

encoding. Finally, a Decision Tree (DT) and a Logistic Regression (LR) are trained supervisedly on this data set and validated on an additional held-out data set.

The results in Tab. 1 (top) document the average accuracy and standard deviation on the held-out validation set over the five random initializations for the different models. We observe that the latent code of CSNs allows for nearly perfect predictive performance and surpasses all variational approaches. Importantly, CSNs’ representations even surpass those of VAE approaches (VAE and *Cat*-VAE) that were trained with the same type of weak supervision as CSNs. As expected, the β -VAE performs worse on average than the weakly-supervised models. Interestingly, however, the Ada-VAE configuration performed worse than the β -VAE. In addition, the discrete latent representations of *Cat*-VAE also perform worse than CSNs. Noticeably, the *Cat*-VAE runs indicate a high deviation in performance, indicating that several *Cat*-VAE runs converged to sub-optimal states. In summary, although the ECR data set only contains variations in individual 2D geometrical objects, the baseline models do not perform as well as CSNs, even when trained with the same amount of information.

Explaining and revising the latent space. An advantage of an iCSN’s semantically bound, discrete latent space, is the straightforward identification of sub-optimal concept representations by a human user *cf.* Fig. 1. Upon identifying correctly or falsely learned concepts, a user can then apply simple logical feedback rules on this discrete concept space.

Specifically, after training via weak supervision, it is recommendable for the machine and human user to discuss the learned concepts and identify whether these coincide with the user’s knowledge or if a revision is necessary. For example, an iCSN can learn to represent a color over several prototype slots or represent two shapes via one slot, indicating that it falsely considers these to belong to the same concept. An iCSN can then convey its learned concepts in two ways. First, it can group novel images that share a concept according to its inferred discrete prototype distance codes and inquire a human user if indeed the grouped images share a common underlying concept *cf.* Fig. 1. Second, utilizing the decoder, it can present the prototypical reconstruction of each learned concept, *e.g.*, presenting an object with a prototypical shade of blue *cf.* Fig. 2.

Having identified potential sub-optimal concept representations, a human user can now interact on the discretized latent space of iCSNs via logical rules and further improve the representations, which we demonstrate via simulated user interactions in the following. For all previous runs of the vanilla CSN configuration, we visually inspect the concept encodings y for one example each of the 32 possible concept combinations and identify those prototype slots which are “activated” in the majority of examples per individual concept (*primary* slots) and, additionally, identify

those prototype slots per concept that are never or rarely activated within our subset of examples (*secondary* slots). We next apply an $L2$ loss on y to never use these *secondary* slots and finetune the previous runs on the original training set with the original reconstruction loss and this additional $L2$ loss. The semantics of this feedback is that concepts should only be represented by their *primary* prototype slots. Additionally, in two runs we revise an observed sub-optimal solution that pentagons and circles are bound to the same prototype slot. Hereby, feedback is provided on all pentagon samples of the training set to bind to an otherwise empty prototype slot, again via an additional $L2$ loss.

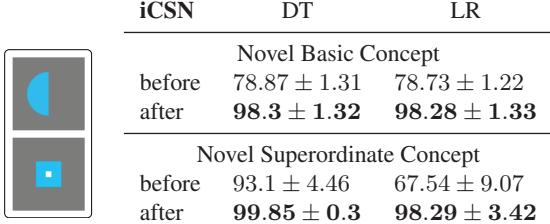
The results of these interactions can be found under *iCSN* in Fig. 5 and Tab. 1 (top) indicating a near-zero latent code variance per ground truth concept and perfect linear probing accuracy, respectively. Thus, indicating the ease of interacting with and revising the latent space of iCSNs.

Interactively learning novel basic concept. Furthermore, the prototype-based representations of iCSNs possess interesting properties for an online learning setting, *e.g.*, when encountering novel concepts which the model has not seen before. Through the decoder of the iCSN, evaluating the reconstruction and reconstruction error can serve as a means for identifying whether the model has a good representation of a novel sample *cf.* Fig. 2.

When teaching an iCSN a novel basic concept like a *half-circle* shape *cf.* Fig. 6, a user can identify an unbound prototype slot of the model’s latent representation and encourage the binding to this slot. To prevent catastrophic forgetting [55, 65], *i.e.*, overriding already learned concepts, we employ a combination of common rehearsal [67] and knowledge distillation methods [32, 47] by letting the model predict the latent codes of past samples and restrict the iCSN to not deviate from those. Specifically, we use a simple $L2$ loss on the known and unknown one-hot concept encodings to encourage binding of the unknown concepts while not forgetting the known ones.

Figure 6 (top) shows the linear model prediction accuracies on the latent space of iCSNs that have been presented a data set containing the novel concept *halfcircle*. The tag *before* indicates the accuracy on the latent code of the *iCSN* runs of Tab. 1 (top) that were trained on the standard ECR concepts, whereas *after* indicates the accuracy on the latent code after additional interactions with a simulated user by providing the information which empty prototype slot of the shape codebook to bind the novel concept to. These results indicate the ease of adding additional knowledge on novel basic concepts to the latent representation of iCSNs.

Interactively learning novel superordinate concept. Next, we showcase how to add a novel superordinate concept. For this setting, we make use of a variation of the ECR data set where white spots were added to the center of roughly half of the objects *cf.* Fig. 6. The other half depicts



iCSN	DT	LR
Novel Basic Concept		
before	78.87 \pm 1.31	78.73 \pm 1.22
after	98.3 \pm 1.32	98.28 \pm 1.33
Novel Superordinate Concept		
before	93.1 \pm 4.46	67.54 \pm 9.07
after	99.85 \pm 0.3	98.29 \pm 3.42

Figure 6. Linear probing via Decision Tree (DT) and Logistic Regression (LR) on the latent codes of iCSN. We evaluate the models with a ECR data set containing a novel shape (top) and a novel superordinate spot concept (bottom), each not seen during initial training (before). After human user interactions (after) this novel information could be easily added to the concept representations.

a solid color as in the original ECR data set.

We consider an online learning setting where spots are unimportant during the initial training but reconsidered important in the second round of interaction. The modularity of iCSNs allows us to easily add a read-out encoder during initial training that learns to represent all the information of the data that is not discretized via the initial paired samples (*cf.* Sec. 3 for details). In the second round of training, this continuous read-out encoder can be replaced with a discretizing read-out encoder and additional codebook. This modularity property further eliminates the danger of catastrophic forgetting in that all previously trained modules can be frozen in the second training round, thus only requiring the novel read-out encoder to be finetuned.

Human user interactions are simulated by assuming that the iCSN has correctly learned the previous concepts with an additional continuous read-out encoder for representing the spotted feature. Subsequently, additional training pairs are introduced that exemplify the novel superordinate concept, and the new read-out encoder is finetuned as in the standard training setting. Results can be seen in Fig. 6 again presenting the linear model accuracy on the latent representations before and after simulated user interactions, indicating that the novel superordinate concept can easily be bound to the model’s internal prototype representations.

Here, we remark on desirable properties of iCSNs for handling confounded data. Assuming an undesired confounding factor within the data generation process that causes spurious features, an iCSN can learn to ignore these features during its training process via the mechanism presented above. This stands in comparison to GANs or variational approaches with Gaussian distributions, which could potentially learn also to model the spurious features.

Ablation studies. To assess the importance of the different components of iCSNs, we conduct an ablation study and depict the linear probing classification performances in Tab. 1 (bottom). Specifically, we test a Cat-VAE that uses swapping of the relevant encoder distributions, rather than

averaging as in previous experiments. And, secondly, we test an iCSN with averaging of the concept encodings ϕ_j , rather than swapping. With these experiments, we wish to (i) compare the discretization via distances to prototypes to discretization via categorical distributions (Cat-VAE w. swapping) and (ii) test the influence of swapping versus averaging of encodings for iCSNs (iCSN w. avg).

The results of Tab. 1 (bottom) when compared with Tab. 1 (top) indicate that discrete representations via distances to prototypes are, in fact, beneficial compared to those of the categorical distributions of Cat-VAEs. Secondly, the swapping procedure appears to be crucial for optimal learning of concept representations in iCSNs.

Limitations. Following assumptions were made in this work: a superordinate concept is divisible into multiple basic concepts and “valid” user feedback was provided in our experiments on interactions. A potential limiting factor of iCSNs is the training reconstruction loss which might be insufficient for learning fine-grained concepts. Additionally, we observed that the choice of τ can influence the quality of the learning process. Setting a small value too early can lead to sub-optimal solutions. Lastly, our approach was tested on ECR due to its object-centric nature and distinct concept distribution of the data set. For more complex settings additional architectures may be required to pre-process the data to *e.g.* extract objects from an image.

6. Conclusion

In this work, we investigated the properties of latent prototype representations for neural concept learning with weak supervision. The results with our novel iCSN framework indicate that these are beneficial for human-understandable concept learning but also human interactions and the incorporation of novel concepts within an online learning setting. Interesting pathways for future research are applying iCSNs to more complex data sets, particularly from critical domains such as medical or scientific data where often relevant concepts are not known in advance, however standard deep learning approaches can learn to focus on confounding factors, *e.g.* for Covid-19 data [18] or plant phenotyping data [70]. We hypothesize the interactive approach of iCSNs to be beneficial in allowing the machine and human to identify relevant and irrelevant concepts within the data jointly.

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