Adaptive Frequency Filters As Efficient Global Token Mixers

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Abstract

Recent vision transformers, large-kernel CNNs and MLPs have attained remarkable successes in broad vision tasks thanks to their effective information fusion in the global scope. However, their efficient deployments, especially on mobile devices, still suffer from noteworthy challenges due to the heavy computational costs of self-attention mechanisms, large kernels, or fully connected layers. In this work, we apply conventional convolution theorem to deep learning for addressing this and reveal that adaptive frequency filters can serve as efficient global token mixers. With this insight, we propose Adaptive Frequency Filtering (AFF) token mixer. This neural operator transfers a latent representation to the frequency domain via a Fourier transform and performs semantic-adaptive frequency filtering via an elementwise multiplication, which mathematically equals to a token mixing operation in the original latent space with a dynamic convolution kernel as large as the spatial resolution of this latent representation. We take AFF token mixers as primary neural operators to build a lightweight neural network, dubbed AFFNet. Extensive experiments demonstrate the effectiveness of our proposed AFF token mixer and show that AFFNet achieve superior accuracy and efficiency trade-offs compared to other lightweight network designs on broad visual tasks, including visual recognition and dense prediction tasks. Code is available at https://github.com/microsoft/TokenMixers.

1. Introduction

Remarkable progress has been made in ever-changing vision network designs to date, wherein effective token mixing in the global scope is constantly highlighted. Three existing dominant network families, *i.e.*, Transformers, CNNs and MLPs achieve global token mixing with their respective ways. Transformers [16, 63, 40, 12, 80, 74] mix tokens with self-attention mechanisms where pairwise correlations between query-key pairs are taken as mixing weights. CNNs achieve competitive performance with transformers by scaling up their kernel sizes [51, 15, 38, 9]. MLPs [60, 25, 35] provide another powerful paradigm via fully connections across all tokens. All of them are effective but computationally expensive, imposing remarkable challenges in practical deployments, especially on edge devices.

Recently, there is increased attention on improving the efficiency of token mixing in transformers. Some works [30, 40, 12, 22, 45, 50, 31, 75] squeeze the scope of token mixing in different ways to compromise the representation capacities of neural networks for their efficiencies. Other works reduce the complexity of the matrix operations in self-attention by making use of the associativity property of matrix products [29] or low-rank approximation methods [20, 76]. These methods all sacrifice the expressiveness of neural networks and lead to unsatisfactory performance of efficient network designs. A general-purpose global token mixing for lightweight networks is still less explored. Better trade-off between accuracy and efficiency for global-scope token mixing is worthy of further study.

In this work, we reveal that *adaptive frequency filters can serve as efficient global token mixers*, inspired by the convolution theorem [44, 53, 48] widely used in conventional signal processing. This theorem states that a convolution in one domain mathematically equals the Hadamard product (also known as elementwise product) in its corresponding Fourier domain. This equivalence allows us to frame

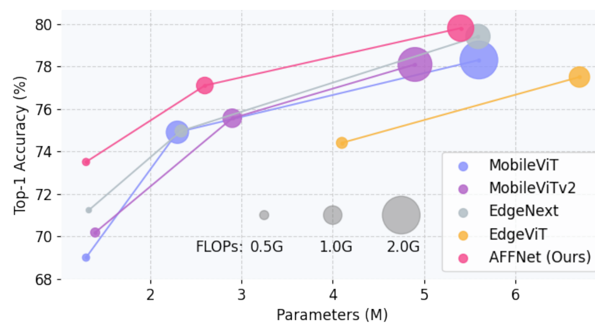

Figure 1. Comparison of Top-1 accuracy on ImageNet-1K [55] between our proposed AFFNet to some state-of-the-art lightweight networks that have global token mixing. The bubble size corresponds to FLOPs.

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global token mixing as a large-kernel convolution in the latent space and efficiently implement this convolution with a Hadamard product operation in the frequency domain by performing Fourier transforms on tokens in the latent space.

Besides large scopes, the adaptability to semantics also matters for token mixing as studied in [13, 8, 70, 1, 72]. This means that the weights for token mixing should be instance-adaptive. Moreover, different semantic attributes of the learned latent representations distribute in different channels [1, 73]. This property poses requirements for channel-specific token mixing wherein the weights of token mixing vary across different channels. From the perspective of framing global adaptive token mixing as a convolution, the kernel of this convolution operation should be not only large but also spatially dynamic. However, it is well known that dynamic convolutions are computationally expensive in common. Large-kernel dynamic convolutions seem extremely prohibitive for efficient/lightweight network designs. In this paper, we propose to adopt frequency filtering in the Fourier domain with learned instance-adaptive masks as a mathematical equivalent of token mixing using large-kernel dynamic convolutions by making use of the aforementioned convolution theorem. This equivalent could reduce the complexity of token mixing from $\mathcal{O}(N^2)$ to $\mathcal{O}(N \log N)$ thanks to adopting Fast Fourier Transforms (FFT), which is more computationally efficient.

With the key insight above, we propose Adaptive Frequency Filtering (AFF) token mixer. In this neural operator, the latent representations (*i.e.*, a set of tokens) are transferred from its original latent space to a frequency space via a 2D discrete Fourier transform applied spatially. In this way, we get the frequency representations whose spatial positions correspond to different frequency components. We adopt an extremely lightweight network to learn instance-adaptive masks from these frequency representations, and then calculate the Hadamard product between the learned masks and the frequency representations for adaptive frequency filtering. The filtered representations are transferred back to the original latent space via an inverse Fourier transform. The features after this inverse transform could be viewed as the results of token mixing with depthwise convolution kernels whose spatial dimensions are as large as those of latent representations (*i.e.*, the token set). According to the convolution theorem [44], our proposed operation mathematically equals to taking the tensors of applying an inverse Fourier transform to the learned masks in the Fourier domain as the corresponding kernel weights and perform convolution with this kernel in the original domain. Detailed introduction, demonstration and analysis are given in subsequent sections.

Furthermore, we take the proposed AFF token mixer as the primary neural operator and assemble it into an AFF block together with a plain channel mixer. AFF blocks serve as the basic units for constructing efficient vision backbone, dubbed AFFNet. We evaluate the effectiveness and efficiency of our proposed AFF token mixer by conducting extensive ablation study and comparison across diverse vision tasks and model scales.

Our contributions can be summarized in the following:

- We reveal that adaptive frequency filtering in the latent space can serve as efficient global token mixing with large dynamic kernels, and propose Adaptive Frequency Filtering (AFF) token mixer.
- We conduct theoretical analysis and empirical study to compare our proposed AFF token mixer with other related frequency-domain neural operators from the perspective of information fusion for figuring out what really matters for the effects of token mixing.
- We take AFF token mixer as the primary neural operator to build a lightweight vision backbone AFFNet. AFFNet achieves the state-of-the-art accuracy and efficiency trade-offs compared to other lightweight network designs across a broad range of vision tasks. An experimental evidence is provided in Fig.1.

2. Related Work

2.1. Token Mixing in Deep Learning

Mainstream neural network families, *i.e.*, CNNs, Transformers, MLPs, differ in their ways of token mixing, as detailed in [71]. CNNs [49, 87] mix tokens with the learned weights of convolution kernels where the spatial kernel size determines the mixing scope. Commonly, the weights are deterministic and the scope is commonly a local one. Transformers [66, 16] mix tokens with pairwise correlations between query and key tokens in a local [40, 12] or global[16, 63] range. These weights are semantic-adaptive but computationally expensive due to the $\mathcal{O}(N^2)$ complexity. MLPs commonly mix tokens with deterministic weights in manually designed scopes [5, 62, 61, 82] wherein the weights are the network parameters. This work aims to design a generally applicable token mixer for lightweight neural networks with three merits: computation-efficient, semantic-adaptive and effective in the global scope.

2.2. Lightweight Neural Networks

Lightweight neural network designs have been of high values for practical deployments. CNNs, Transformers, and MLPs have their own efficient designs. MobileNets series [27, 56, 26] introduce depthwise and pointwise convolutions as well as modified architectures for improving the efficiency. Shufflenet series [85, 41] further improve pointwise convolution via shuffle operations. MobileViT [45] combines lightweight MobileNet block and multi-head self-attention blocks. Its follow-up versions further improve it with a linear-complexity self-attention method [46].

Besides, there are many works reducing the complexity of self-attention via reducing the region of token mixing [40, 12, 50, 31] or various mathematical approximations [20, 76, 42]. Many efficient MLPs limit the scope of token mixing to horizontal and vertical stripes [83, 25, 59] or a manually designed region [6].

2.3. Frequency-domain Deep Learning

Frequency-domain analysis has been a classical tool for conventional signal processing [2, 52] for a long time. Recently, frequency-domain methods begin to be introduced to the field of deep learning for analyzing the optimization [78, 79] and generalization [67, 77] capabilities of Deep Neural Networks (DNNs). Besides these, frequency-domain methods are integrated into DNNs to learn non-local [11, 54, 34, 19] or domain-generalizable [36] representations. Our proposed method might be similar to them at first glance but actually differs from them in both modelling perspectives and architecture designs. These five works propose different frequency-domain operations by introducing convolutions [11], elementwise multiplication with trainable network parameters [54], matrix multiplication with trainable parameters [34], groupwise MLP layers [19] and elementwise multiplication with spatial instance-adaptive masks [36] to frequency-domain representations, respectively. All of them are not designed for the same purpose with ours. We provide detailed mathematical analysis on their shortcomings as token mixers and conduct extensive experimental comparisons in the following sections.

3. Method

We first describe a unified formulation of token mixing, then introduce our proposed Adaptive Frequency Filtering (AFF) token mixer. We further analyze what properties matter for a frequency-domain operation in terms of its effects on token mixing. We finally introduce AFFNet which is a lightweight backbone with AFF token mixer as its core.

3.1. Unified Formulation of Token Mixing

Token mixing is of high importance since learning non-local representations is critical for visual understanding [69, 16, 63]. In most mainstream neural networks, the input image is firstly patchified into a feature tensor $\mathbf{X} \in \mathbb{R}^{H\times W\times C}$ whose spatial resolution is $H \times W$ and the number of channels is C. This feature tensor could be viewed as a set of tokens, in which each token can be denoted as $\mathbf{x} \in \mathbb{R}^{1\times 1\times C}$. The updated token for a query $\mathbf{x}^q$ after token mixing in its contextual region $\mathcal{N}(\mathbf{x}^q)$ can be formulated in a unified form:

$$\hat{\mathbf{x}^q} = \sum_{i\in\mathcal{N}(\mathbf{x}^q)} \boldsymbol{\omega}^{i\to q} \times \phi(\mathbf{x}^i), \tag{1}$$

where $\hat{\mathbf{x}^q}$ refers to the updated $\mathbf{x}^q$ and $\mathbf{x}^i$ refers to the tokens in $\mathcal{N}(\mathbf{x}^q)$. $\phi(\cdot)$ denotes the embeding functions. $\boldsymbol{\omega}^{i\to q}$ represents the weights of information fusion from token $\mathbf{x}^i$ to the updated $\mathbf{x}^q$. The symbol $\times$ could be Hadamard product or matrix multiplication.

We revisit the prevailing token mixing methods in different types of network architectures in terms of their effectiveness and efficiency. For CNNs, tokens are mixed by matrix multiplication with deterministic network parameters as the mixing weights. Here, the kernel sizes of convolutions determine the scopes of token mixing. This makes mixing in a global scope quite costly due to the quadratically increased parameters and FLOPs as the kernel size increases. Transformers mix tokens with pairwise correlations between query and key tokens. Its computational complexity is $\mathcal{O}(N^2)$ (N is the total number of tokens), limiting its applications in lightweight networks. Like CNNs, MLPs also mix tokens with deterministic network parameters. The scope of token mixing in advanced MLPs [5, 62, 61, 82] are commonly manually design, where the globality comes at the cost of huge computational complexity. They are all not specifically designed for lightweight neural networks.

This work aims to deign a *computationally efficient*, *semantically adaptive* and *global-scope* token mixer for lightweight networks. This requires a large $\mathcal{N}(\mathbf{x}^q)$ and instance-adaptive $\boldsymbol{\omega}^{i\to q}$ with less network parameters and low computation costs as possible.

3.2. Adaptive Frequency Filtering Token Mixer

We apply the convolution theorem [44, 53, 48] to deep learning for designing a token mixer with aforementioned merits for lightweight neural networks. Based on this theorem, we reveal that *adaptive frequency filters can serve as efficient global token mixers*. In the following, we introduce its mathematical modelling, architecture design and the equivalence between them for our proposed token mixer.

Modelling. To simplify understanding, we frame token mixing in the form of global convolution, succinctly denoted by $\hat{\mathbf{X}} = \mathcal{K}*\mathbf{X}$. For the query token at position (h, w), *i.e.*, $\mathbf{X}(h, w)$, Eq.(1) can be reformulated as:

$$\hat{\mathbf{X}}(h,w) = \sum_{h'=-\lfloor \frac{H}{2}\rfloor}^{\lfloor \frac{H}{2}\rfloor} \sum_{w'=-\lfloor \frac{W}{2}\rfloor}^{\lfloor \frac{W}{2}\rfloor} \mathcal{K}(h',w')\mathbf{X}(h-h',w-w'), \tag{2}$$

where $\hat{\mathbf{X}}(h, w)$ represents the updated token for $\mathbf{X}(h, w)$ after token mixing. H and W are the height and weight of the input tensor, respectively. $\mathcal{K}(h', w')$ denotes the weights for token mixing, implemented by a global convolution kernel which has the same spatial size with $\mathbf{X}$. The padding operation for $\mathbf{X}$ is omitted here for simplicity and the specific padding method is introduced in the subsequent parts.

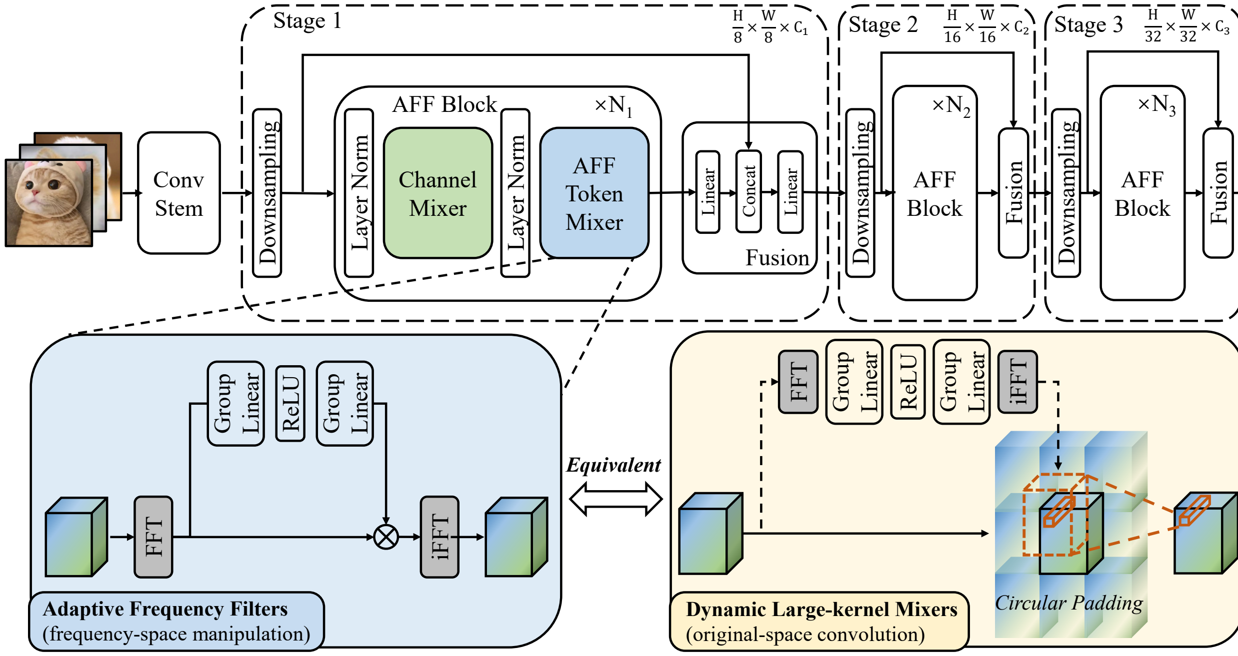

Figure 2. Illustration of our proposed AFF token mixer and its corresponding network AFFNet. The AFF token mixer is implemented by adaptive frequency filters at the bottom left and mathematically equals to the mixing operation at the bottom right. It can be viewed as mixing tokens with a large-kernel dynamic convolution whose weights are inferred by a sub-network as shown at the bottom right.

With the expectation for our proposed token mixer as a *semantic-adaptive* and *global-scope* one, the weights $\mathcal{K}$ for token mixing should be adaptive to $\mathbf{X}$ and of a large spatial size. As illustrated by the bottom-right subfigure in Fig.2, a straightforward way for enabling $\mathcal{K}$ adaptive to $\mathbf{X}$ is to use a dynamic convolution kernel [28, 8, 23, 84], *i.e.*, inferring weights of $\mathcal{K}$ with $\mathbf{X}$ as the inputs of a sub-network. However, adopting dynamic convolutions is usually computational costly, even more so, when using large-kernel ones. This thus imposes big challenges in designing an efficient token mixer for lightweight networks along this way. Next, we introduce an efficient method as its equivalent implementation by making use of the convolution theorem [44].

Architecture. The convolution theorem [44, 48, 53] for inverse Fourier transform states that a convolution in one domain mathematically equals the Hadamard product in its corresponding Fourier domain. This inspires us to propose a lightweight and fast architecture (illustrated by the lower left part of Fig.2) as an extremely efficient implementation of our modelling above.

Given feature $\mathbf{X} \in \mathbb{R}^{H\times W\times C}$, *i.e.*, a set of tokens in the latent space, we adopt Fast Fourier Transform (FFT) to obtain the corresponding frequency representations $\mathbf{X}_F$ by $\mathbf{X}_F = \mathcal{F}(\mathbf{X})$. The detailed formulation of $\mathcal{F}(\cdot)$ is:

$$\mathbf{X}_F(u,v) = \sum_{h=0}^{H-1}\sum_{w=0}^{W-1} \mathbf{X}(h,w)e^{-2\pi i(uh+vw)}. \tag{3}$$

As indicated by Eq.(3), features of different spatial positions in $\mathbf{X}_F$ correspond to different frequency components of $\mathbf{X}$. They incorporate global information from $\mathbf{X}$ with a transform of $\mathcal{O}(N\log N)$ complexity.

We apply the aforementioned convolution theorem to achieve efficient global token mixing for $\mathbf{X}$ by filtering its frequency representation $\mathbf{X}_F$ with a learnable instance-adaptive mask. We further adopt inverse FFT to the filtered $\mathbf{X}_F$ for getting the updated feature representations $\hat{\mathbf{X}}$ in the original latent space. This process can be formulated as:

$$\hat{\mathbf{X}} = \mathcal{F}^{-1}[\mathcal{M}(\mathcal{F}(\mathbf{X}))\odot\mathcal{F}(\mathbf{X})], \tag{4}$$

where $\mathcal{M}(\mathcal{F}(\mathbf{X}))$ is the mask tensor learned from $\mathbf{X}_F$, which has the same shape with $\mathbf{X}_F$. As shown in the lower left subfigure in Fig.2, to make the network lightweight as possible, $\mathcal{M}(\cdot)$ is efficiently implemented by a group 1×1 convolution (linear) layer, followed by a ReLU function and another group linear layer. $\odot$ denotes Hadamard product, also known as elementwise multiplication, and $\mathcal{F}^{-1}(\cdot)$ denotes inverse Fourier transform. Here, $\hat{\mathbf{X}}$ can be viewed as the results of global adaptive token mixing for $\mathbf{X}$, which is mathematically equivalent to adopting a large-size dynamic convolution kernel as the weights for token mixing. The equivalence is introduced in the following.

Equivalence. The convolution theorem still applies to the latent representations of neural networks. The multiplication of two signals in the Fourier domain equals to the

Fourier transform of a convolution of these two signals in their original domain. When applying this to the frequency-domain multiplication in Fig.(2), we know that:

$$\mathcal{M}(\mathcal{F}(\mathbf{X})) \odot \mathcal{F}(\mathbf{X}) = \mathcal{F}\{\mathcal{F}^{-1}[\mathcal{M}(\mathcal{F}(\mathbf{X}))] * \mathbf{X}\}. \tag{5}$$

Combining Eq.(4) and Eq.(5), it is easy to get that:

$$\hat{\mathbf{X}} = \mathcal{F}^{-1}[\mathcal{M}(\mathcal{F}(\mathbf{X}))] * \mathbf{X}, \tag{6}$$

where $\mathcal{F}^{-1}(\mathcal{M}(\mathcal{F}(\mathbf{X})))$ is a tensor of the same shape with $\mathbf{X}$, which could be viewed as a dynamic depthwise convolution kernel as large as $\mathbf{X}$ in spatial. This kernel is adaptive to the contents of $\mathbf{X}$. Due to the property of Fourier transform [44], a circular padding is adopted to $\mathbf{X}$ here as shown in Fig.2. So far, we understand why the operation in Eq.(4) mathematically equals to a global-scope token mixing operation with semantic-adaptive weights.

3.3. Analysis

As introduced in Sec.2.3, there have been some studies applying frequency-domain methods to DNN for learning non-local or domain-generalizable representations in previous works [11, 54, 34, 19, 36]. They are all designed for different purposes with ours. In this section, we revisit the frequency-domain operations in these works from the perspective of token mixing and compare our design with them.

FFC [11] and AFNO [19] adopt linear (also known as 1×1 convolution) layers with non-linear activation functions to the representations in the frequency domain. Specifically, AFNO [19] adopt a linear layer followed by a ReLU function, another linear layer and a SoftShrink[1] function to the frequency representations after Fourier transforms, which can be briefly described as *FFT→Linear→ReLU→Linear→SoftShrink→iFFT*. Here, linear layer and Fourier transform are in fact commutative, *i.e.*, $\text{Linear}(\mathcal{F}(\mathbf{X})) = \mathcal{F}(\text{Linear}(\mathbf{X}))$, which can be proved with the distributive property of matrix multiplication by:

$$\begin{aligned} &\mathbf{W}_{Linear} \sum_{h=0}^{H-1} \sum_{w=0}^{W-1} \mathbf{X}(h,w) e^{-2\pi i(uh+vw)} \\ &= \sum_{h=0}^{H-1} \sum_{w=0}^{W-1} \left(\mathbf{W}_{Linear}\mathbf{X}(h,w)\right) e^{-2\pi i(uh+vw)}, \end{aligned} \tag{7}$$

where $\mathbf{W}_{Linear}$ denotes the parameters of a linear layer. We know that successive Fourier transform and its inverse transform equal to an identity function. Thus, the architecture of AFNO could be rewrote as: *FFT→Linear→ReLU→(iFFT→FFT)→Linear→SoftShrink→iFFT*. Upon the commutative law proved in Eq.(7), we can know this architecture is in fact equivalent to *Linear→**FFT→ReLU→iFFT** →Linear→**FFT→SoftShrink→iFFT***. Now, it is easy to find that only ReLU and SoftShrink functions remain in the Fourier domain. These two deterministic functions cannot achieve semantic-adaptive filtering as our proposed AFF token mixer does. The same problem also exists in FFC [11].

[1] https://pytorch.org/docs/stable/generated/torch.nn.Softshrink.html

Properties	FFC	AFNO	GFNet	FNO	DFF	Ours
Semantic-adaptive	✗	✗	✗	✗	✓	✓
Variable-size input	✓	✓	✗	✗	✓	✓
Channel-wise mix	✓	✓	✓	✓	✗	✓

Table 1. Comparisons of our proposed AFF token mixer with other frequency-domain neural operators in terms of three important properties for token mixing.

GFNet [54] and FNO [34] multiply the representations after Fourier transforms with trainable network parameters. GFNet [54] adopts elementwise multiplication while FNO [34] uses matrix multiplication. Both of them are not semantic-adaptive since the masks implemented by network parameters are shared over different instances and fixed after training. Besides, they cannot support for variable-size inputs since the shapes of these masks are fixed, leading to the lack of flexibility in their practical using.

DFF [36] learns a spatial mask to filter out frequency components that are not conductive to domain generalization. It is proposed for domain generalization problems in which only spatial mask is needed as studied in [36] since different spatial position of the features after a Fourier transform correspond to different frequency components. However, it is not competent as a token mixer since the learned mask is shared along the channel dimension. This means that the weights for its equivalent token mixing are shared for different channels. However, different channels commonly represent different semantic attributes [1, 73], thus requiring adaptive weights in token mixing.

We summarize the comparisons of different frequency-domain designs in terms of three important properties for token mixing in Table 1. The results of experimental verification are in Table 5 as follows.

3.4. Network Architectures

With our AFF token mixer as the core neural operator, we introduce its corresponding module and network design.

AFF Block For the output $\mathbf{X}^{l-1}$ of the $(l-1)$-th AFF Block, we adopt the commonly used module MBConv [46, 57, 58, 45, 64] with Layer Normalization (LN) for channel mixing, then feed it to our proposed AFF token mixer for global token mixing to get the output of l-th AFF block. Skip-connections for channel mixing and token mixing are adopted to facilitate model training. The entire architecture of AFF Block can be formulated as:

$$
\begin{aligned}
\hat{\mathbf{X}}^l &= \mathrm{MBConv}^l\left(\mathrm{LN}\left(\mathbf{X}^{l-l}\right)\right) + \mathbf{X}^{l-l} \\
\mathbf{X}^l &= \mathrm{AFF}^l\left(\mathrm{LN}\left(\hat{\mathbf{X}}^l\right)\right) + \hat{\mathbf{X}}^l
\end{aligned}
\tag{8}
$$

AFFNet We stack multiple AFF blocks for constructing a lightweight backbone network, namely AFFNet, as shown in Fig.2. Following the common practices [45, 46], we employ a convolution stem for tokenization and a plain fusion for combining local and global features at each stage. We build three versions of AFFNet with different numbers of channels, yielding different parameter scales. AFFNet and its tiny (AFFNet-T) and extremely tiny (AFFNet-ET) versions have 5.5M, 2.6M and 1.4M parameters, respectively. Their detailed configurations are in the Supplementary.

4. Experiments

We evaluate our proposed AFF token mixer by conducting comparisons with the state-of-the-art lightweight networks and extensive ablation studies for its design.

4.1. Image Classification

Settings. We train different versions of our proposed lightweight networks AFFNet as backbones on ImageNet-1k dataset [55] from scratch. All models are trained for 300 epochs on 8 NVIDIA V100 GPUs with a batch size of 1024. More implementation details are in the Supplementary.

Results. We report the comparison results between our proposed AFFNet and other SOTA lightweight models in Table 2. We observe that our AFFNet outperforms other lightweight networks with comparable model sizes in Top-1 accuracy. The AFFNet reaches 79.8% Top-1 accuracy with 5.5M parameters and 1.5G FLOPs. Our extremely tiny model AFFNet-ET attains 73% Top-1 accuracy with sorely 1.4M and 0.4G FLOPs. As a result, AFFNet achieves the best trade-offs between accuracy and efficiency. To show the comparison results more intuitively, we illustrate the accuracy and efficiency trade-offs of our AFFNet and some advanced lightweight models with global token mixers in Fig. 1. Thanks to AFF token mixer, AFFNet is superior to them by a clear margin across different model scales. Its superiority is especially significant when the model is extremely tiny, which demonstrates the effectiveness of AFF token mixer on information fusion at very low costs. AFFNet, AFFNet-T, and AFFNet-ET models achieve 4202, 5304, and 7470 images/s thoughtput on ImageNet-1K tested with one NVIDIA A100 GPU, respectively, which is 13.5%, 8.2%, and 14.9% faster than MobileViT-S/XS/XXS. More detailed results are in the Supplementary.

Model	Pub.	Res.	Param. (M)	FLOPs (G)	Top-1
MNetv1-0.5 [27]	arXiv17	224^2	1.3	0.2	63.7
MViT-XXS [45]	ICLR22	256^2	1.3	0.4	69.0
EdgeNext-XXS [43]	ECCV22	256^2	1.3	0.3	71.2
MViTv2-0.5 [46]	TMLR23	256^2	1.4	0.5	70.2
AFFNet-ET	-	256^2	1.4	0.4	73.0
MNetv3-L-0.5 [26]	ICCV19	224^2	2.6	0.1	68.8
MFormer-52 [7]	CVPR22	224^2	3.6	0.1	68.7
PVTv2-B0 [68]	CVM22	224^2	3.7	0.6	70.5
MViT-XS [45]	ICLR22	256^2	2.3	1.0	74.8
EdgeNext-XS [43]	ECCV22	256^2	2.3	0.5	75.0
EFormer-S0 [32]	arXiv22	224^2	3.5	0.4	75.7
MViTv2-0.75 [46]	TMLR23	256^2	2.9	1.0	75.6
AFFNet-T	-	256^2	2.6	0.8	77.0
MNetv2 [56]	CVPR18	224^2	6.9	0.6	74.7
ShuffleNetV2 [41]	ECCV18	224^2	5.5	0.6	74.5
MNetv3 [26]	ICCV19	224^2	5.4	0.2	75.2
T2T-ViT [81]	ICCV21	224^2	6.9	1.8	76.5
DeiT-T [63]	ICML21	224^2	5.7	1.3	72.2
CoaT-Lite-T [14]	ICCV21	224^2	5.7	1.6	77.5
LeViT-128 [18]	ICCV21	224^2	9.2	0.4	78.6
GFNet-Ti [54]	NeurIPS21	224^2	7.0	1.3	74.6
EFormer-L1 [33]	NeurIPS22	224^2	12.3	1.3	79.2
EFormer-S1 [32]	arXiv22	224^2	6.1	0.7	79.0
Mformer [7]	CVPR22	224^2	9.4	0.2	76.7
EfficientViT [3]	arXiv22	224^2	7.9	0.4	78.6
EdgeViT-XS [10]	ECCV22	256^2	6.7	1.1	77.5
MOne-S3 [65]	arXiv22	224^2	10.1	1.9	78.1
MViT-S [45]	ICLR22	256^2	5.6	2.0	78.4
EdgeNext-S [43]	ECCV22	256^2	5.6	1.3	79.4
MViTv2-1.0 [46]	TMLR23	256^2	4.9	1.8	78.1
AFFNet	-	256^2	5.5	1.5	79.8

Table 2. Comparisons of our proposed AFFNet with other state-of-the-art lightweight networks on ImageNet-1K classification over different model scales (*i.e.*, <2M, 2M $\sim$ 4M and $>$ 4M). For conciseness, Pub., Res., Param., MNet, MOne, MFormer, EFormer and MViT are short for Publication, Resolution, Parameters, MobileNet, MobileOne, MobileFormer, EfficientFormer and MobileViT, respectively.

4.2. Object Detection

Settings. We conduct object detection experiments on MS-COCO dataset [37], Following the common practices in [27, 56, 45, 46, 43], we compare different lightweight backbones upon the Single Shot Detection (SSD) [39] framework wherein separable convolutions are adopted to replace the standard convolutions in the detection head for evaluation in the lightweight setting. In the training, we load ImageNet-1K pre-trained weights as the initialization of the backbone network, and fine-tune the entire model on the training set of MS-COCO with the AdamW optimizer for 200 epochs. The input resolution of the images is 320×320. Detailed introduction for the used dataset and more implementation details are in the Supplementary.

Results. As shown in Table 3, the detection models equipped with AFFNet consistently outperforms other lightweight CNNs or transformers based detectors in mAP

Model	Detection		Segmentation		
	Param.	mAP(%) COCO	Param.	mIOU(%) ADE20K	VOC
MViT-XXS [45]	1.9	18.5	1.9	-	73.6
MViTv2-0.5 [46]	2.0	21.2	3.6	31.2	75.1
AFFNet-ET	1.9	21.8	2.2	33.0	76.1
MViT-XS [45]	2.7	24.8	2.9	-	77.1
MViTv2-0.75 [46]	3.6	24.6	6.2	34.7	75.1
AFFNet-T	3.0	25.3	3.5	36.9	77.8
ResNet-50 [24]	22.9	25.2	68.2	36.2	76.8
MNetv1 [27]	5.1	22.2	11.2	-	75.3
MNetv2 [56]	4.3	22.1	18.7	34.1	75.7
MViT-S [45]	5.7	27.7	6.4	-	79.1
MViTv2-1.0 [46]	5.6	26.5	9.4	37.0	78.9
EdgeNext [43]	6.2	27.9	6.5	-	80.2
AFFNet	5.6	28.4	6.9	38.4	80.5

Table 3. Comparisons of our AFFNet with other state-of-the-art models for object detection on COCO dataset, and segmentation on ADE20k and VOC dataset. Here, Param., MNet and MViT are short for Paramters, MobileNet and MobileViT, respectively.

across different model scales. Specifically, AFFNet surpasses the second-best EdgeNext [43] by 0.5% in mAP with 0.6M fewer parameters, and surpasses the model with ResNet-50 backbone by 3.2% in mAP using about 1/4 of parameters. Our smallest model AFFNet-ET outperforms the second-best model with comparable parameters MobileViTv2-0.5 [46] by 0.6% in mAP with fewer parameters. These results demonstrate the effectiveness of our proposed method on capturing spatial location information required by the task of object detection at low costs.

4.3. Semantic Segmentation

Settings. We conduct semantic segmentation experiments on two benchmarks datasets ADE20k [86] and PASCAL VOC 2012 [17] (abbreviated as VOC). For the experiments on VOC dataset, we follow the common practices in [4, 47] to extend the training data with more annotations and data from [21] and [37], respectively. The widely used semantic segmentation framework DeepLabv3 [4] is employed for experiments with different backbones. The input resolution of the images is set to 512×512 and the ImageNet-1K pretrained weights are loaded as model initialization. All models were trained for 120 and 50 epochs on ADE20K and VOC dataset, respectively. Please see our Supplementary for more detailed introduction.

Results. As the results shown in Table 3, AFFNet performs clearly better than other lightweight networks on these two datasets. Our AFFNet outperforms the second-best lightweight network MobileViTv2-1.0 [46] by 1.4% in mIOU on ADE20K, and outperforms the second-best lightweight model EdgeNext [43] by 0.3% in mIOU on VOC. Besides, it achieves large improvements (2.2% mIOU on ADE20K, 3.7% mIOU on VOC) relative to the representative CNN model (*i.e.*, ResNet-50) with about 10% of the parameters of ResNet-50. These exhibit the effectiveness of our proposed method on dense prediction tasks.

Method	Param (M)	FLOPs (G)	Top-1
Base.	5.2	1.3	77.9
Base. + Conv-mixer (3×3)	10.7	2.7	78.6
Base. + AFF w/o FFT	5.5	1.5	78.4
Base. + AFF (Our AFFNet)	5.5	1.5	79.8

Table 4. Comparisons of our proposed model with baseline (no spatial token mixer) and models with other token mixers in the original domain on ImageNet-1K classification. "Base." denotes the baseline model discarding all AFF token mixers. "Conv-Mixer (3×3)" refers to adopting token mixers implemented by 3×3 convolutions in the original space. "AFF w/o FFT" denotes performing adaptive filtering in the original space with the same networks by discarding the Fourier transforms where "w/o" and "AFF" are short for "without" and "AFF token mixer", respectively.

4.4. Ablation Study

Effectiveness and complexity of AFF token mixer. We analyze the effectiveness and complexity of our proposed AFF token mixer by comparing AFFNet with the *Base.* model in which all AFF token mixers are replaced with identity functions. As shown in Table 4, all AFF token mixers in AFFNet only requires 0.3M parameter increase ($< 6\%$) and 0.2G FLOPs increase ($\sim 15\%$) relative to the baseline and improves the Top-1 accuracy on ImageNet-1K by 1.9%. Comparing to the model with one 3×3 convolution layer as the token mixer, *i.e.*, *Base.+Conv-Mixer (3×3)*, AFFNet delivers 1.2% Top-1 accuracy improvements with about only half of parameters and FLOPs. This strongly demonstrates the effectiveness and efficiency of our proposed method for token mixing in lightweight networks.

Original vs. frequency domain. We compare applying the same adaptive filtering operations in original domain and in frequency domain. We discard the all Fourier and inverse Fourier transforms and remain others the same as AFFNet, *i.e.*, *Base.+AFF w/o FFT* in Table 4. Our AFFNet clearly outperforms it by 1.4% Top-1 accuracy with the same model complexity. Applying adaptive filtering in the original domain is even weaker than convolutional token mixer, which indicates that only adaptive ***frequency*** filters can serve as effeicient global token mixers.

Comparisons of different frequency operations. We compare the frequency operation design in AFF token mixer with those in previous works [34, 54, 11, 36, 19] in terms of their effects as token mixers. The results are in Table 5. As analyzed in Sec.3.3, FFC [11] and AFNO [19] actually perform filtering with deterministic functions, resulting in the lack of the adaptivity to semantics. The frequency-

Method	Param (M)	FLOPs (G)	Top-1
Base.	5.2	1.3	77.9
Base. + AFNO [19]	5.5	1.5	78.8
Base. + GFN [54]	6.5	1.5	79.1
Base. + FFC [11]	7.7	1.7	79.1
Base. + DFF [36]	7.7	1.7	79.3
Base. + FNO [34]	141.0	1.5	79.7
Base. + AFF w. SUM	5.5	1.5	78.8
Base. + AFF (AFFNet)	5.5	1.5	79.8

Table 5. Comparisons of our design for AFF token mixer and other frequency-domain operations in previous works [34, 54, 11, 36, 19] in terms of their roles for token mixing on ImageNet-1K. "AFF w. SUM" denotes replacing the Hadamard product with a summation operation, "w." is short for "with".

domain operations in them are both obviously inferior to ours. Moreover, our operation design is also clearly better than those in GFN [54] and FNO [34] since they perform filtering with network parameters implemented masks. These masks are fixed after training and lead to a large increase in parameters (*Base.+FNO* has more than 25$\times$ parameters as ours). Note that the implementation of FNO [34] with unshared fully connected layers for each frequency component results in a significant increase in the number of parameters. DFF [36] is designed for filtering out the frequency components adverse to domain generalization, thus requiring a spatial mask only. Our AFFNet is superior to *Base.+DFF* by 0.5% with fewer parameters and FLOPs, demonstrating the importance of channel-wise mixing. This will be further verified with a fairer comparison. These existing frequency-domain operations might be similar with our proposed one at the first glance, but they are designed for different purposes and perform worse than ours as token mixers. When replacing the Hadamard product in our method with a summation operation, the Top-1 accuracy drops by 1.0% since the equivalence introduced in Sec.3.2 no longer holds.

The importance of channel-specific token mixing. We demonstrate this by comparing the frequency-domain operations in DFF [36] with ours. Considering their masks are learned with different networks, here, we further conduct a fairer comparison by adopting an average pooling along the channels of the learned masks in AFF token mixer. As shown in Table 6, frequency filtering with the masks of a shape of 1$\times$H$\times$W lead to 0.5% accuracy drop, verifying the importance of channel-specific token mixing. This is because different semantic attributes of the learned latent representations distribute in different channels [1, 73], thus requiring channel-specific weights for token mixing. Besides, it delivers the same accuracy with *Base.+DFF* in Table 5. This indicates that the network architectures for inferring masks are not dominating factors for the effects of token mixing, allowing us to use a lightweight one.

Mask Shape	Param (M)	FLOPs (G)	Top-1
1$\times$H$\times$W	5.5	1.5	79.3
C$\times$H$\times$W	5.5	1.5	79.8

Table 6. Experiments of verifying the importance of channel-specific token mixing on ImageNet-1K. Here, we adopt an average pooling operation along the channel dimension of the masks learned in AFFNet, yielding the mask with a shape of 1$\times$H$\times$W. This mask is shared across channels.

Spatial K-Size	N_{group}	Param. (M)	FLOPs (G)	Top-1
1×1	C	5.3	1.4	79.4
1×1	1	7.7	2.0	79.9
3×3	8	7.9	2.0	79.8
1×1	8	5.5	1.5	79.8

Table 7. Comparisons of different hyper-parameter choices in the sub-network for learning the filtering masks in AFFNet on ImageNet-1K. "Spatial K-Size" refers to the spatial size of convolution kernels. N_{group} denotes the number of groups for group linear or convolution layers. C is the total number of channels.

Comparisons of hyper-parameter choices. As shown in Fig.2, we adopt two group linear layers (also known as 1$\times$1 convolution layers) with ReLU to learn the masks for our proposed adaptive frequency filtering. As shown in Table 7, improving the kernel size cannot further improve the performance but leads to larger model complexities. Moreover, we keep the spatial kernel size as 1$\times$1 while using different group numbers. When N_{group}=C, the Top-1 accuracy drops by 0.4%, in which depthwise convolutions are used so that the contexts among different channels are under-exploited for inferring the weights of token mixing. When N_{group}=1, it means that regular convolution/linear layers are used, which slightly improve the Top-1 accuracy by 0.1% at the expense of 40% parameters increase and 33.3% FLOPs increase. This setting explores more contexts but results in a worse accuracy and efficiency trade-off.

5. Conclusion

In this work, we reveal that *adaptive frequency filters can serve as efficient global token mixers* in a mathematically equivalent manner. Upon this, we propose Adaptive Frequency Filtering (AFF) token mixer to achieve low-cost adaptive token mixing in the global scope. Moreover, we take AFF token mixers as primary neural operators to build a lightweight backbone network, dubbed AFFNet. AFFNet achieves SOTA accuracy and efficiency trade-offs compared to other lightweight network designs across multiple vision tasks. Besides, we revisit the existing frequency-domain neural operations for figuring out what matters in their designs for token mixing. We hope this work could inspire more interplay between conventional signal processing and deep learning technologies.

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