Supplementary Materials of SC^2-PCR

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1. Derivations

1.1. Derivation of ambiguity probability

Notations. Suppose there are N pairs of correspondences, in which the inlier rate is α. The second order spatial compatibility SC^2 measure between correspondence i and j is defined as follows:

$$SC^2_{ij} = C_{ij} \cdot \sum_{k=1}^{N} C_{ik} \cdot C_{kj}. \tag{1}$$

C ($C \in \mathbb{R}^{N\times N}$) is the hard compatibility matrix as follows:

$$C_{ij} = \begin{cases} 1; d_{ij} \leq d_{thr} \\ 0; d_{ij} > d_{thr} \end{cases}. \tag{2}$$

d_{ij} is the distance difference between two correspondences. When $i = j$, C_{ij} is set to 0. The distance difference between two inliers $d_{in,in}$ follows the uniform distribution, and the probability density function of $d_{in,in}$ is as follows:

$$\mathrm{PDF}_{in,in}(l) = 1/d_{thr}, 0 \leq l \leq d_{thr} \tag{3}$$

For the distance difference between an inlier and an outlier $d_{in,out}$, or two outliers $d_{out,out}$, their probability density functions are both $F(\cdot)$:

$$\mathrm{PDF}_{in,out}(l) = F(l), \mathrm{PDF}_{out,out}(l) = F(l); 0 \leq l \leq d_r. \tag{4}$$

$F(l)$ is a constant within $(0, d_{thr})$ as follows:

$$F(l) = f_0, 0 \leq l \leq d_{thr}. \tag{5}$$

Remark 1.1: The ambiguity probability of SC^2 measure, i.e., $\mathrm{P}(SC^2_{in,out} > SC^2_{in,in})$, can be written as follows:

$$\begin{gathered} \mathrm{P}(SC^2_{in,out} > SC^2_{in,in}) = p \cdot \mathrm{P}(X > (N \cdot \alpha - 2)), \\ X \sim S((N\alpha - 1)p + (N(1-\alpha) - 1)p^2, N(1-\alpha)p^2), \\ p = d_{thr} \cdot f_0. \end{gathered} \tag{6}$$

Derivation of Remark 1.1. We first reformulate Eq. (1) as follows:

$$SC^2_{ij} = C_{ij} \cdot M_{ij}, \tag{7}$$

where M_{ij} is computed as follows:

$$M_{ij} = \sum_{k=1}^{N} C_{ik} \cdot C_{kj}. \tag{8}$$

M_{ij} counts the quantity of the commonly compatible correspondences of i and j in the global set. According to Eq. (2) and (3), we can obtain that:

$$\mathrm{P}(C_{in,in} = 1) = 1. \tag{9}$$

According to Eq. (2), (4) and (5), we can get that

$$\mathrm{P}(C_{in,out} = 1) = \int_0^{d_{thr}} F(l)dl = d_{thr} \cdot f_0 = p \tag{10}$$

$$\mathrm{P}(C_{out,out} = 1) = \int_0^{d_{thr}} F(l)dl = d_{thr} \cdot f_0 = p. \tag{11}$$

According to Eq. (7), to make $SC^2_{in,out} > SC^2_{in,in}$ hold, two conditions need to be met: $C_{in,out} = 1$ and $M_{in,out} > M_{in,in}$. According to Eq. (10), we can obtain the following equation:

$$\begin{aligned} &\mathrm{P}(SC^2_{in,out} > SC^2_{in,in}) \\ =&\mathrm{P}(C_{in,out} = 1) \cdot \mathrm{P}(M_{in,out} > M_{in,in}) \\ =&p \cdot \mathrm{P}(M_{in,out} > M_{in,in}). \end{aligned} \tag{12}$$

Next, we compute the distribution of $M_{in,out}$ and $M_{in,in}$. Since inliers have different distribution with outliers, we consider them separately and reformulate Eq. 8 as follows:

$$M_{ij} = \sum_{m\in\mathcal{I}} C_{im} \cdot C_{mj} + \sum_{n\in\mathcal{O}} C_{in} \cdot C_{nj}. \tag{13}$$

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where $\mathcal{I}$ is the inlier set while $\mathcal{O}$ is the outlier set. (For conveniecne we use this notation in the following part).

We first discuss the value in M matrix between two inliers, i.e. $M_{in,in}$. According to Eq. (9), we can find that any two inliers are compatible. Thus, when correspondence i and j are inliers, the number of correspondences compatible with both of them in the inlier set is the number of inliers excluding themselves ($C_{ii}=0, C_{jj}=0$), i.e.:

$$\sum_{m\in\mathcal{I}} C_{im}\cdot C_{mj} = N\cdot\alpha - 2; i\in\mathcal{I}, j\in\mathcal{I}, \tag{14}$$

where α is the inlier rate. For outliers, according to Eq. (10), the probability that an outlier is compatible with an inlier is p. Then the probability that an outlier is compatible with both i and j is p^2. The number of outliers in the whole correspondence set is $N(1-\alpha)$. So the number of correspondences compatible with both of them in the outlier set is in a Bernoulli distribution [2] as follows:

$$\sum_{n\in\mathcal{O}} C_{in}\cdot C_{nj} \sim B(N(1-\alpha), p^2); i\in\mathcal{I}, j\in\mathcal{I}, \tag{15}$$

where $B(\cdot,\cdot)$ is the Bernoulli distribution. Thus, $M_{in,in}$ is in the following distribution:

$$M_{in,in} \sim N\cdot\alpha - 2 + B(N(1-\alpha), p^2). \tag{16}$$

After that, we discuss the distribution of the value in M matrix between an inlier and an outlier, i.e. $M_{in,out}$. For convenience, we assume correspondence i is inlier while j is outlier. For the inlier set except correspondence i ($C_{ii}=0$), any of them is compatible with i (Eq. 9), and the probability that one of them is compatible with j is p (Eq. 10). So the number of correspondences compatible with both of correspondence i and j in the inlier set is in following distribution:

$$\sum_{m\in\mathcal{I}} C_{im}\cdot C_{mj} \sim B(N\alpha - 1, p); i\in\mathcal{I}, j\in\mathcal{O}. \tag{17}$$

Meanwhile, for each outlier except correspondence j ($C_{jj}=0$), the probabilities that it is compatible with i or j are both p (Eq. 10 and 11). So the probability that an outlier is both compatible with i and j is p^2. Thus, we can get the following distribution:

$$\sum_{n\in\mathcal{O}} C_{in}\cdot C_{nj} \sim B(N(1-\alpha)-1, p^2); i\in\mathcal{I}, j\in\mathcal{O}. \tag{18}$$

So the distribution of $M_{in,out}$ is as follows:

$$M_{in,out} \sim B(N\alpha-1, p) + B(N(1-\alpha)-1, p^2); \tag{19}$$

Since p is a small value, the Binomial distribution in Eq. (16) and (19) can be approximately equivalent to the Poisson distribution [2], i.e.:

$$\begin{aligned} M_{in,in} &\sim N\cdot\alpha - 2 + \pi(N(1-\alpha)p^2), \\ M_{in,out} &\sim \pi((N\alpha-1)p) + \pi((N(1-\alpha)-1)p^2), \end{aligned} \tag{20}$$

where $\pi(\cdot)$ is the Poisson distribution. Furthermore, for two Poisson distribution: $X_1\sim\pi(\lambda_1)$ and $X_2\sim\pi(\lambda_2)$, their sum is also in the Poisson distribution [2] as follows:

$$X1 + X_2 \sim \pi(\lambda_1+\lambda_2). \tag{21}$$

So we can convert $M_{in,out}$ in Eq. (20) into following form:

$$M_{in,out} \sim \pi((N\alpha-1)p + (N(1-\alpha)-1)p^2). \tag{22}$$

Meanwhile, we can convert $\mathrm{P}(M_{in,out} > M_{in,in})$ into following form:

$$\begin{aligned} &\mathrm{P}(M_{in,out} > M_{in,in}) \\ =&\mathrm{P}(M_{in,out} - M_{in,in} > 0) \\ =&\mathrm{P}(X > N\cdot\alpha - 2), \end{aligned} \tag{23}$$

where X is in following distribution:

$$X \sim \pi((N\alpha-1)p + (N(1-\alpha)-1)p^2) - \pi(N(1-\alpha)p^2) \tag{24}$$

For two Poisson distribution: $X_1\sim\pi(\lambda_1)$ and $X_2\sim\pi(\lambda_2)$, their difference is in the Skellam distribution [5–7], i.e.:

$$X1 - X_2 \sim S(\lambda_1, \lambda_2). \tag{25}$$

So the distribution of X in Eq. (24) can be converted as follows:

$$S((N\alpha-1)p + (N(1-\alpha)-1)p^2, N(1-\alpha)p^2). \tag{26}$$

Combining Eq. (12), (23) and (26), we compute the value of $\mathrm{P}(SC^2_{in,out} > SC^2_{in,in})$ as Remark 1.1.

2. Additional Experiments

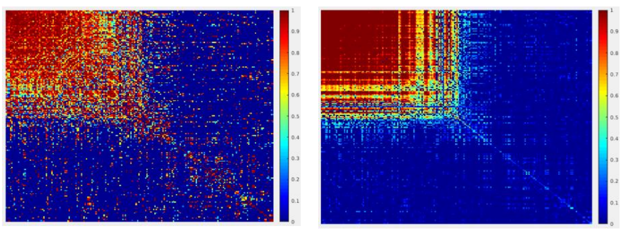
Parameter K_1 and K_2. In the proposed method, when some seeds are selected, we use a two-stage selection strategy to extend each seed into a consensus set. In the first stage, K_1 correspondences are selected by finding top-K_1 neighbors of seed. In the second stage, a local SC^2 matrix is rebuilt to further filter potential outliers and reserve K_2 correspondences. In order to show the effect of these two parameters, we report the registration results with respect to these parameters in Tab. 1.

As shown in Tab. 1, our method is parameter insensitive. The proposed method with different parameters can all lead to acceptable results combined with both FPFH and FCGF descriptors. In the final version, we choose K_1=30 and K_2=20 for its best registration recall.

Heat Map. Compared with the spatial compatibility (SC) measure [1, 8], the second order spatial compatibility (SC^2) measure can reduce the probability of ambiguity event. In order to show the difference between these two measures, we report the heat maps of them on a real data in

Param		FPFH			FCGF			
K_1	K_2	RR(%)	RE($\circ$)	TE(cm)	RR(%)	RE($\circ$)	TE(cm)	Time(s)
10	3	82.13	2.15	6.67	92.91	2.05	6.52	0.11
10	5	82.69	2.10	6.59	93.04	2.05	6.52	0.11
20	10	83.79	2.13	6.56	93.10	2.06	6.54	0.11
30	20	83.98	2.18	6.56	93.28	2.08	6.55	0.11
40	30	83.67	2.15	6.71	93.22	2.05	6.52	0.12
50	40	83.67	2.16	6.78	93.16	2.06	6.54	0.12
60	50	83.92	2.18	6.68	93.10	2.05	6.49	0.12
70	60	83.55	2.16	6.69	92.98	2.04	6.49	0.13

Table 1. The registration results with varying parameters on 3DMatch dataset.



(a) SC measure (b) SC^2 measure

Figure 1. The heat maps of spatial compatibility (SC) and second order spatial compatibility (SC^2) measures.

Fig. 1. Specifically, we sort all the correspondences, placing inliers first and outliers behind to build the SC and SC^2 matrices respectively. Then we normalize these two matrices to [0,1], and plot the corresponding heat maps. As shown in Fig. 1, there are high values on the right and bottom in the heat map of SC measure, showing that many outliers are compatible with inliers. By contrast, the right and bottom sides are clean in the heat map of SC^2 measure, showing that outliers have low compatibility with inliers.

Scene-wise Results. Following [1, 3], we also report the scene-wise registration results of the proposed method on 3DMatch dataset, combing both FPFH and FCGF descriptors as shown in Tab. 2.

	FPFH			FCGF		
	RR(%)	RE($\circ$)	TE(cm)	RR(%)	RE($\circ$)	TE(cm)
Kitchen	88.34	1.95	5.41	99.21	1.69	5.15
Home1	89.74	1.82	6.23	96.79	1.79	6.37
Home2	73.56	2.80	7.46	83.17	3.48	7.50
Hotel1	92.04	2.20	7.01	98.67	1.89	6.08
Hotel2	81.73	2.08	6.49	91.35	1.94	5.61
Hotel3	90.74	2.01	5.79	92.59	2.08	5.80
Study	76.03	2.31	8.94	88.36	2.31	9.21
Lab	76.62	1.67	5.99	80.52	1.91	8.44

Table 2. Scene-wise registration results on 3DMatch Dataset.

More Qualitative Results. We show the more registration results in Fig. 2 and 3.

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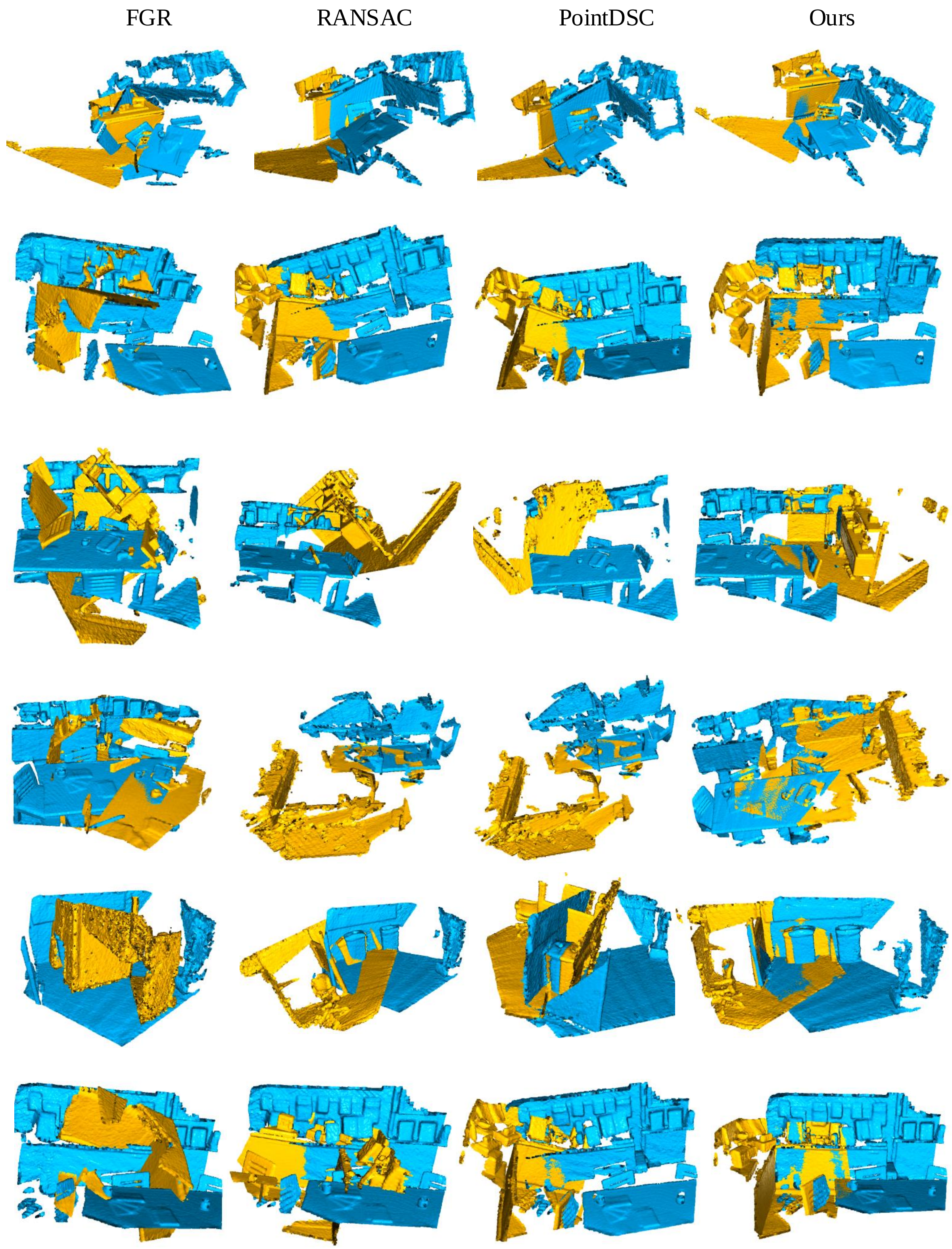

Figure 2. Qualitative comparison on 3DMatch and 3DLoMatch dataset. From left to right are: FGR [9], RANSAC [4], PointDSC [1] and Ours

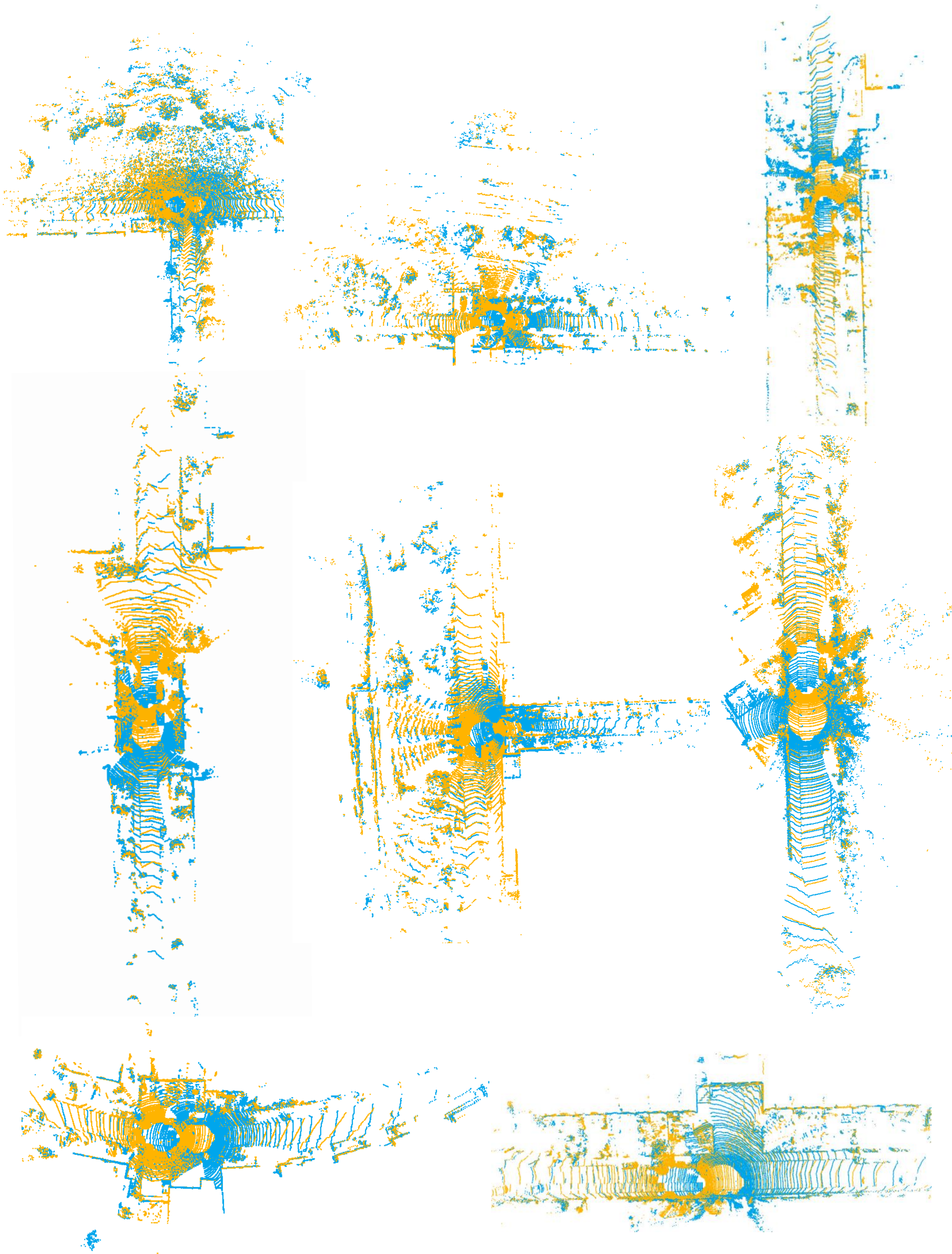

Figure 3. Qualitative results of our method on KITTI dataset.