

Rotation-invariant Hierarchical Segmentation on Poincaré Ball for 3D Point Cloud Supplemental Material

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1. Remapping of labels to compute Intersection over Union (IoU)

In this section, we describe more in details the algorithm we used to match prediction labels, from clustering, to ground truth labels and compute in this way the IoU score. Let $\mathcal{P} = \{p_1, \dots, p_N\}$ be a point cloud of N points, whose ground truth segmentation is represented by the set $\mathcal{P}_L = \{\ell_1, \dots, \ell_N\}$. Without loss of generality, we assume that $\ell_n \in L = \{1, \dots, M\}$. Now, let $\mathcal{P}_C = \{c_1, \dots, c_N\}$ be a clustering of \mathcal{P} in K clusters. As before, we can assume that $c_n \in C = \{1, \dots, K\}$. To compute, the $\text{IoU}(\mathcal{P}_L, \mathcal{P}_C)$ between the ground truth and the prediction, we need to find a matching function between the two sets C and L . Namely, since the number K of clusters could be higher than the number M of segments, we want to find $f : C \rightarrow \bar{L}$, where $\bar{L} = L \cup \{\infty\}$ is the set of M labels plus an extra label ∞ that we use as *trash label*. The idea is that the matching function f maps to the subset L only the top M most similar clusters, and sends to ∞ all the rest. In this way, we penalize clustering whose $K \gg M$. To score, the similarity between the class m and the cluster k , we first need to compute the matrix $S \in \mathbb{R}^{M, K}$, defined as

$$s_{m,k} = \text{IoU}(\mathbb{1}_{\{\mathcal{P}_L=m\}}, \mathbb{1}_{\{\mathcal{P}_C=k\}}) \quad \forall 1 \leq m \leq M, \forall 1 \leq k \leq K,$$

where $\mathbb{1}_A$ is the characteristic function of a set. Then, the mapping function is defined as follows:

$$f(k) = \begin{cases} m, & \text{if } k = \text{argmax}_{c \in C} s_{m,c} \\ \infty, & \text{otherwise} \end{cases}$$

where $\mathcal{C} = \text{argmax}_{j \in \{1, \dots, K\}} (s_{m,j})$. Basically, the function f maps the cluster k to label m , if k is the cluster that achieves the maximum IoU score with class m . Using this function, we can obtain a remapped clustering $\mathcal{P}_{f(C)} = \{f(c_1), \dots, f(c_N)\}$ such that $f(c_n) \in \bar{L}$, and the final IoU score is defined as

$$\text{IoU}(\mathcal{P}_L, \mathcal{P}_C) = \text{IoU}(\mathcal{P}_L, \mathcal{P}_{f(C)})$$

We report in Algorithm 1 the pseudocode, implementing the proposed remapping.

Algorithm 1 Clusters remapping

```
1: function IOUWITHREMAP( $\mathcal{P}_L, \mathcal{P}_C$ )
2:    $S \leftarrow \mathbf{0} \in \mathbb{R}^{M, K}$ 
3:   for  $k = 1, 2, \dots, K$  do ▷ Filling the matrix  $S$ 
4:     for  $m = 1, 2, \dots, M$  do
5:        $s_{m, k} \leftarrow \text{IoU}(\mathbb{1}_{\{\mathcal{P}_L=m\}}, \mathbb{1}_{\{\mathcal{P}_C=k\}})$ 
6:     end for
7:   end for
8:    $c' \leftarrow (\infty, \dots, \infty) \in \mathbb{R}^K$  ▷  $c'[j] \in \bar{L}$  is the array of remapped labels of clusters
9:   for  $m = 1, 2, \dots, M$  do ▷ Compute remapping
10:     $k \leftarrow \arg \max_{c \in \{1, \dots, K\}} (s_{m, c})$ 
11:    if  $m \in \arg \max_{i \in \{1, \dots, M\}} s_{i, k}$  then
12:       $c'[k] \leftarrow m$ 
13:    end if
14:  end for
15:   $\mathcal{P}_{C'} \leftarrow (0, \dots, 0) \in \mathbb{R}^n$  ▷ Vector containing remapped clustering
16:  for  $n = 1, 2, \dots, N$  do
17:     $\mathcal{P}_{C'}[n] \leftarrow c'[\mathcal{P}_C[n]]$  ▷ Assign remapped labels
18:  end for
19:  return  $\text{IoU}(\mathcal{P}_L, \mathcal{P}_{C'})$ 
20: end function
```

2. ShapeNet Airplane

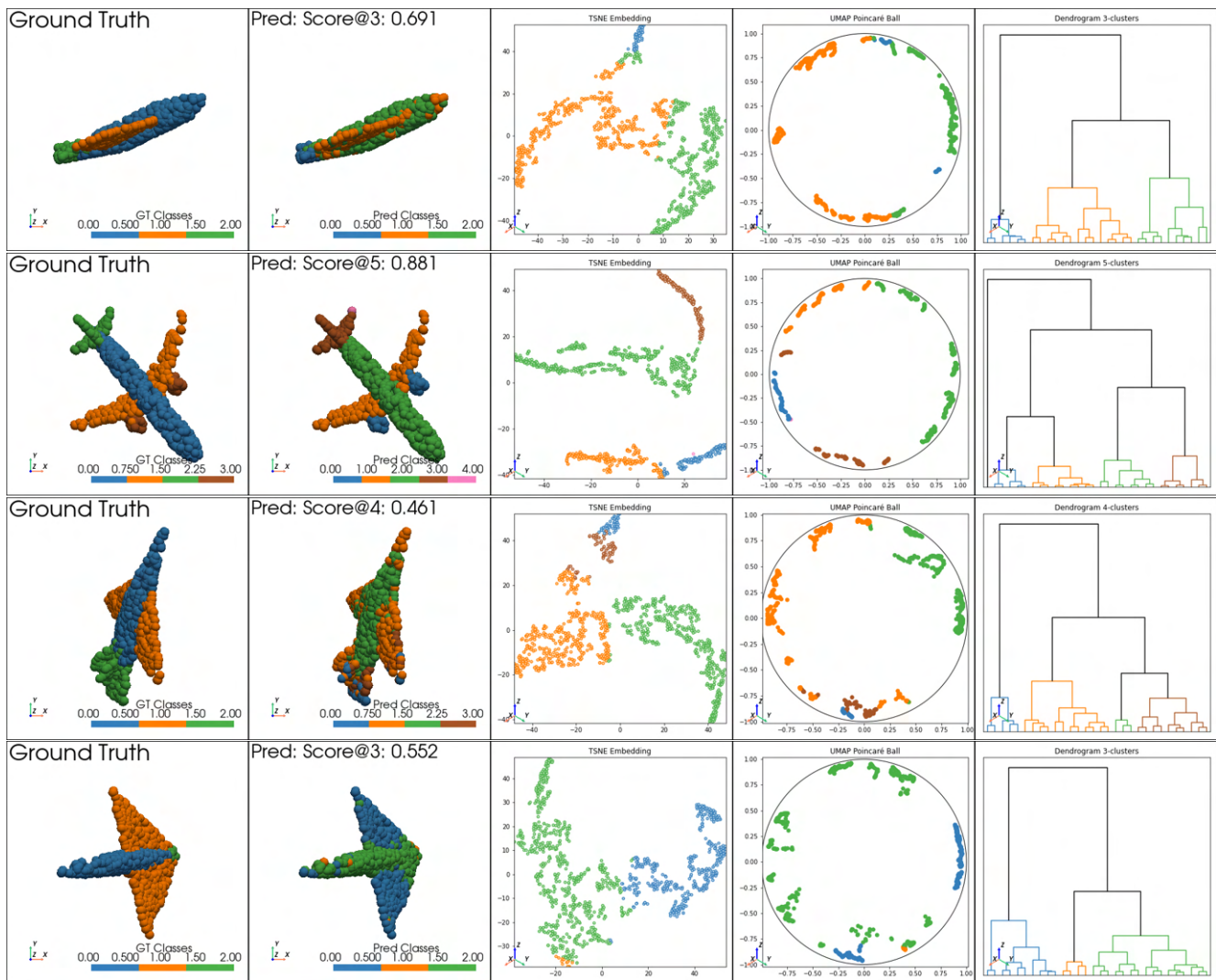


Figure 1. Hierarchical segmentation of airplane examples in ShapeNet dataset.

Bag

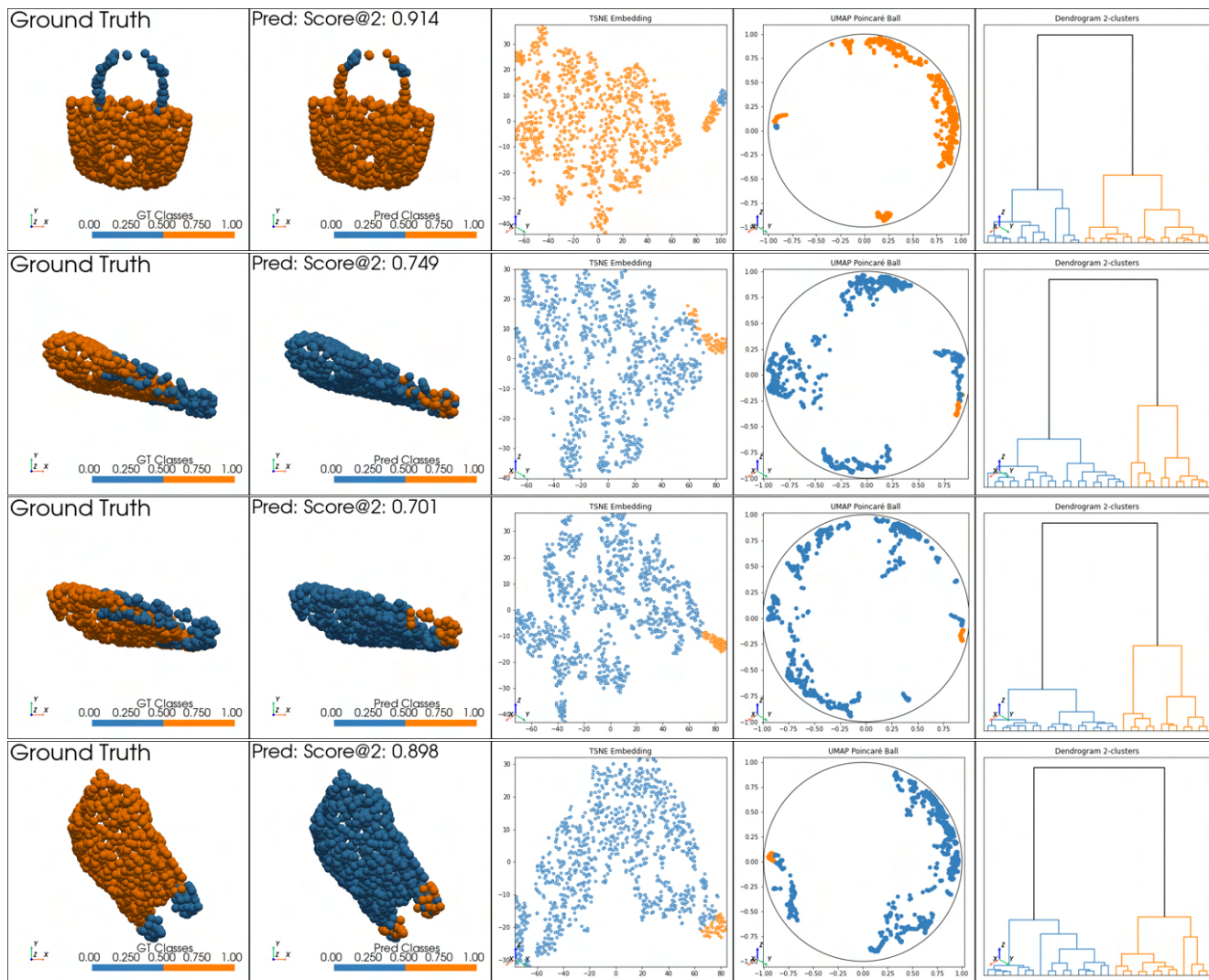


Figure 2. Hierarchical segmentation of bag examples in ShapeNet dataset.

Cap

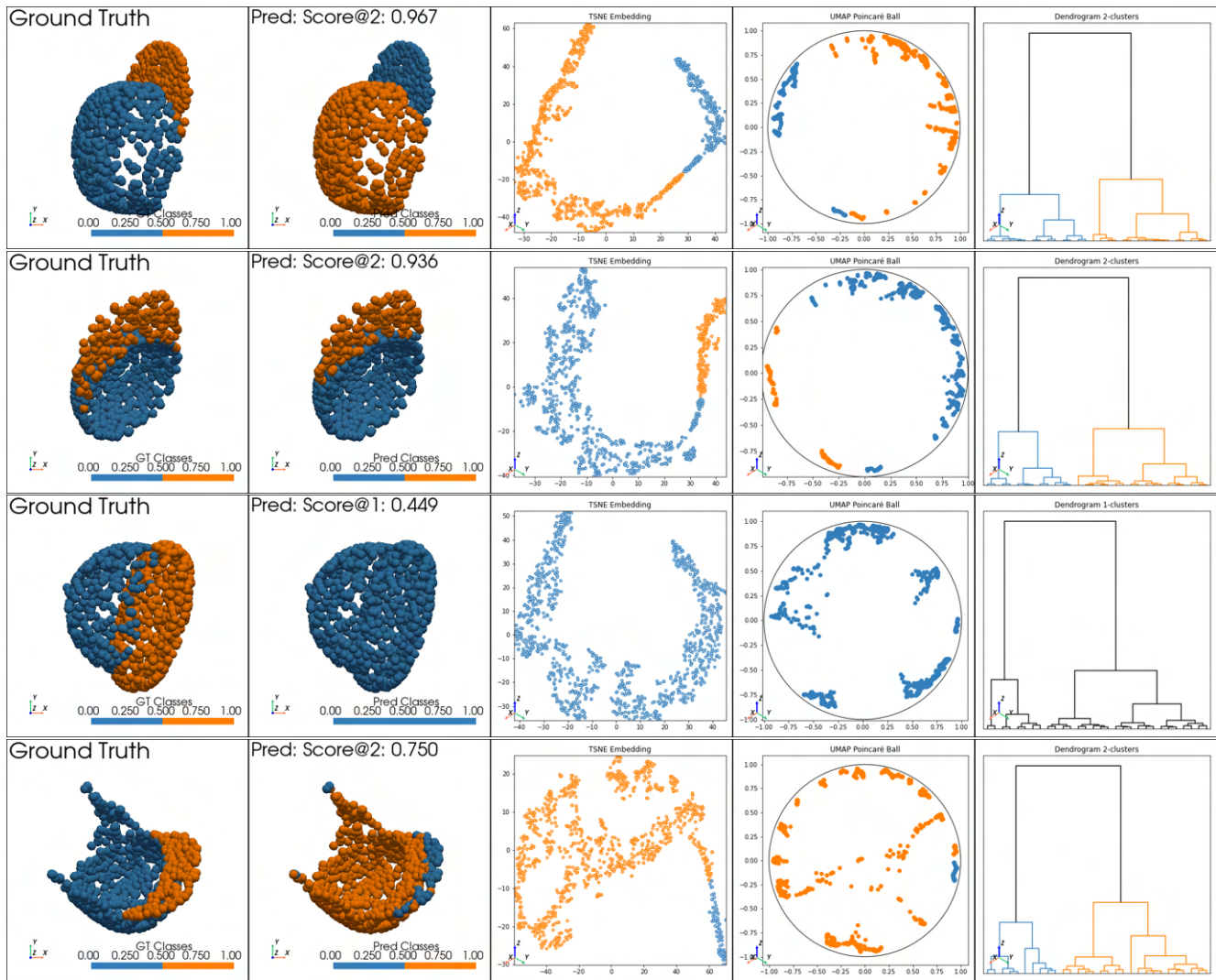


Figure 3. Hierarchical segmentation of cap examples in ShapeNet dataset.

Car

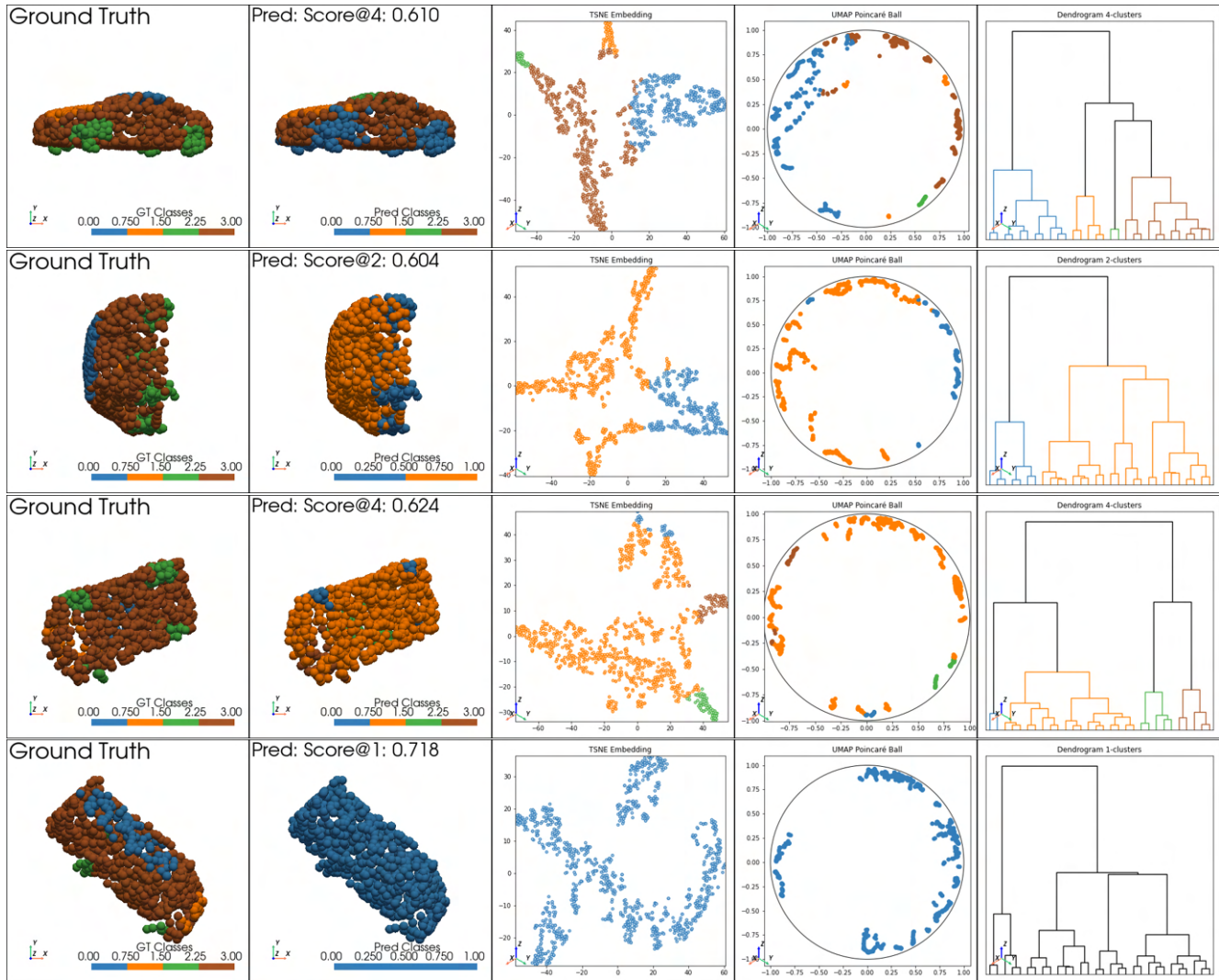


Figure 4. Hierarchical segmentation of cap examples in ShapeNet dataset.

Chair

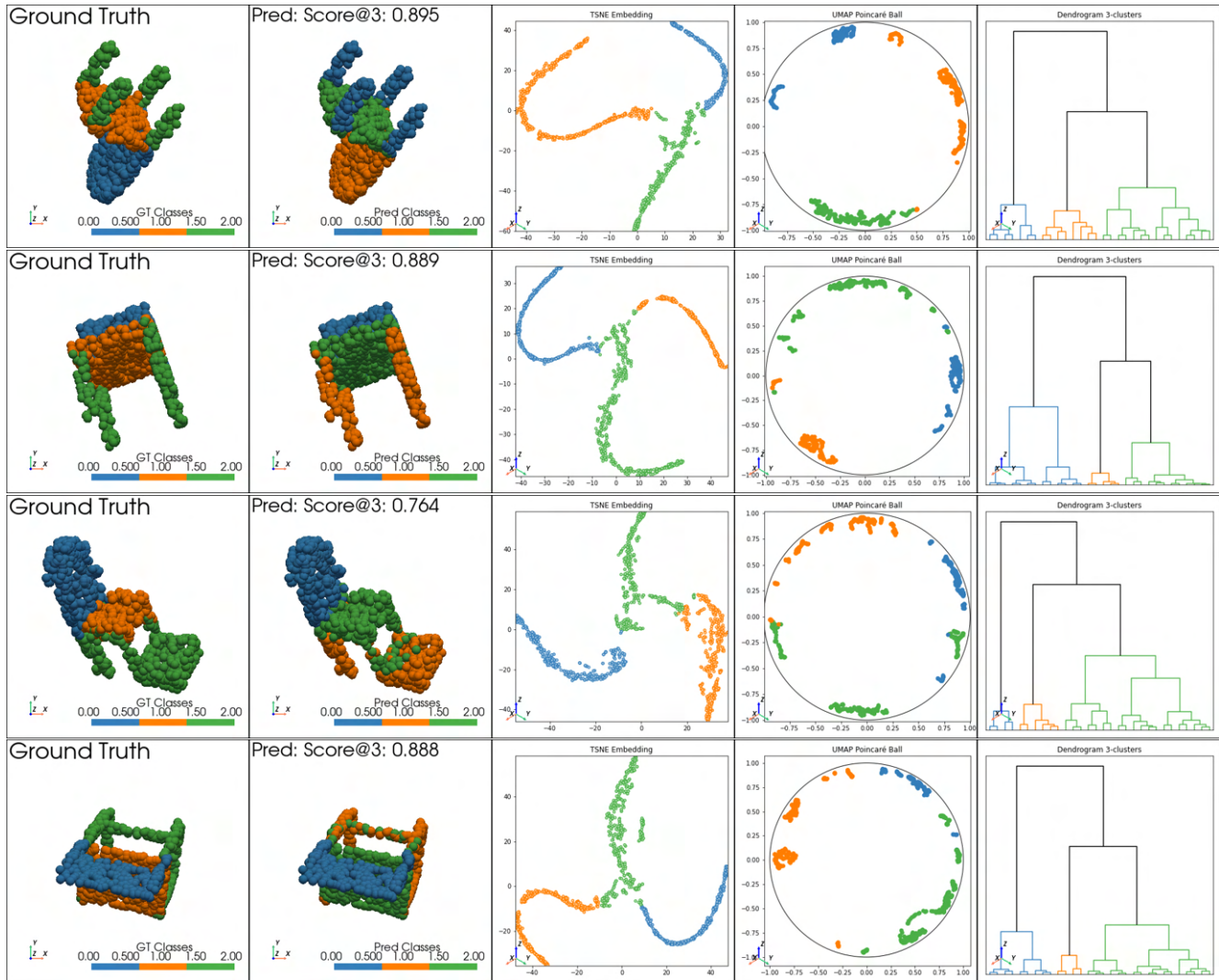


Figure 5. Hierarchical segmentation of bag examples in ShapeNet dataset.

Earphone

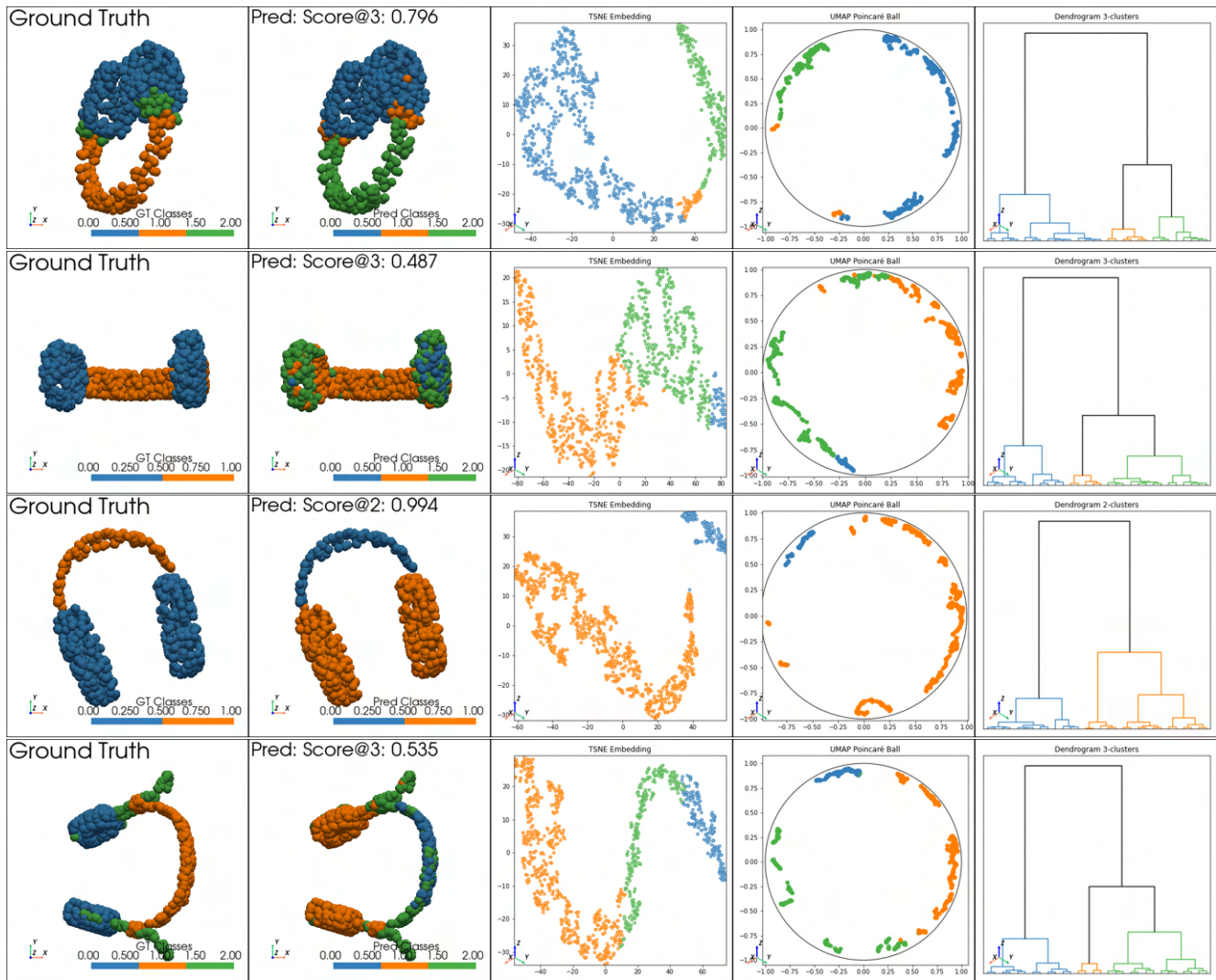


Figure 6. Hierarchical segmentation of earphone examples in ShapeNet dataset.

Guitar

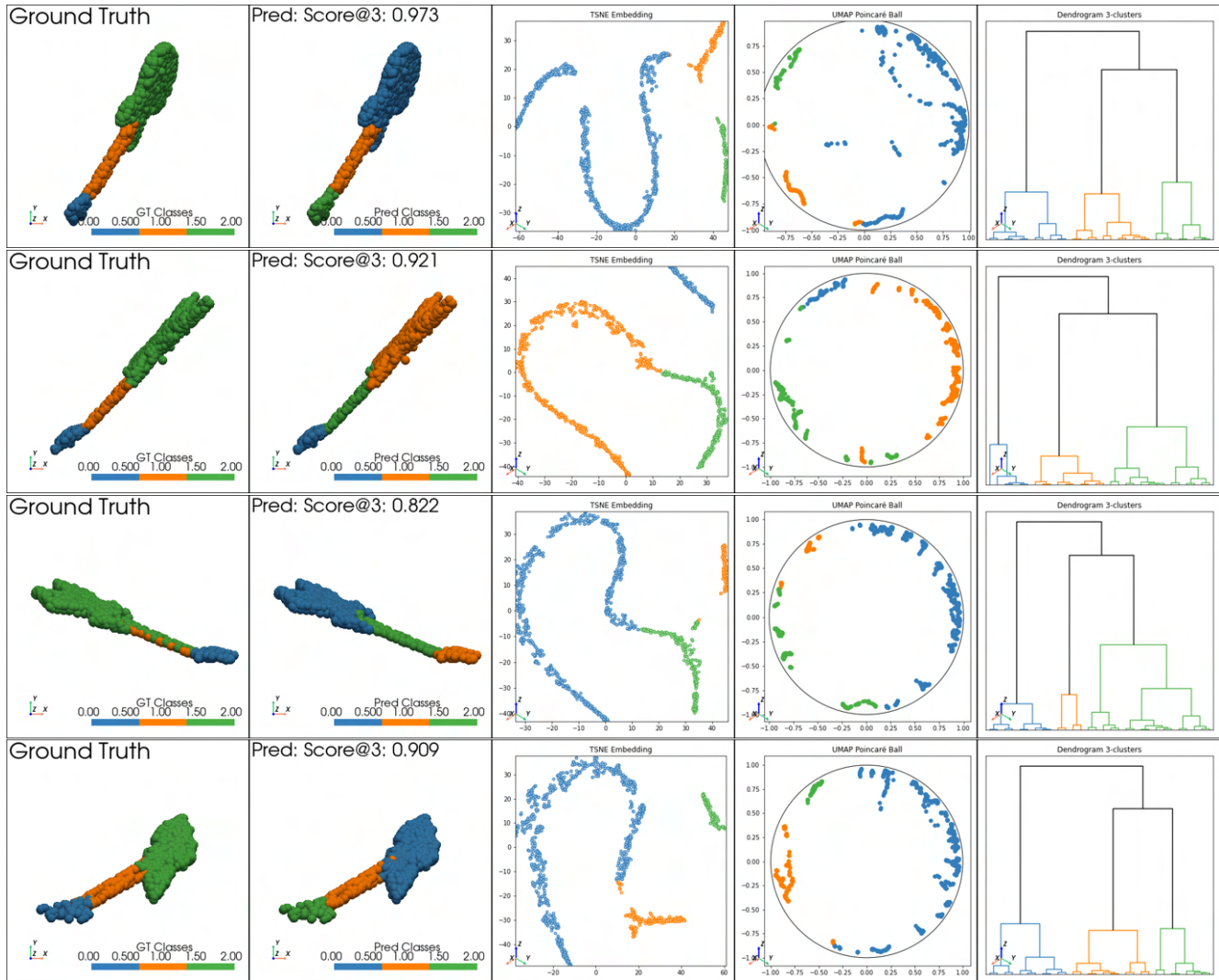


Figure 7. Hierarchical segmentation of guitar examples in ShapeNet dataset.

Knife

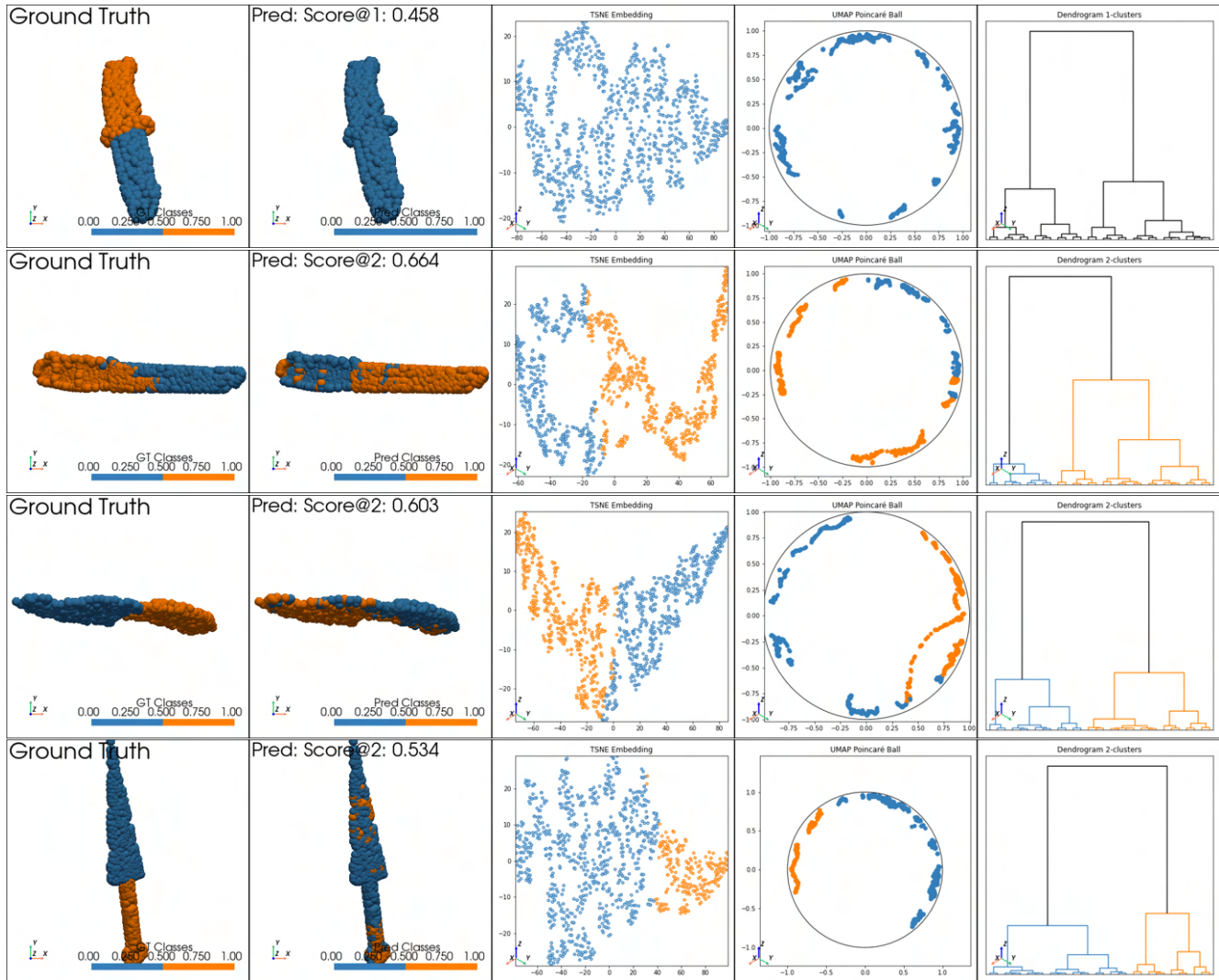


Figure 8. Hierarchical segmentation of knife examples in ShapeNet dataset.

Lamp

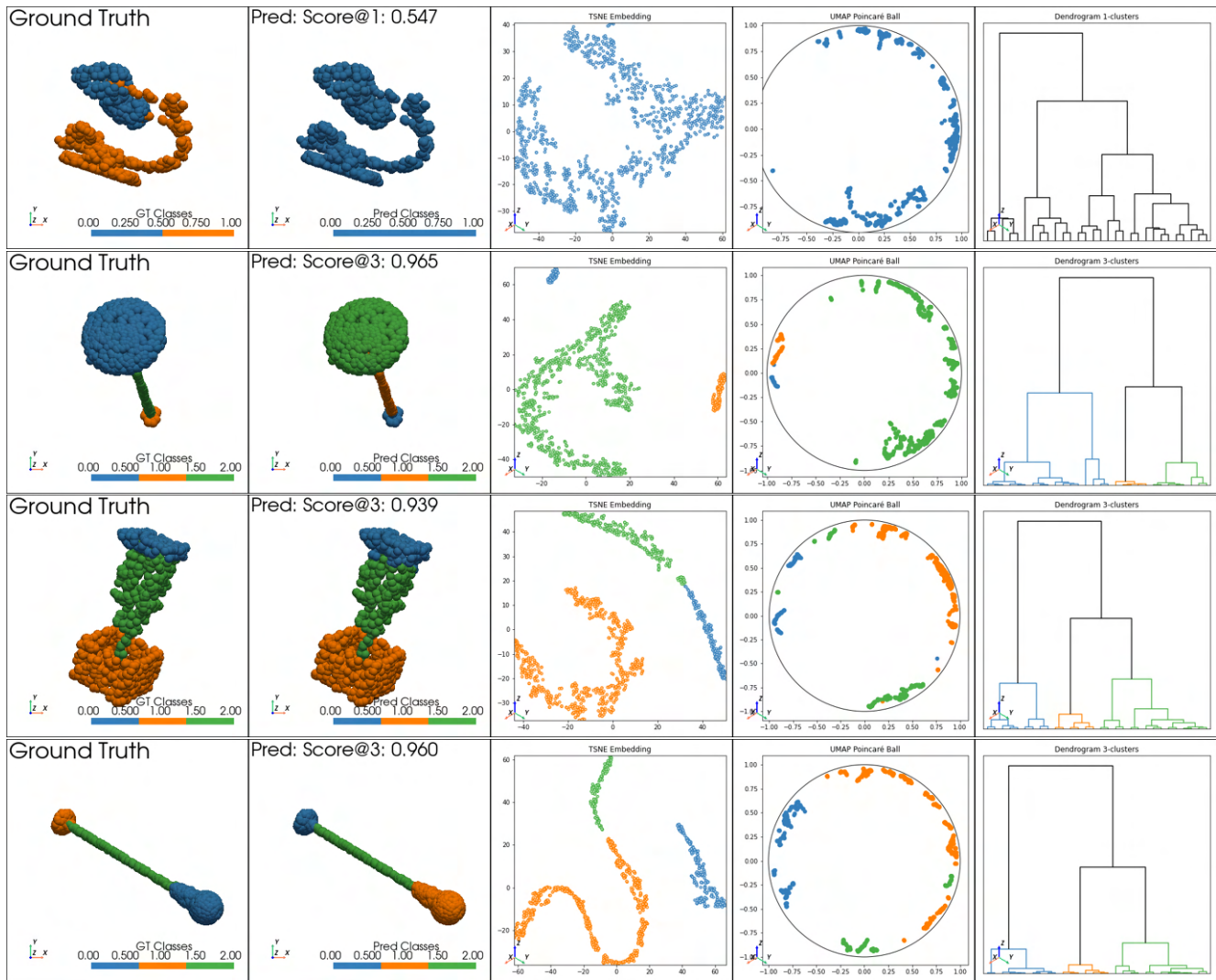


Figure 9. Hierarchical segmentation of lamp examples in ShapeNet dataset.

Laptop

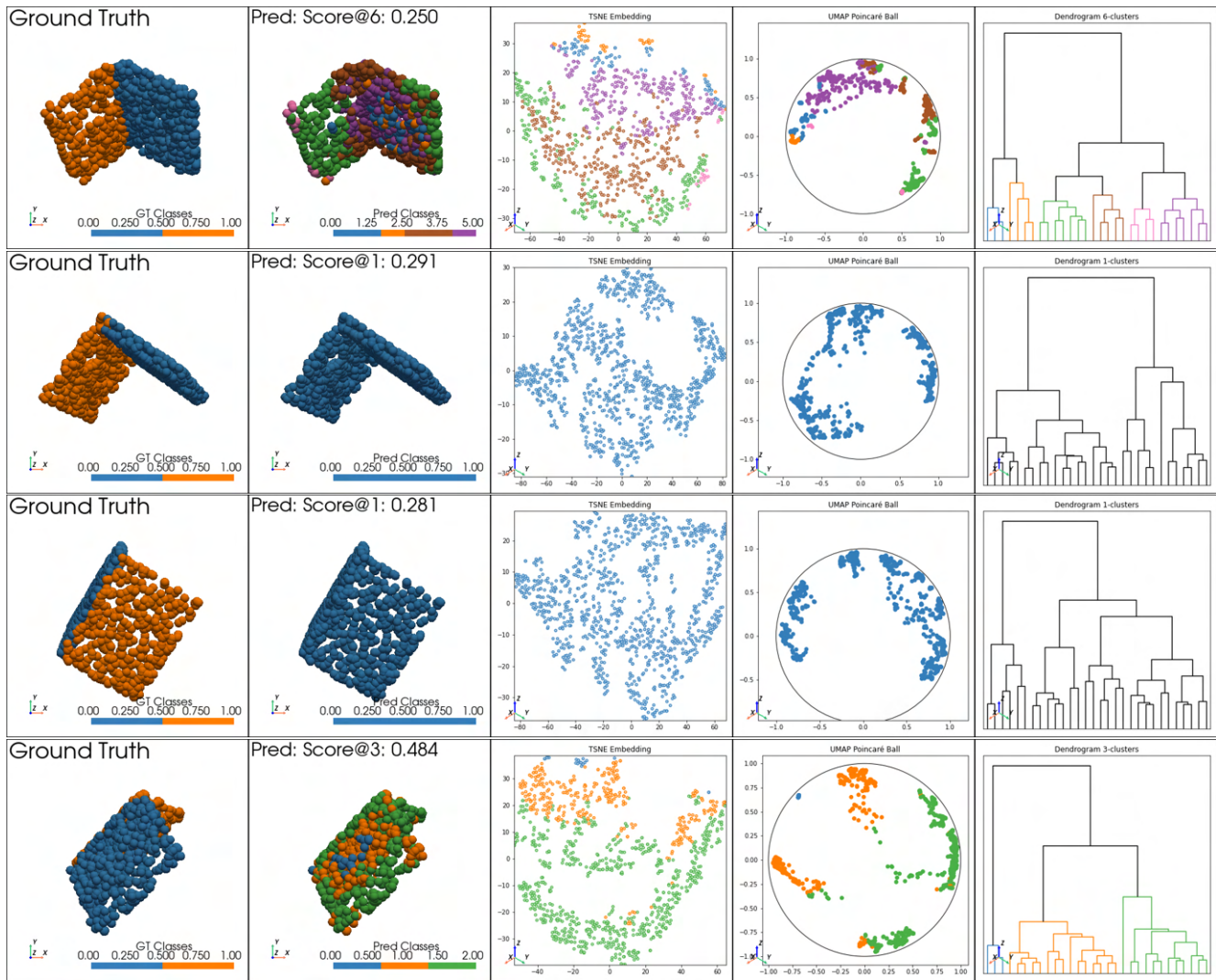


Figure 10. Hierarchical segmentation of laptop examples in ShapeNet dataset.

Motorbike

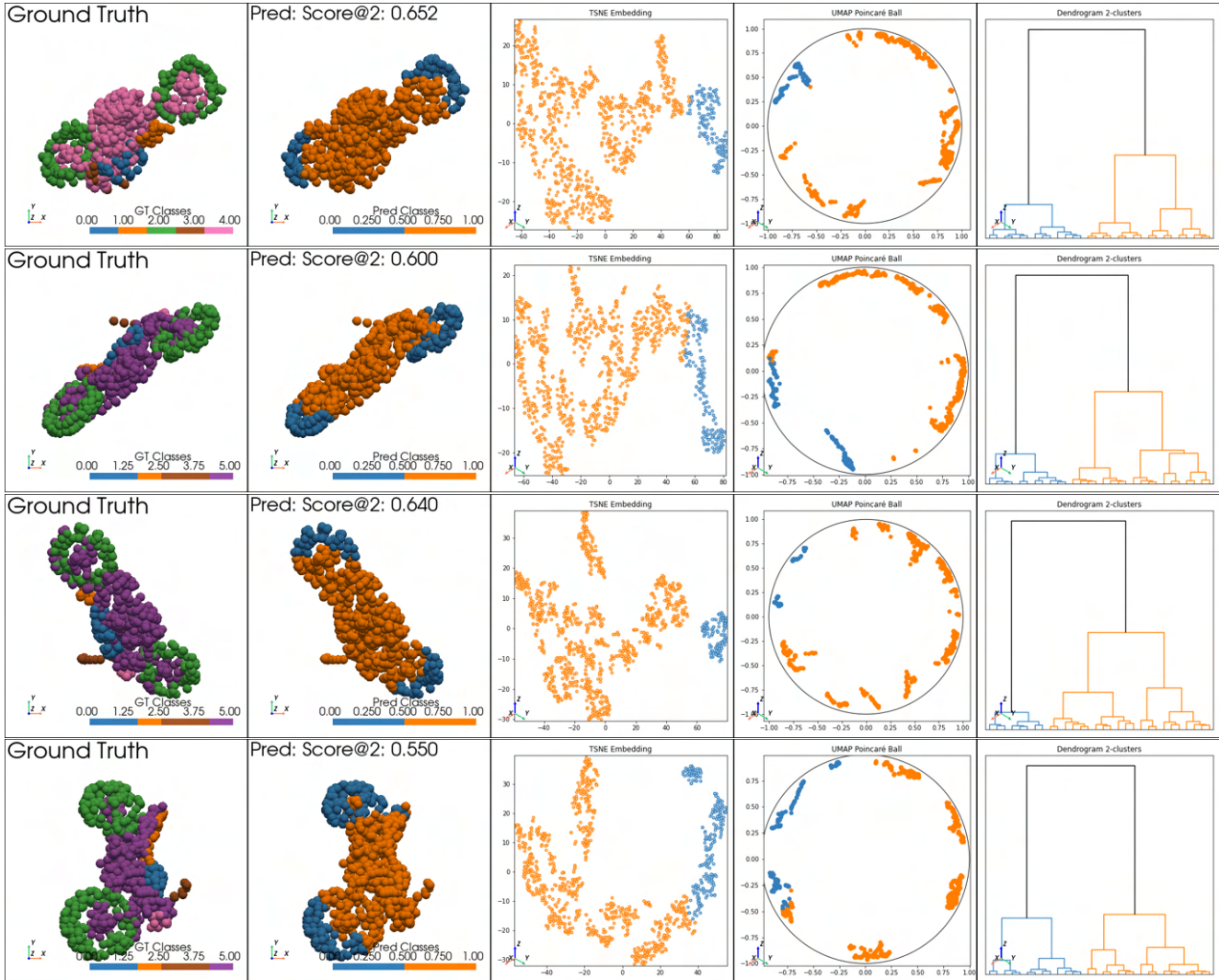


Figure 11. Hierarchical segmentation of motorbike examples in ShapeNet dataset.

Mug

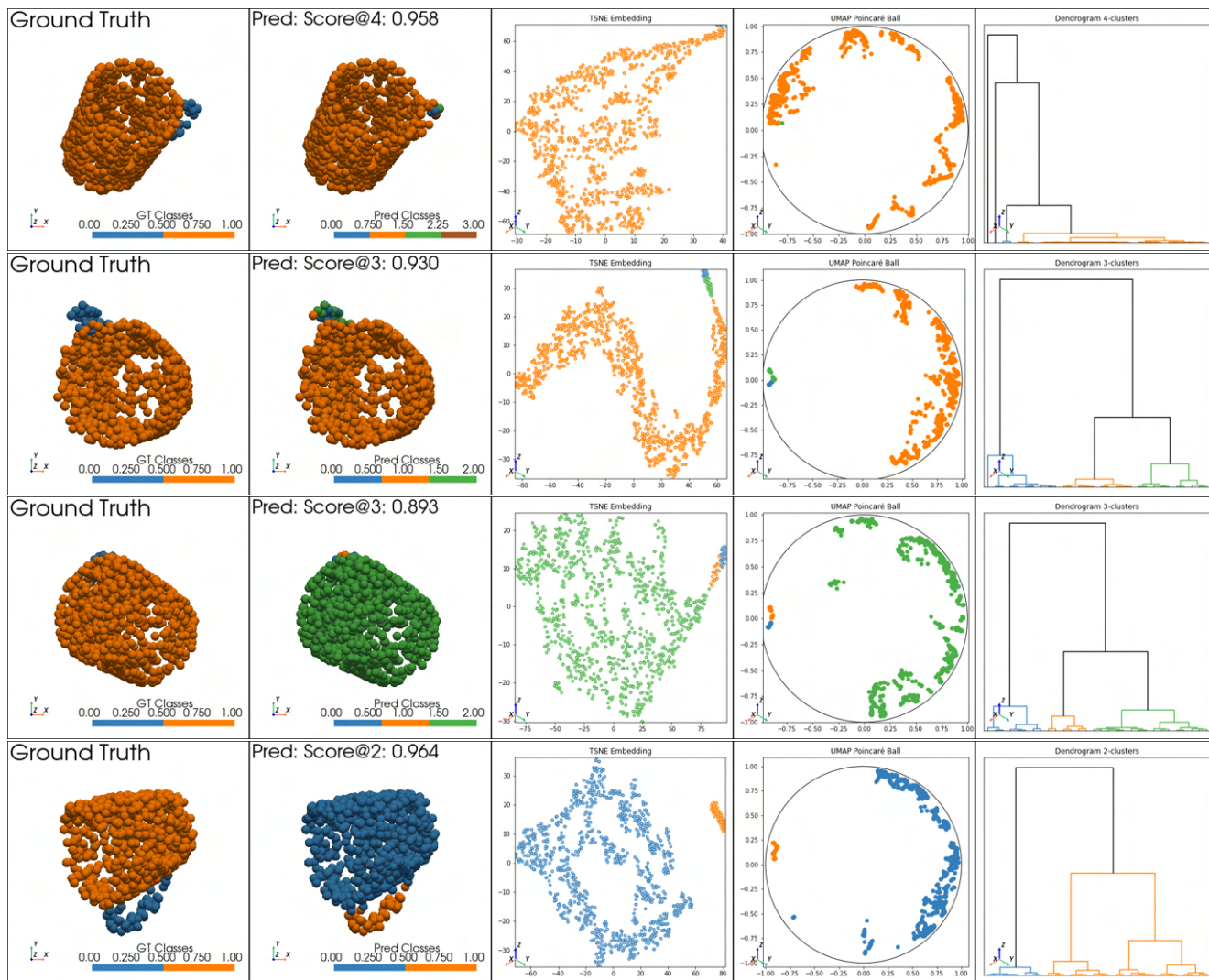


Figure 12. Hierarchical segmentation of mug examples in ShapeNet dataset.

Pistol

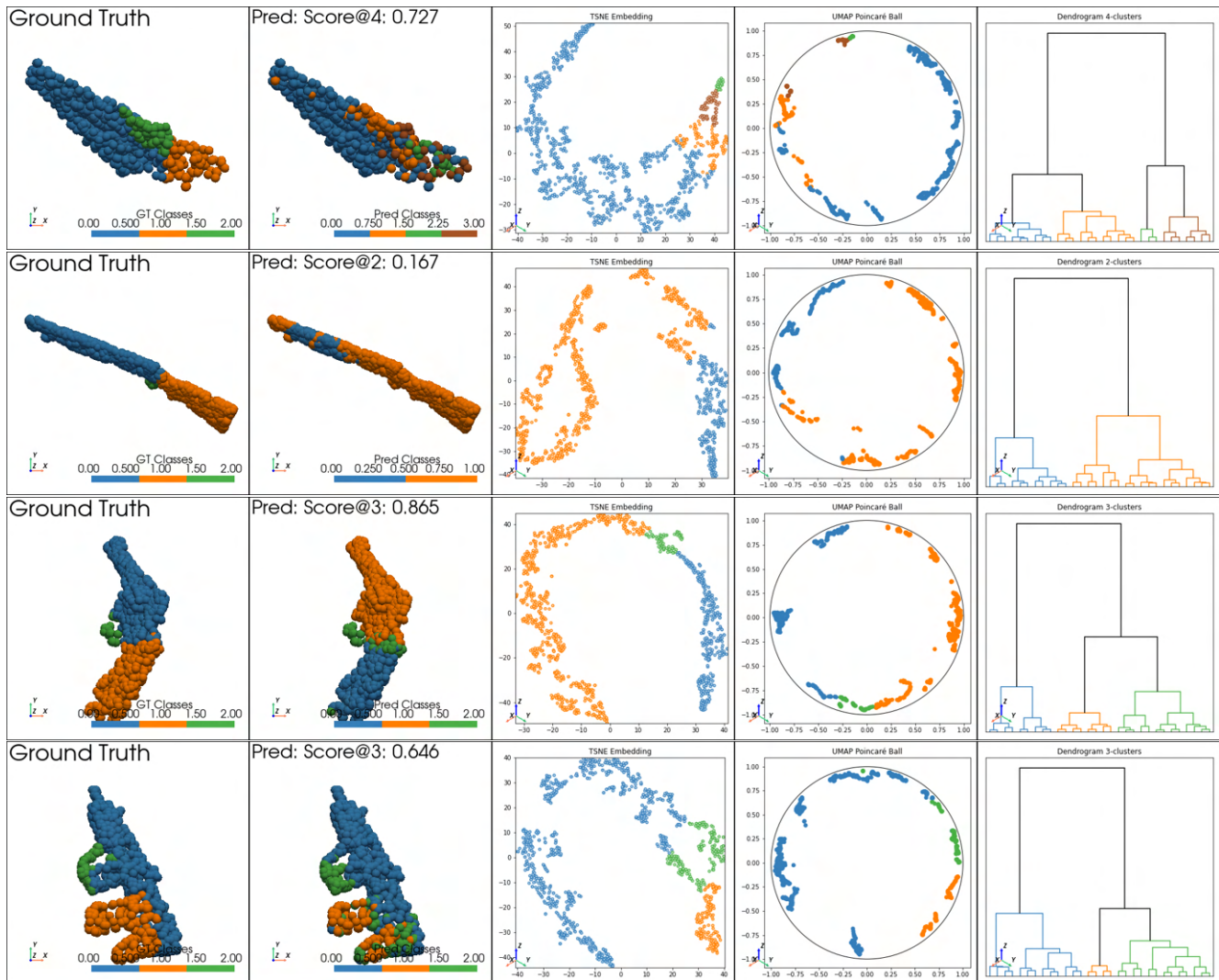


Figure 13. Hierarchical segmentation of pistol examples in ShapeNet dataset.

Rocket

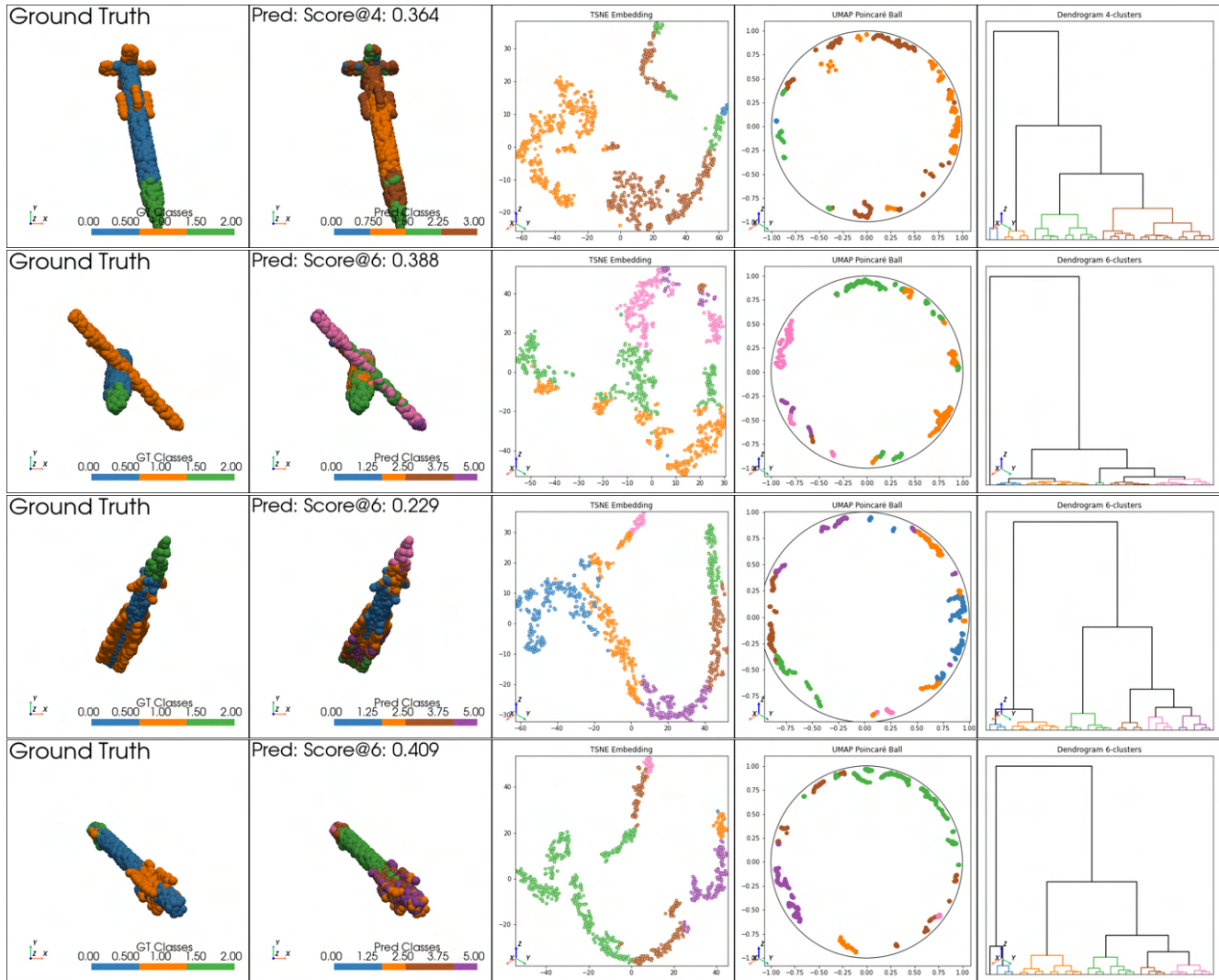


Figure 14. Hierarchical segmentation of rocket examples in ShapeNet dataset.

Skateboard

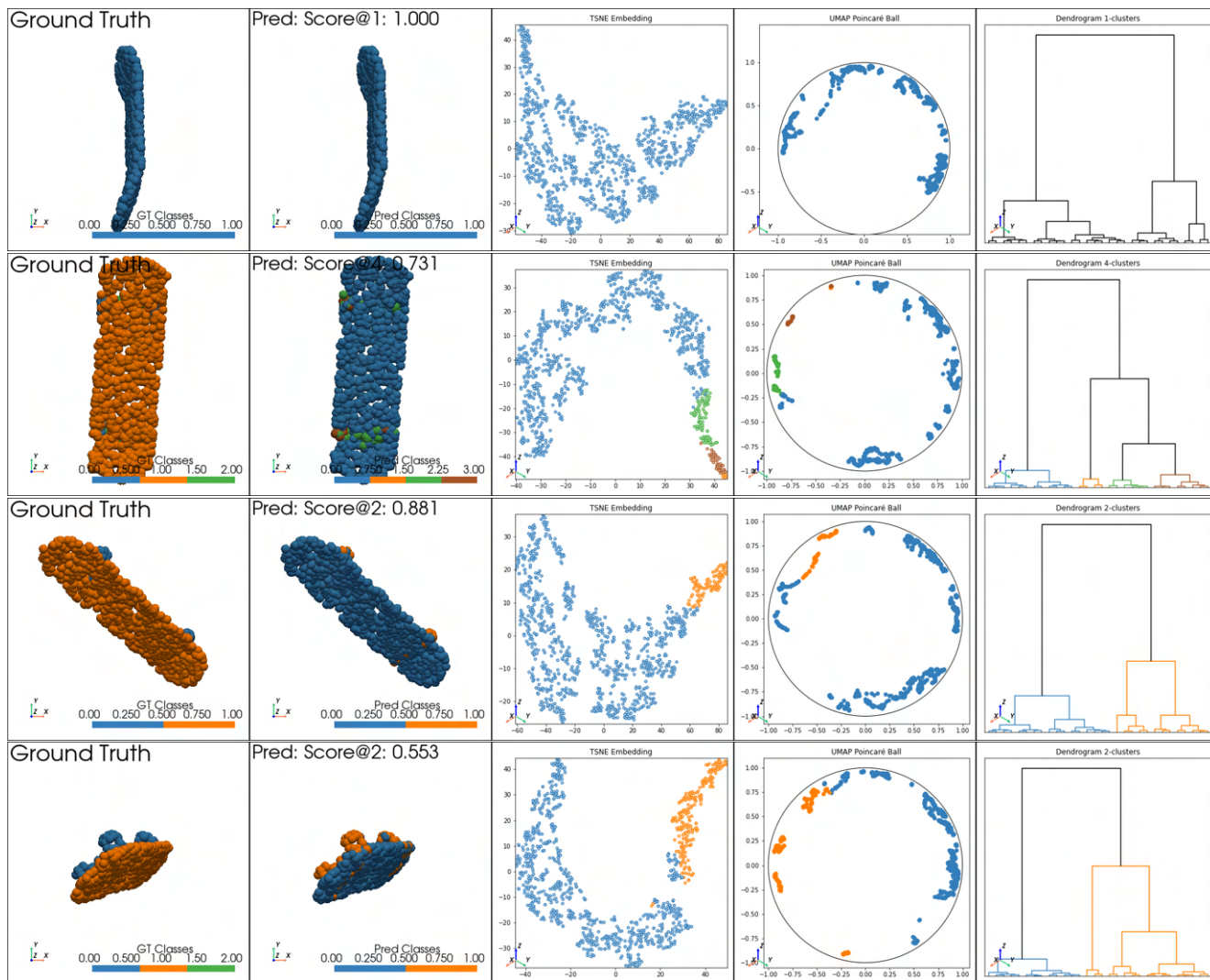


Figure 15. Hierarchical segmentation of skateboard examples in ShapeNet dataset.

Table

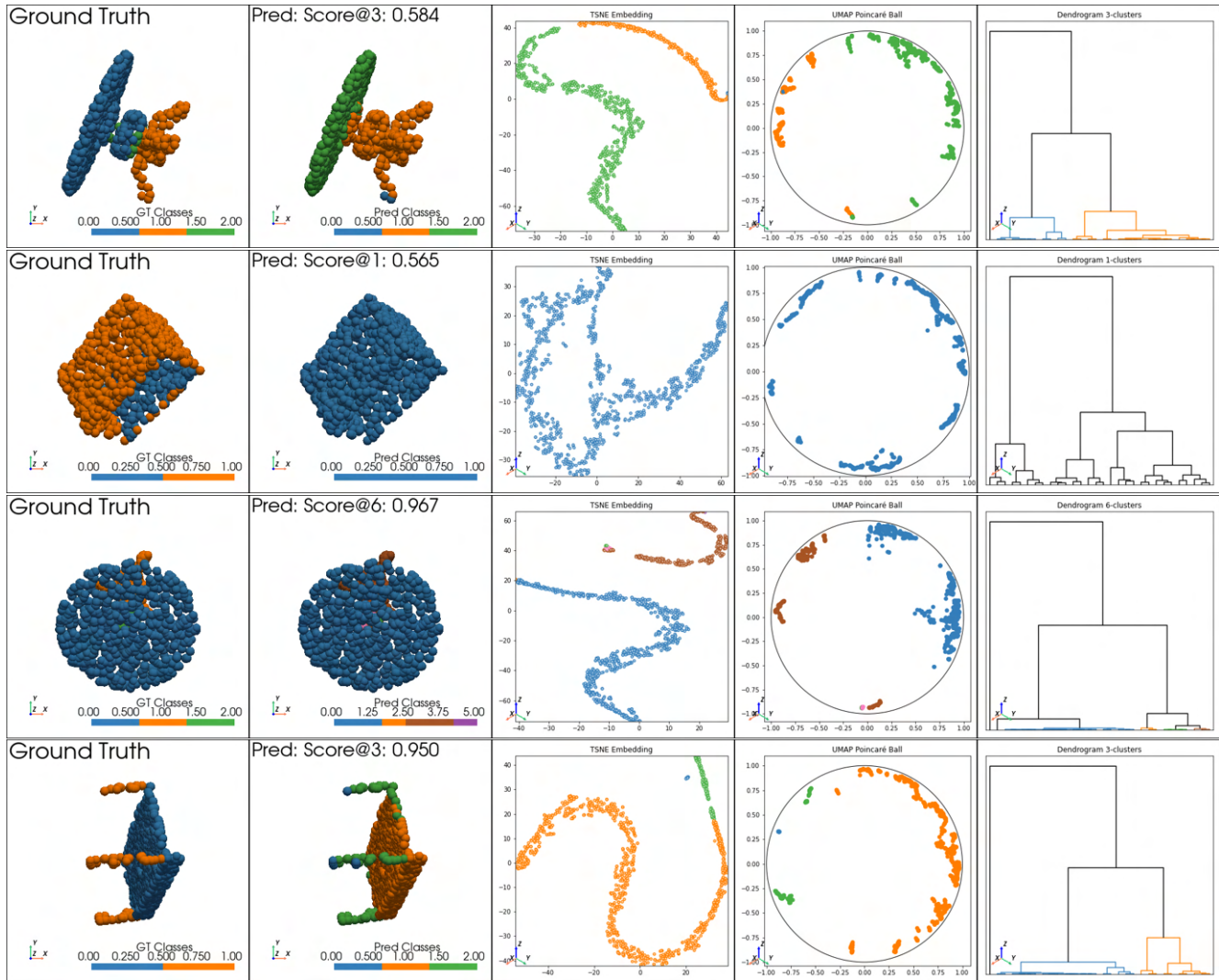


Figure 16. Hierarchical segmentation of table examples in ShapeNet dataset.

3. PartNet

Bed

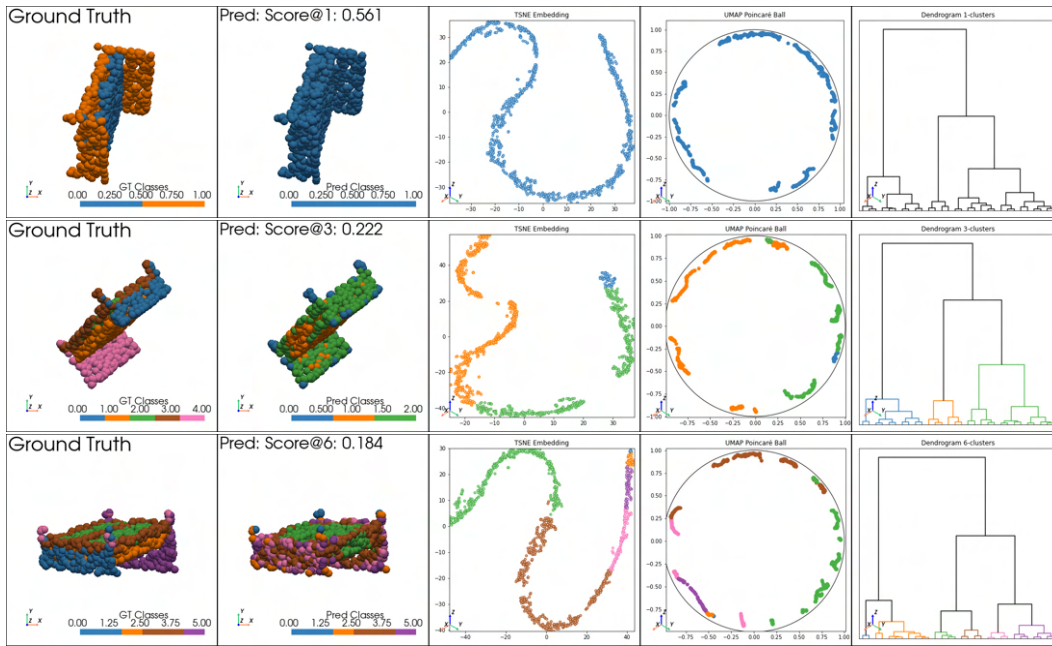


Figure 17. Hierarchical segmentation of 1st bed example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

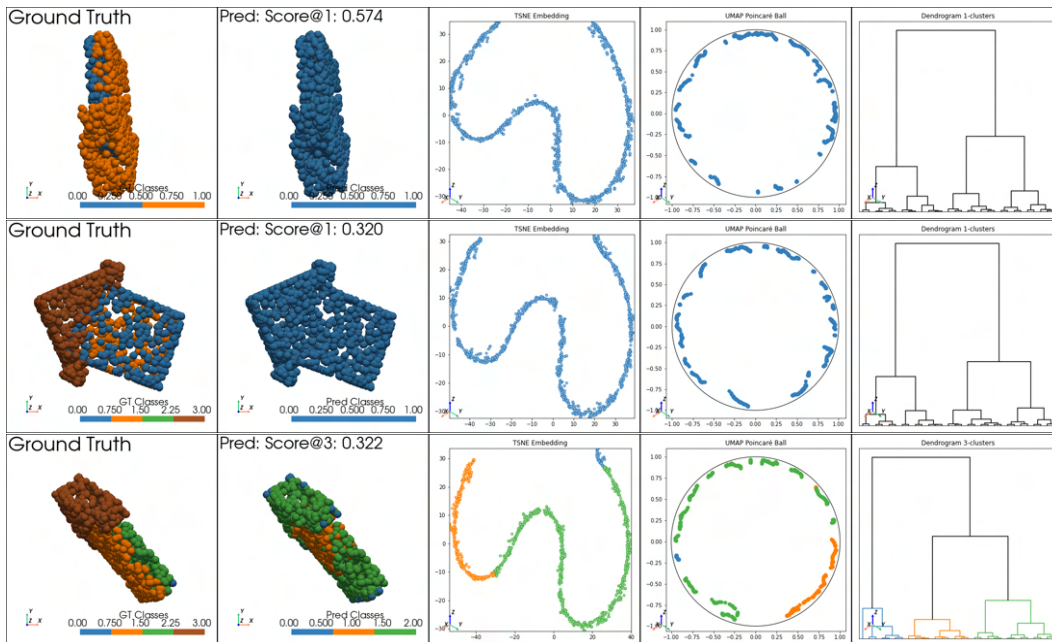


Figure 18. Hierarchical segmentation of 2nd bed example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

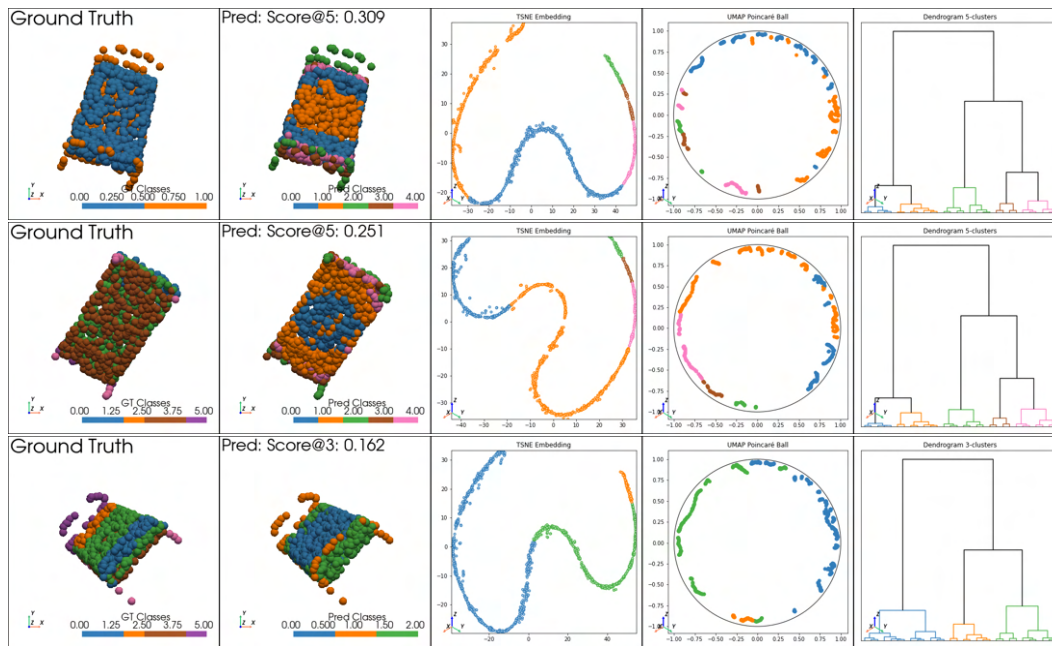


Figure 19. Hierarchical segmentation of 3rd bed example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

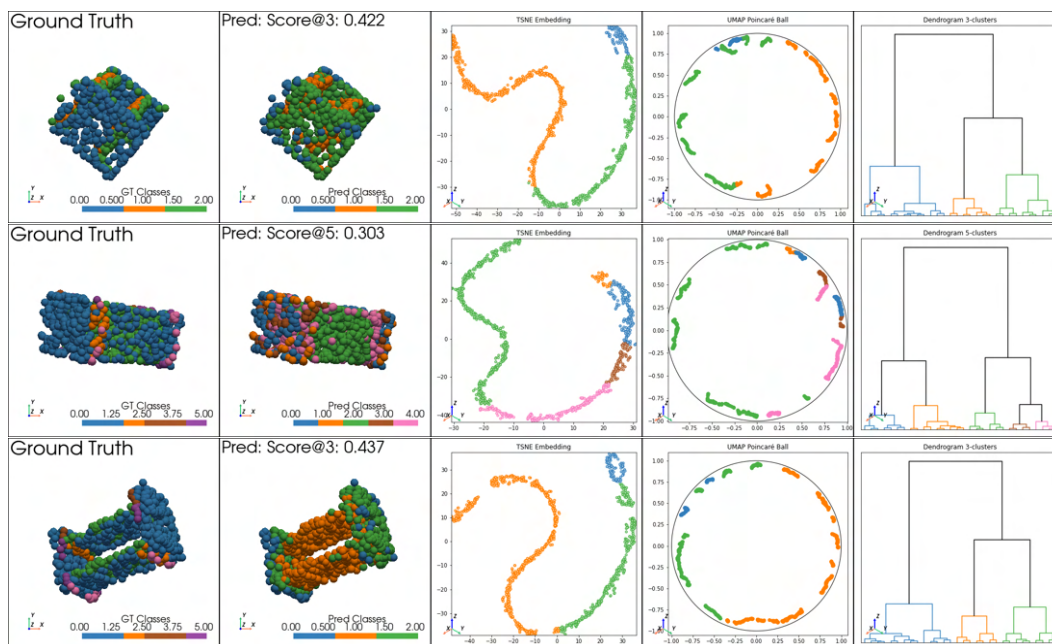


Figure 20. Hierarchical segmentation of 4th bed example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

Bottle

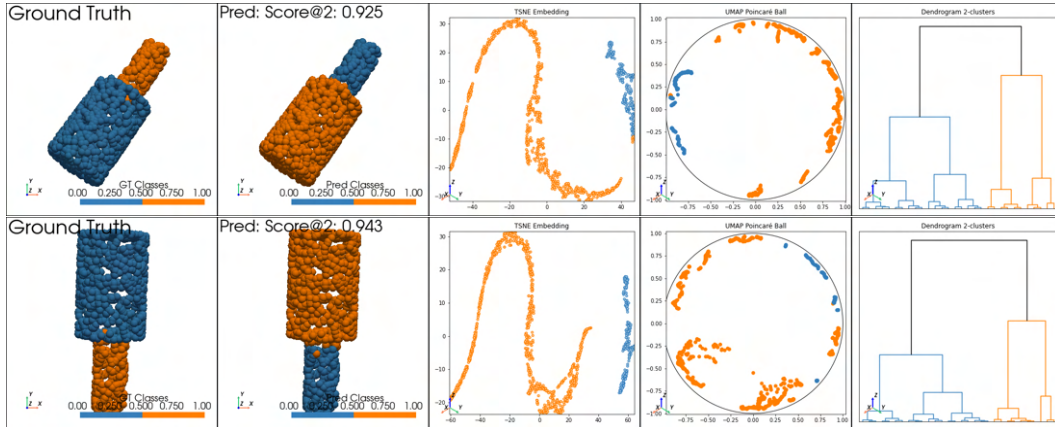


Figure 21. Hierarchical segmentation of 1st bottle example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

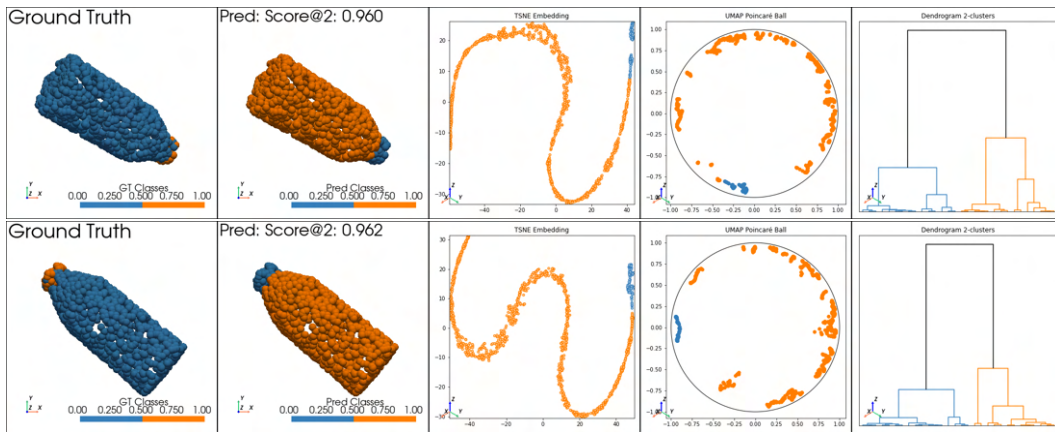


Figure 22. Hierarchical segmentation of 2nd bottle example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

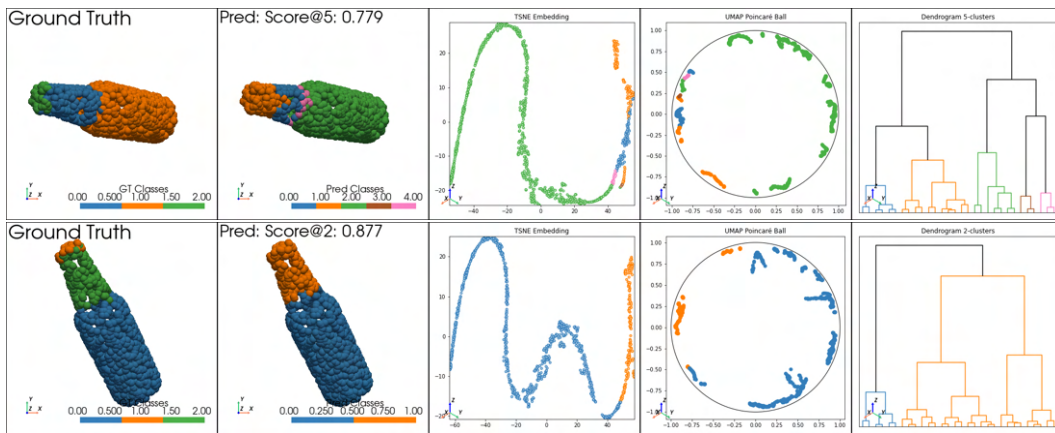


Figure 23. Hierarchical segmentation of 3rd bottle example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

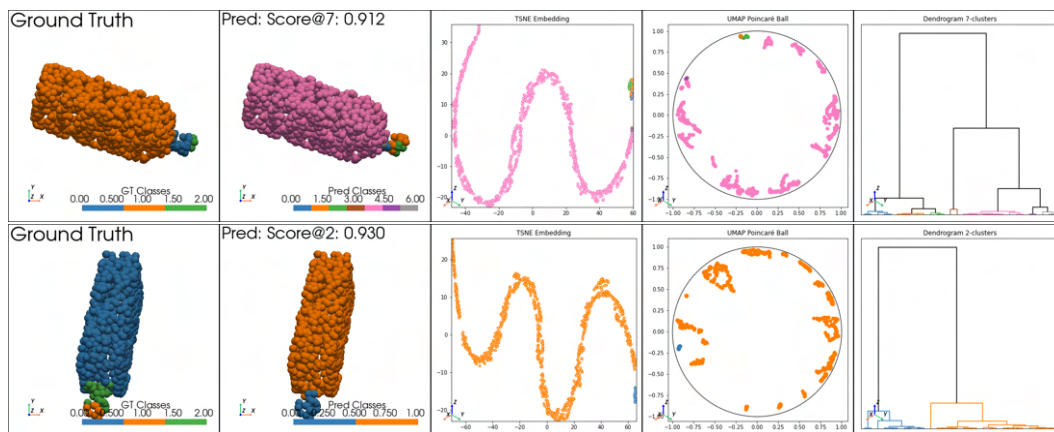


Figure 24. Hierarchical segmentation of 4th bottle example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

Chair

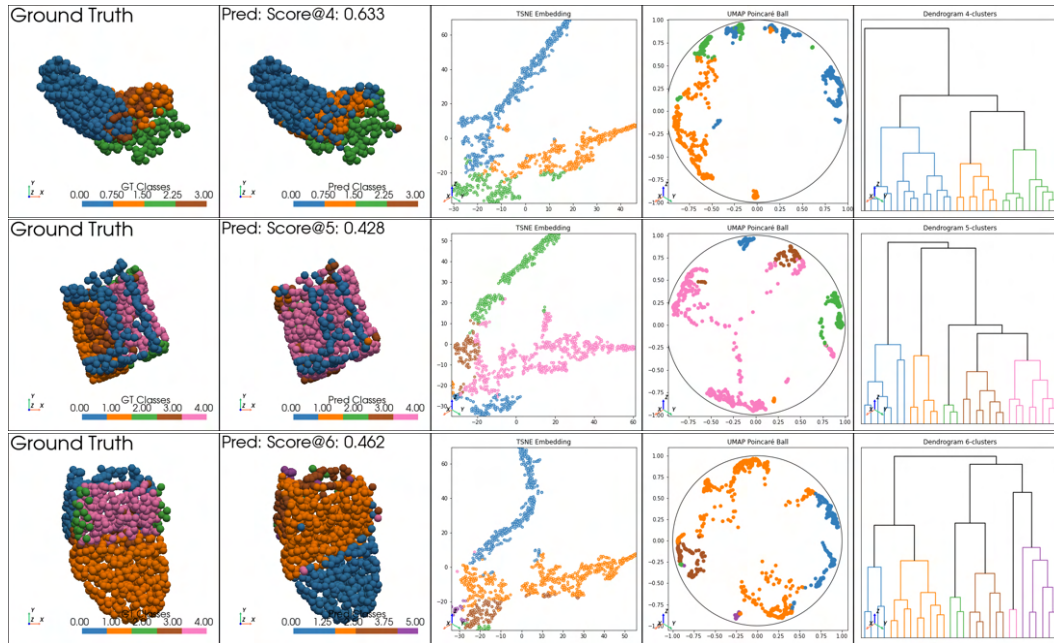


Figure 25. Hierarchical segmentation of 1st chair example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

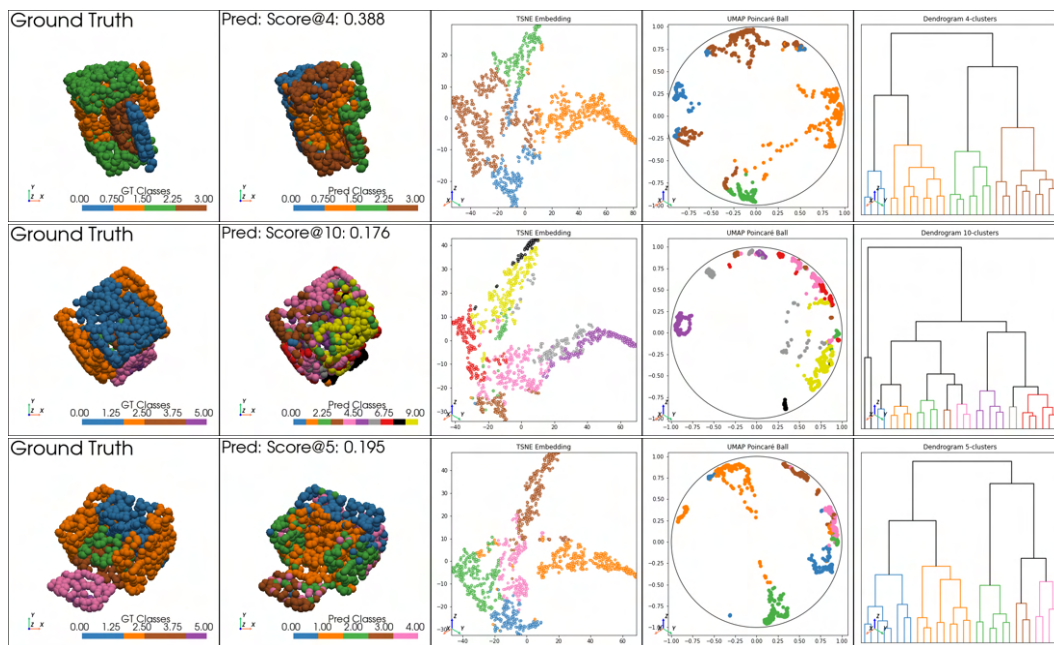


Figure 26. Hierarchical segmentation of 2nd chair example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

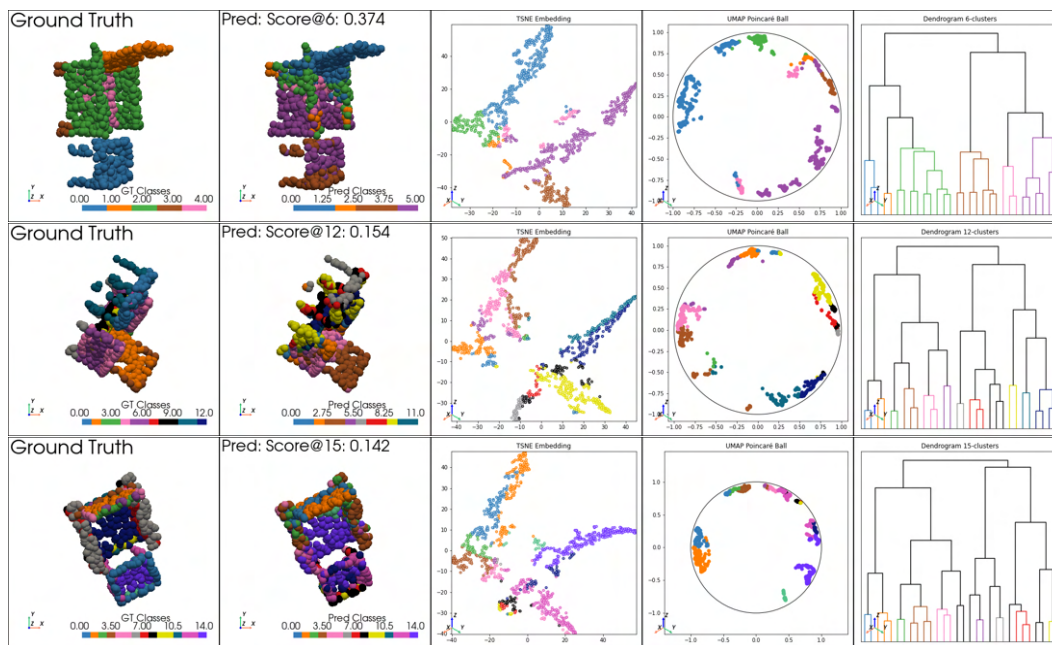


Figure 27. Hierarchical segmentation of 3rd chair example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

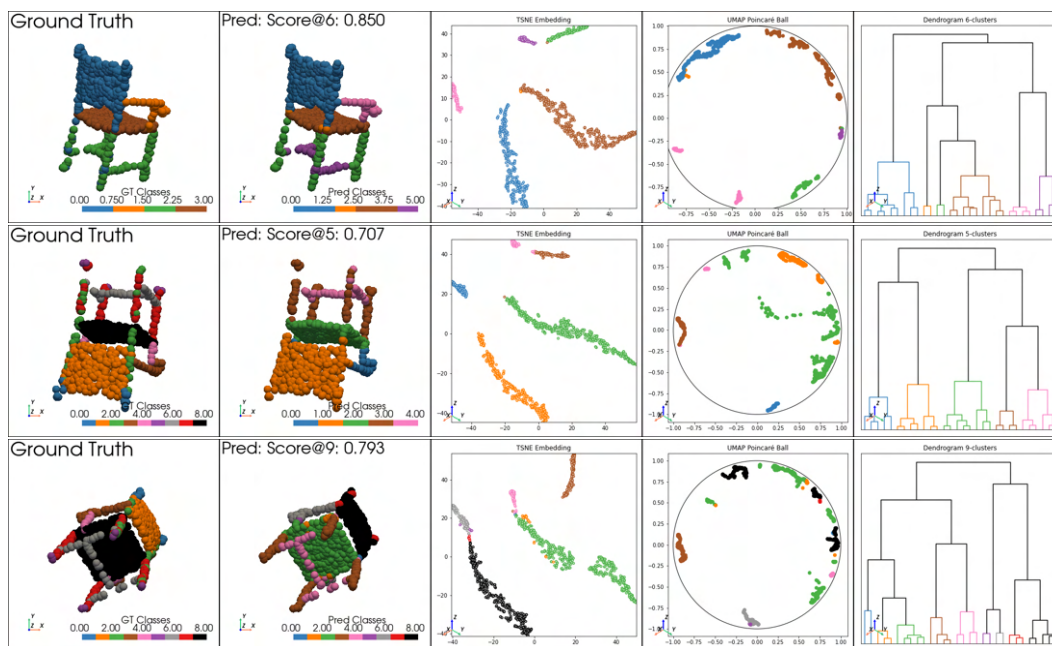


Figure 28. Hierarchical segmentation of 4th chair example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

Clock

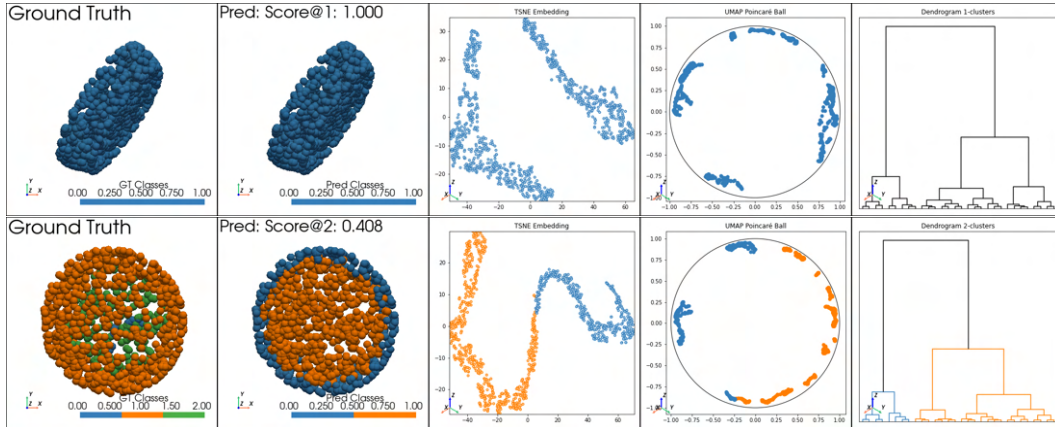


Figure 29. Hierarchical segmentation of 1st clock example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

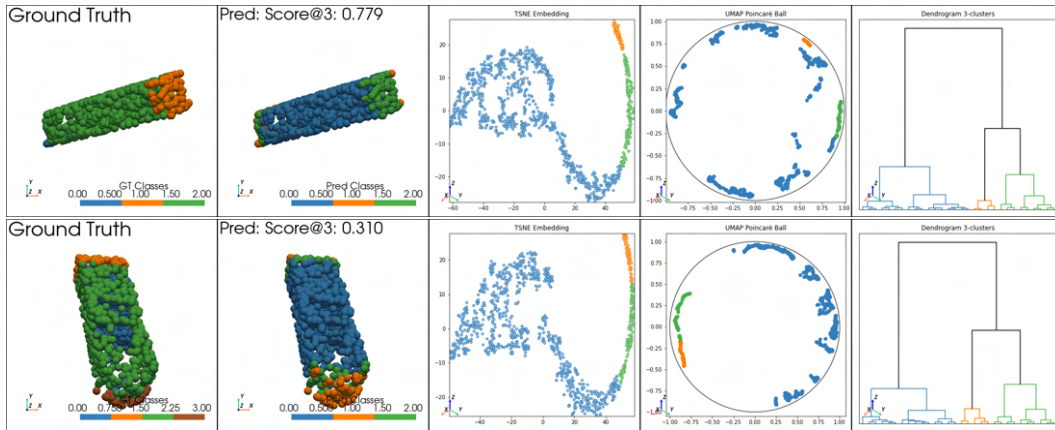


Figure 30. Hierarchical segmentation of 2nd clock example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

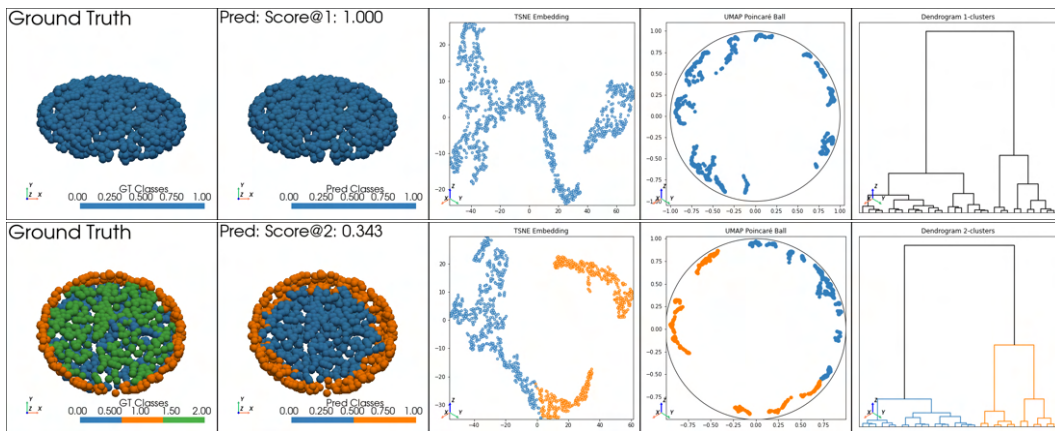


Figure 31. Hierarchical segmentation of 3rd clock example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

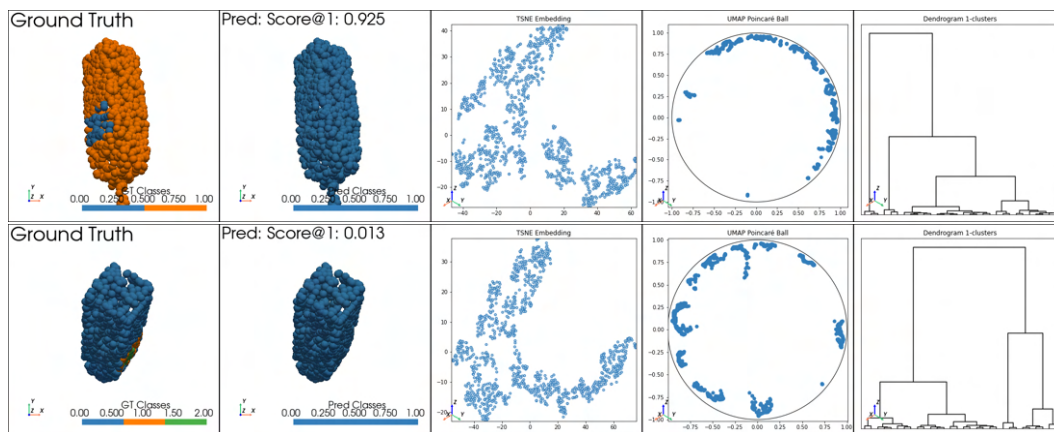


Figure 32. Hierarchical segmentation of 4th clock example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

Dishwasher

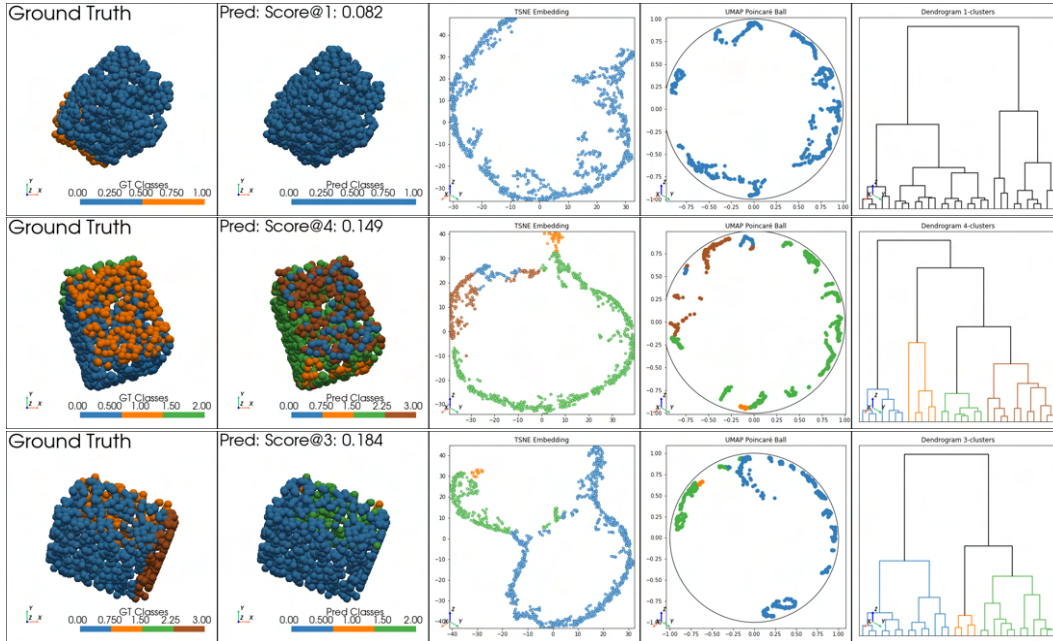


Figure 33. Hierarchical segmentation of 1st dishwasher example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

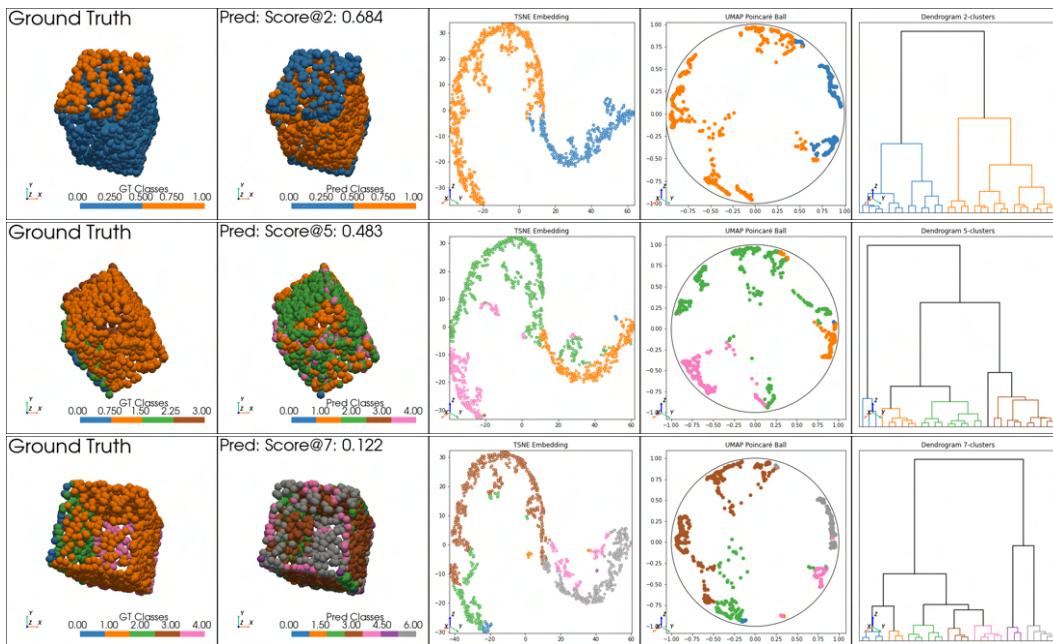


Figure 34. Hierarchical segmentation of 2nd dishwasher example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

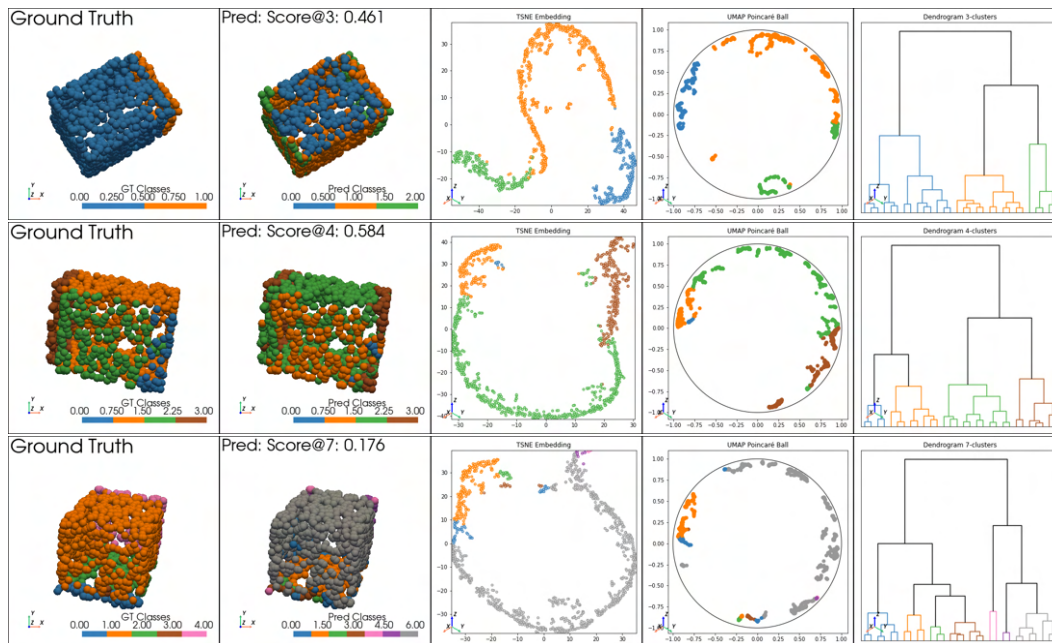


Figure 35. Hierarchical segmentation of 3rd dishwasher example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

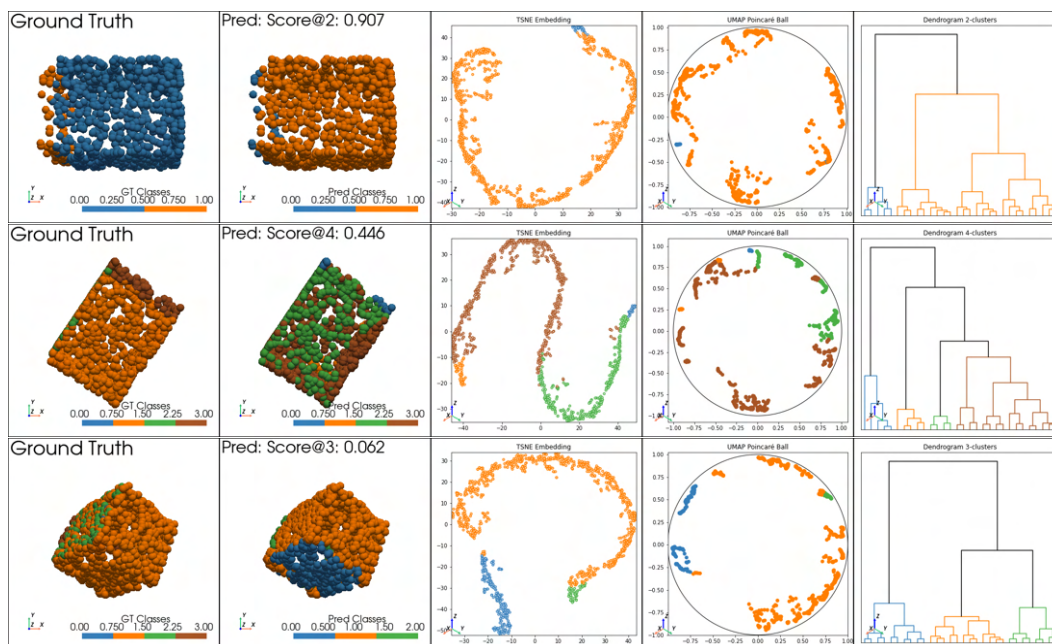


Figure 36. Hierarchical segmentation of 4th dishwasher example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

Display

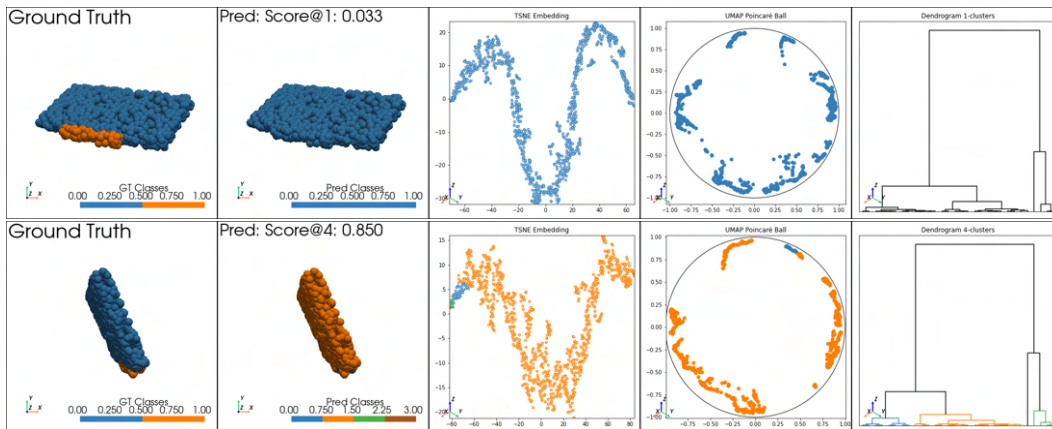


Figure 37. Hierarchical segmentation of 1st display example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

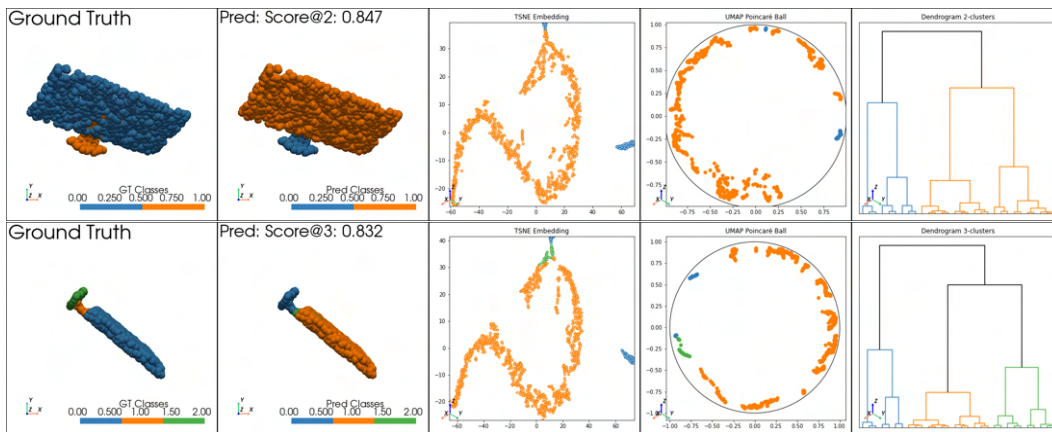


Figure 38. Hierarchical segmentation of 2nd display example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

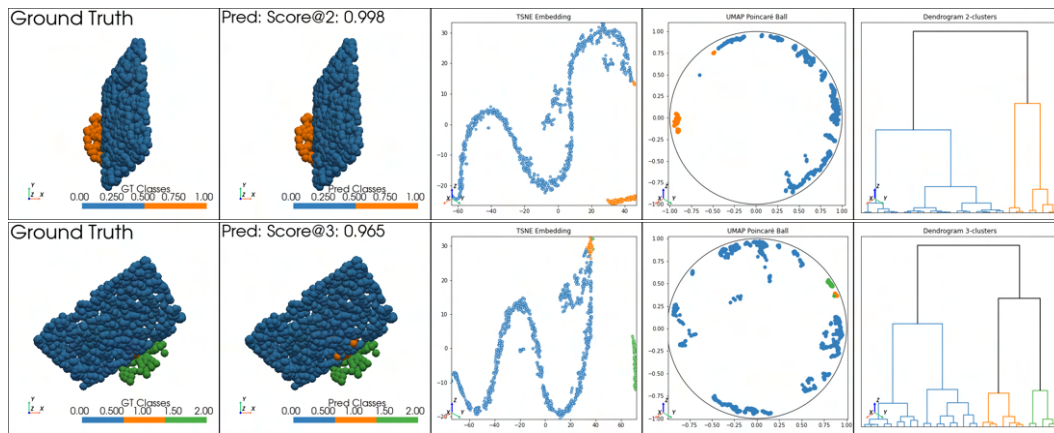


Figure 39. Hierarchical segmentation of 3rd display example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

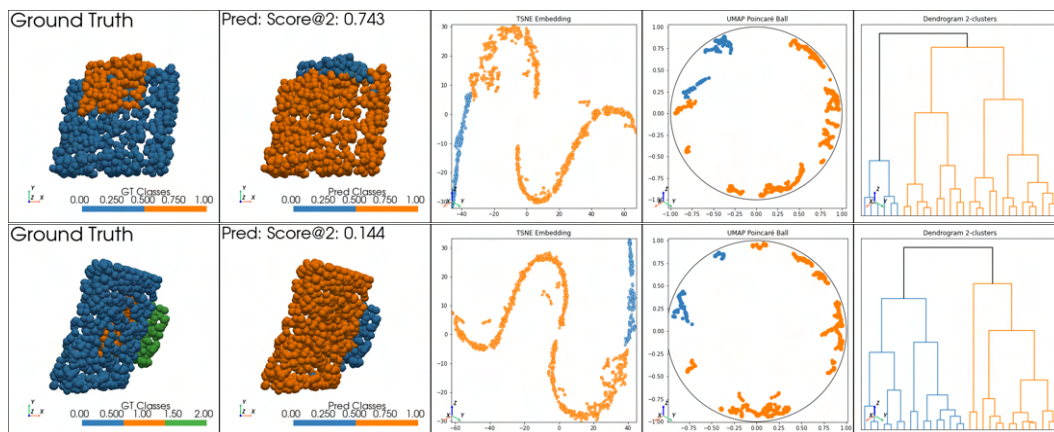


Figure 40. Hierarchical segmentation of 4th display example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

Door

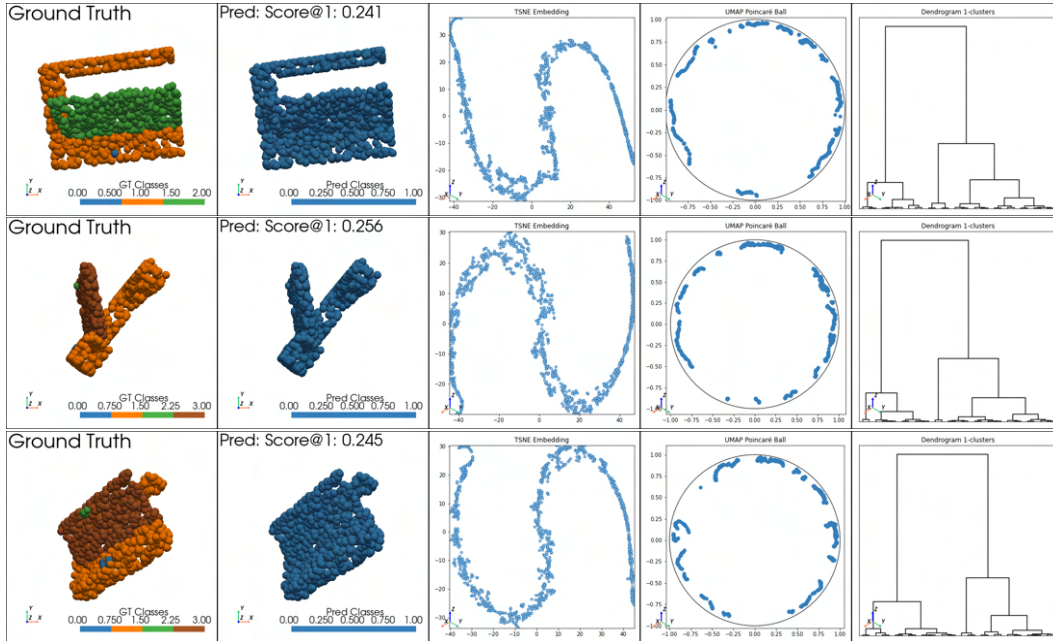


Figure 41. Hierarchical segmentation of 1st door example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

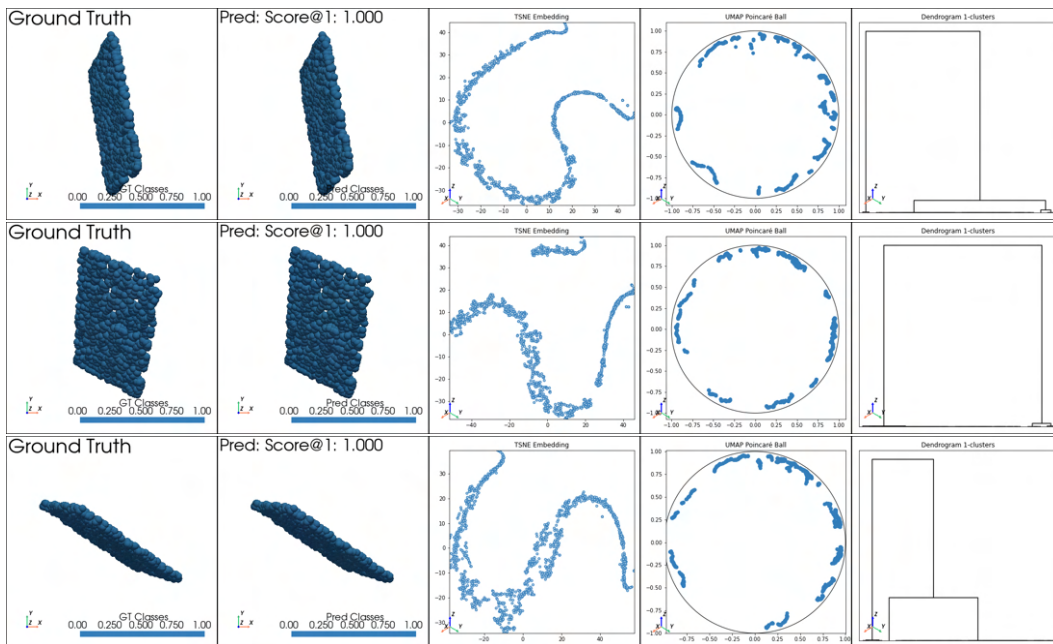


Figure 42. Hierarchical segmentation of 2nd door example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

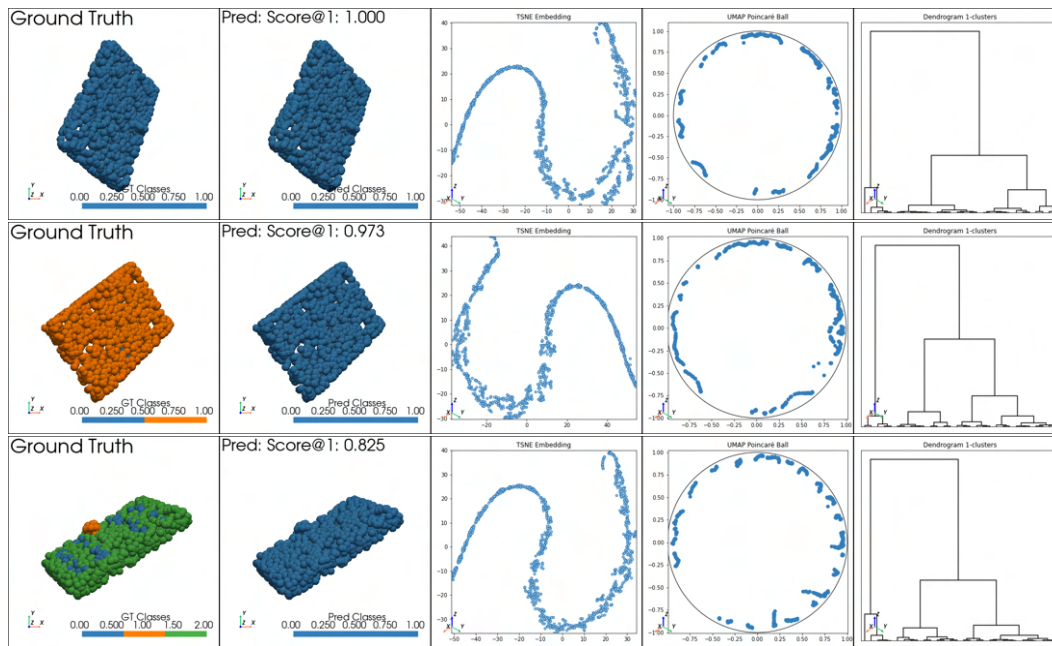


Figure 43. Hierarchical segmentation of 3rd door example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

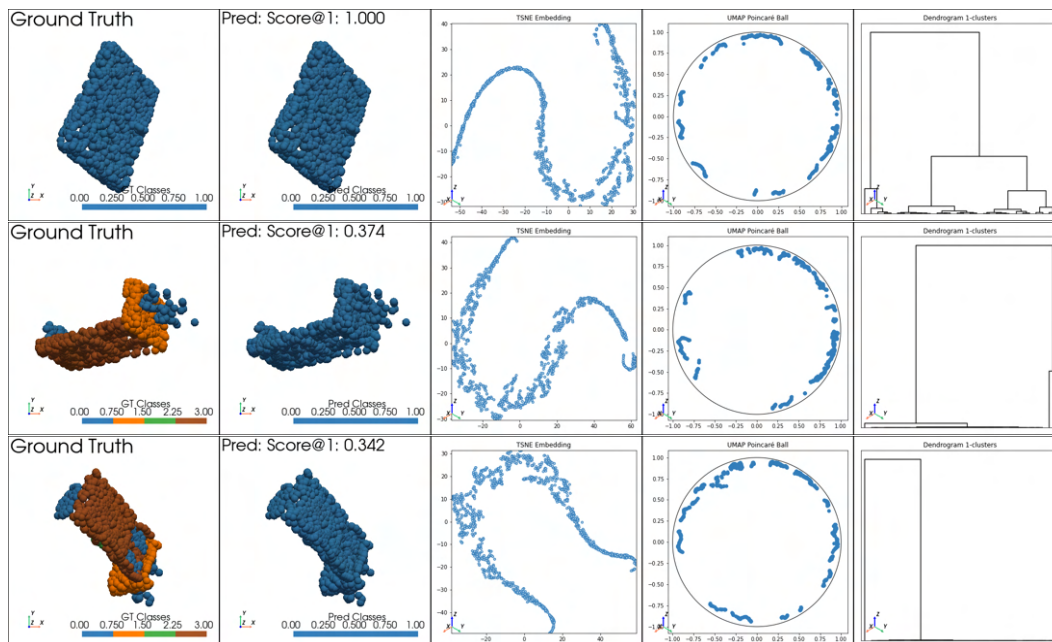


Figure 44. Hierarchical segmentation of 4th door example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

Earphone

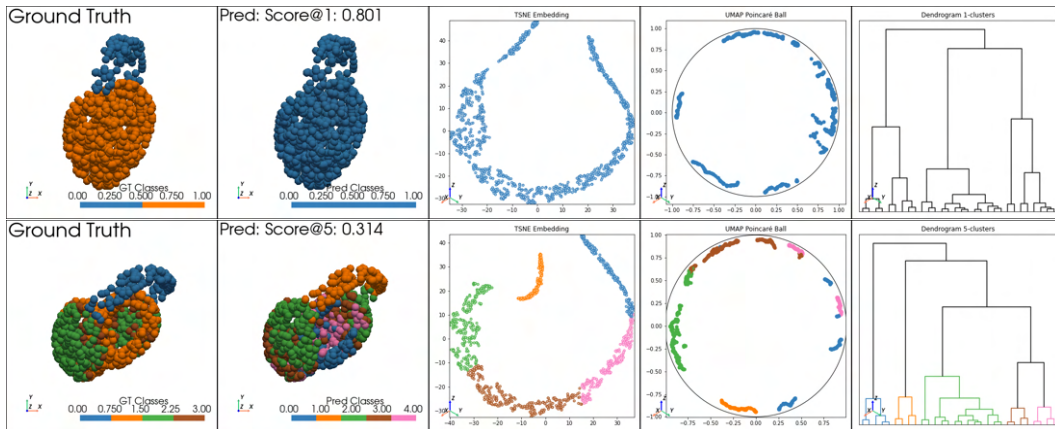


Figure 45. Hierarchical segmentation of 1st earphone example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

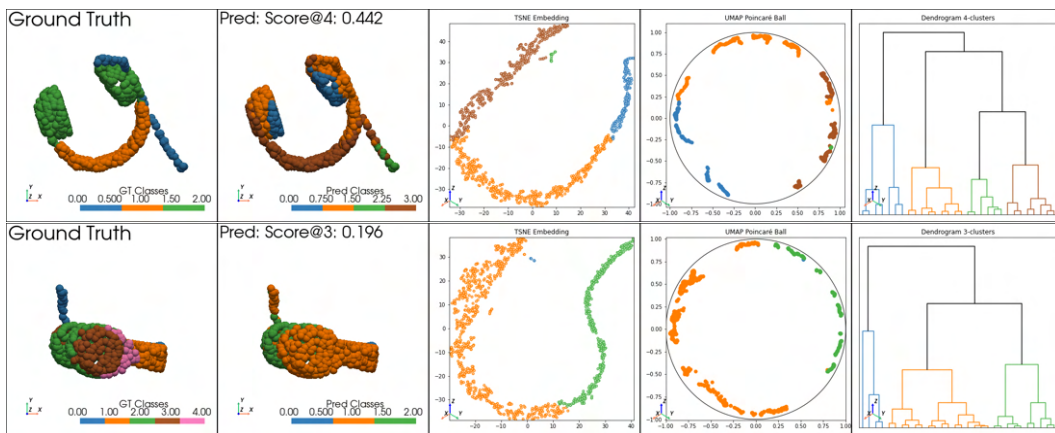


Figure 46. Hierarchical segmentation of 2nd earphone example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

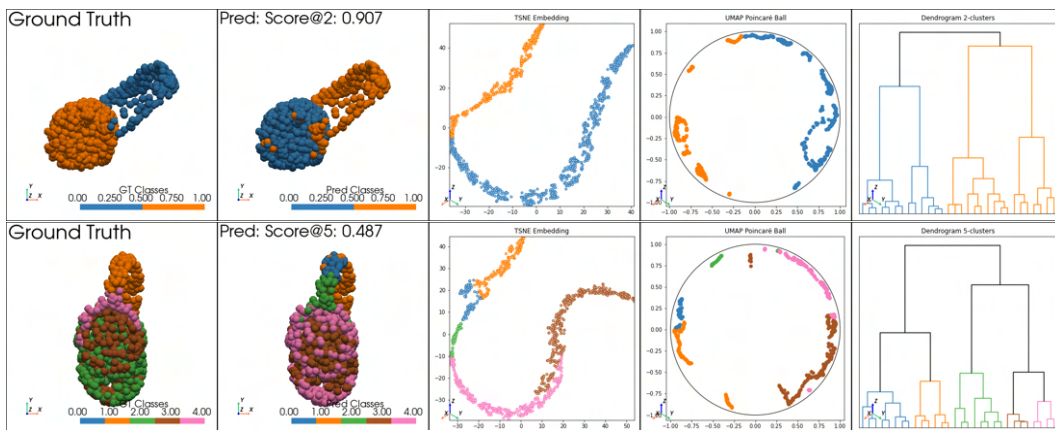


Figure 47. Hierarchical segmentation of 3rd earphone example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

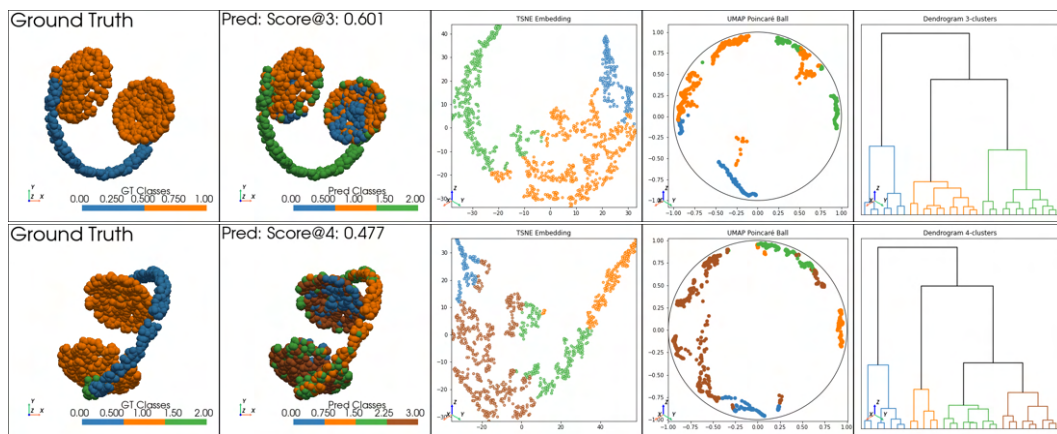
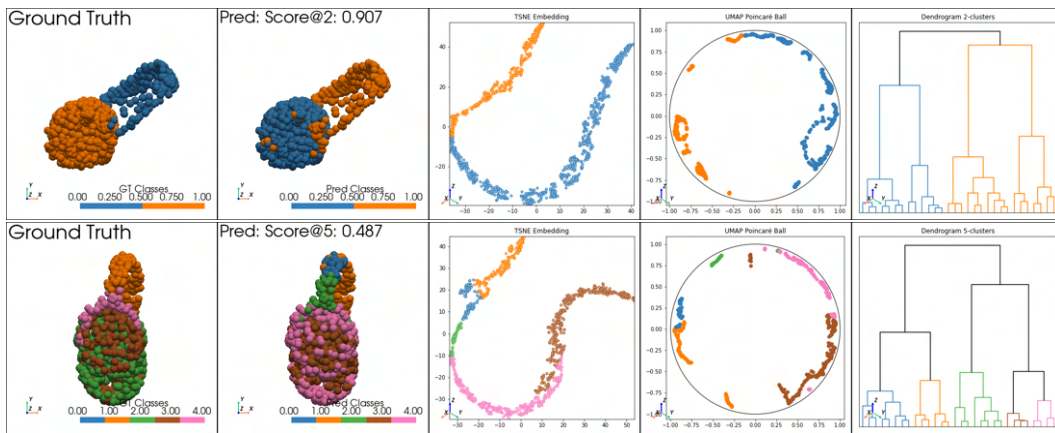
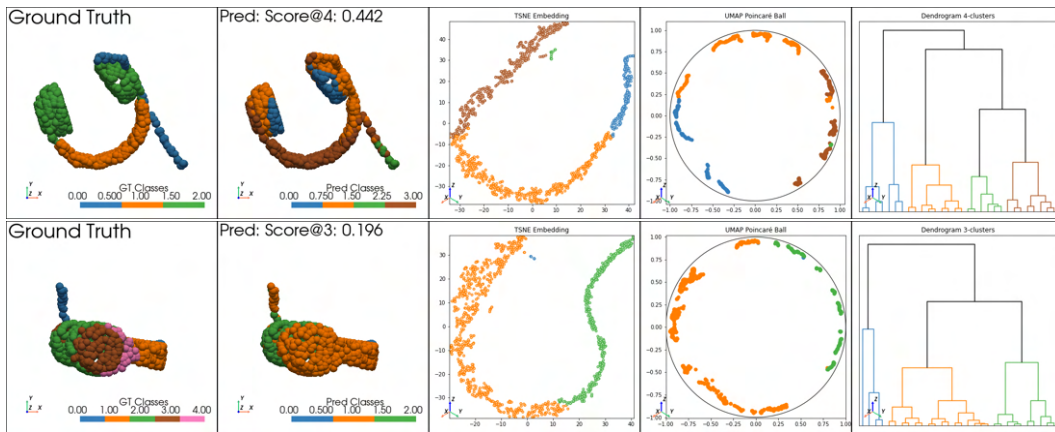
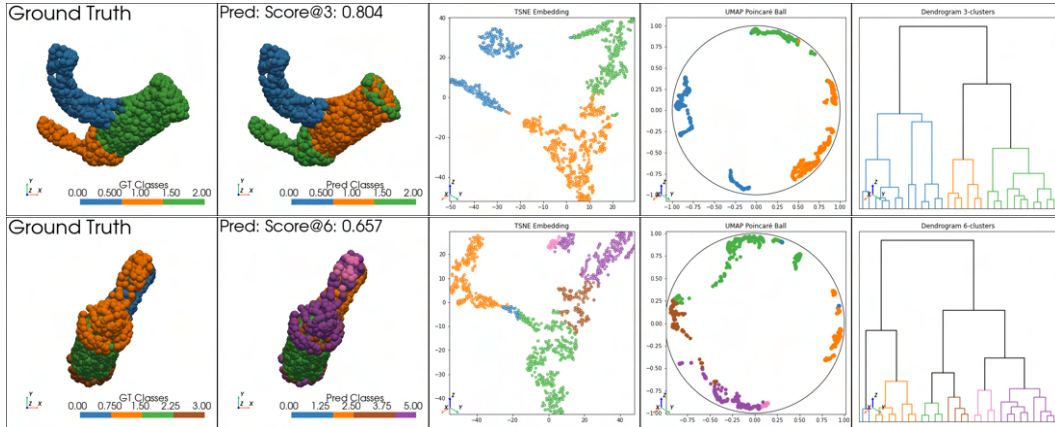


Figure 48. Hierarchical segmentation of 4th earphone example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

Faucet



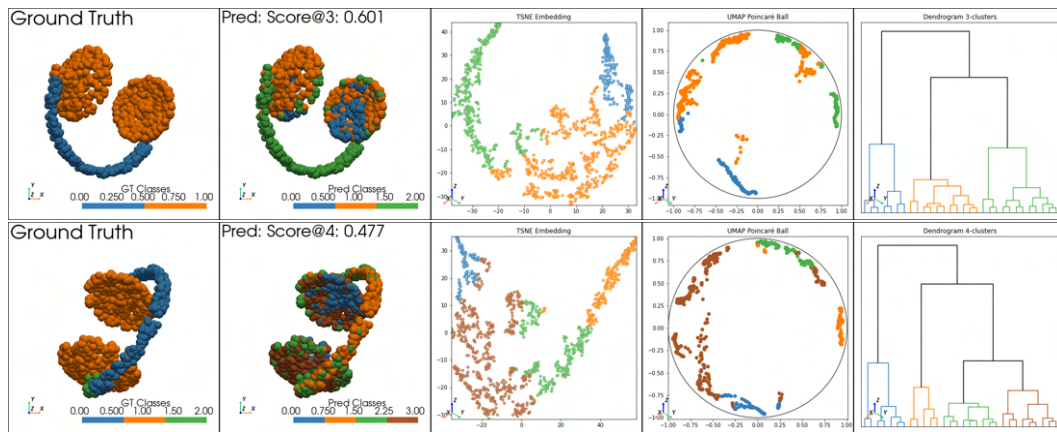


Figure 52. Hierarchical segmentation of 4th faucet example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

Knife

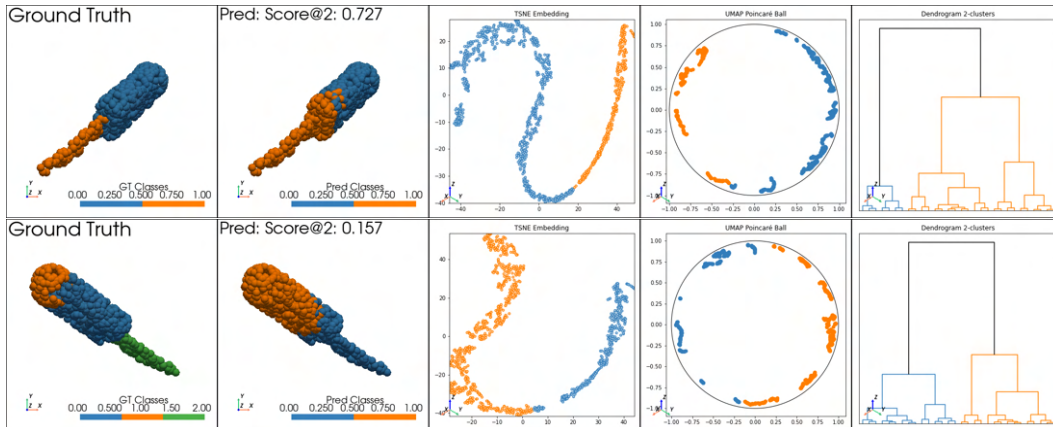


Figure 53. Hierarchical segmentation of 1st knife example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

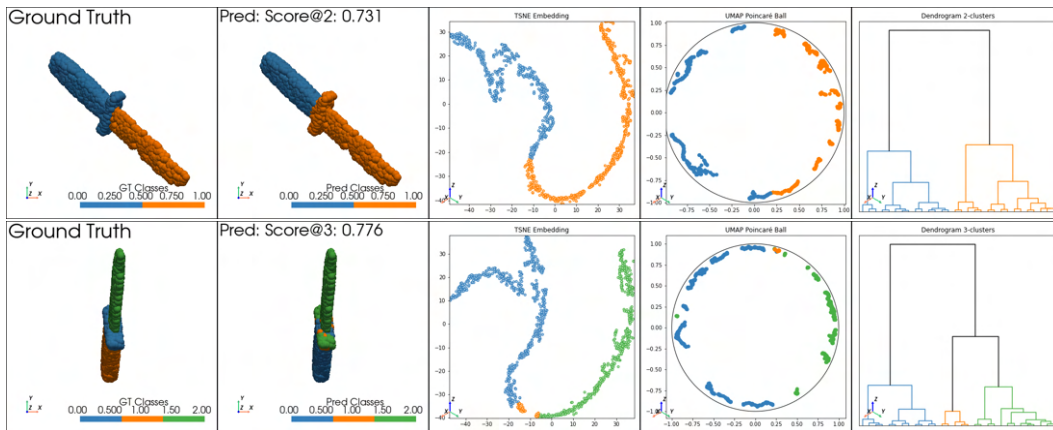


Figure 54. Hierarchical segmentation of 2nd knife example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

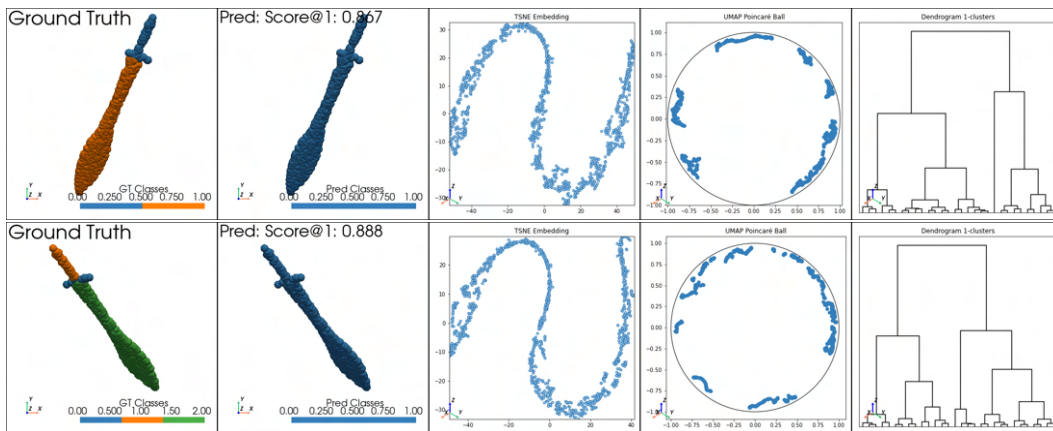


Figure 55. Hierarchical segmentation of 3rd knife example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

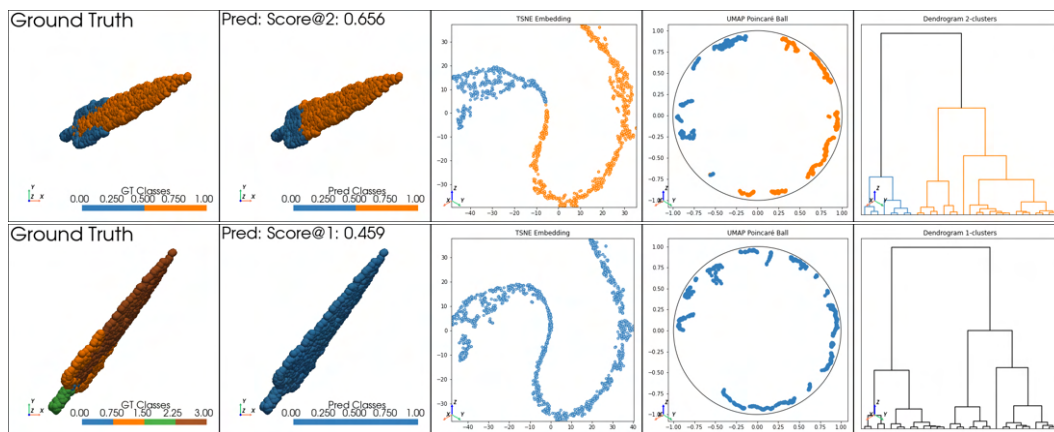


Figure 56. Hierarchical segmentation of 4th knife example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

Lamp

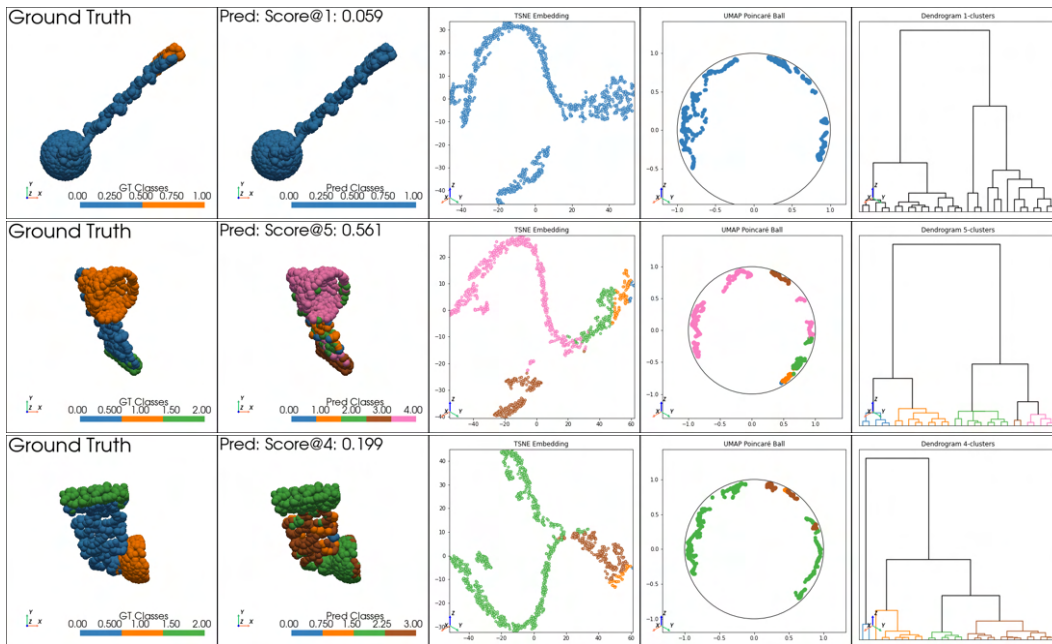


Figure 57. Hierarchical segmentation of 1st lamp example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

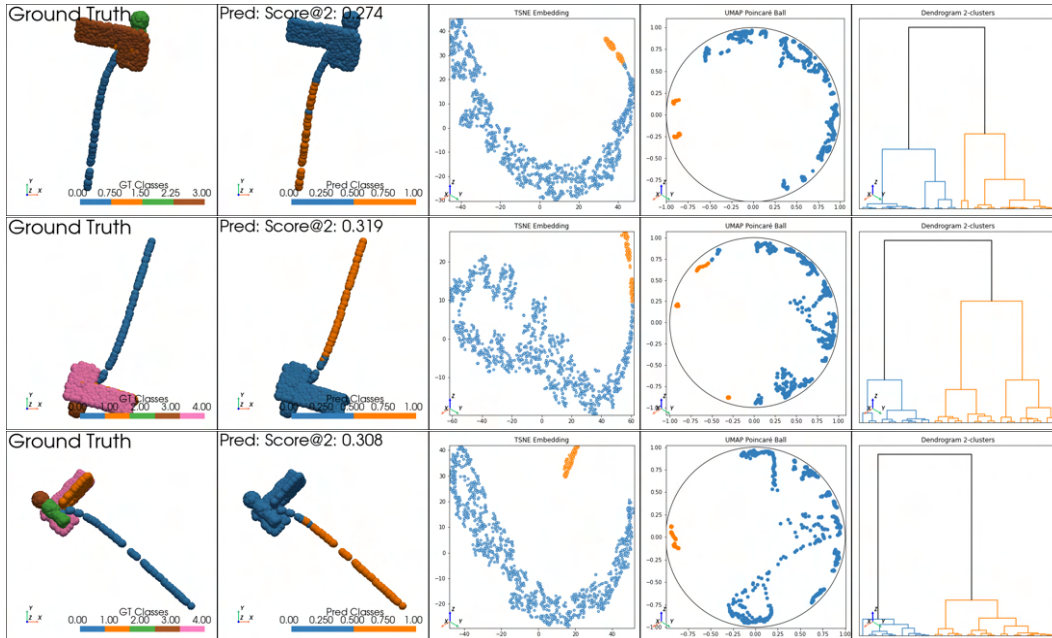


Figure 58. Hierarchical segmentation of 2nd lamp example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

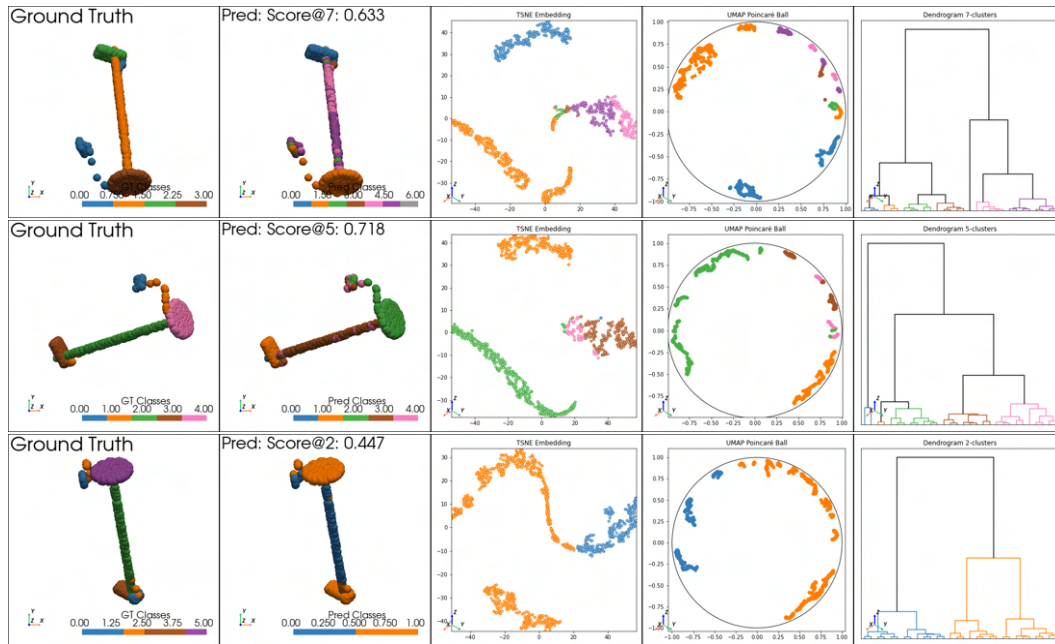


Figure 59. Hierarchical segmentation of 3rd lamp example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

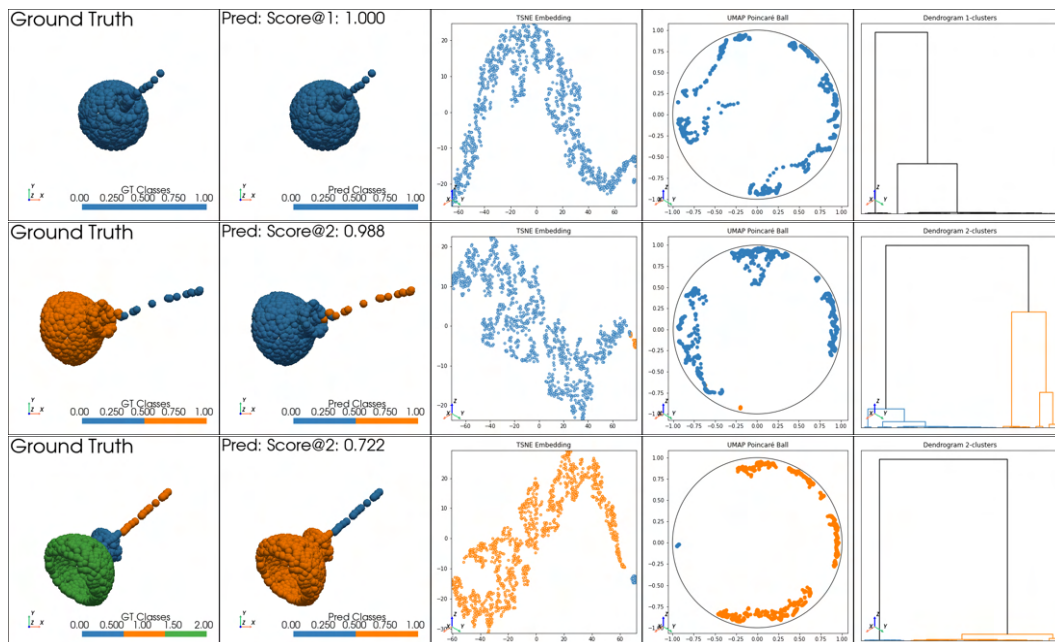


Figure 60. Hierarchical segmentation of 4th lamp example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

Microwave

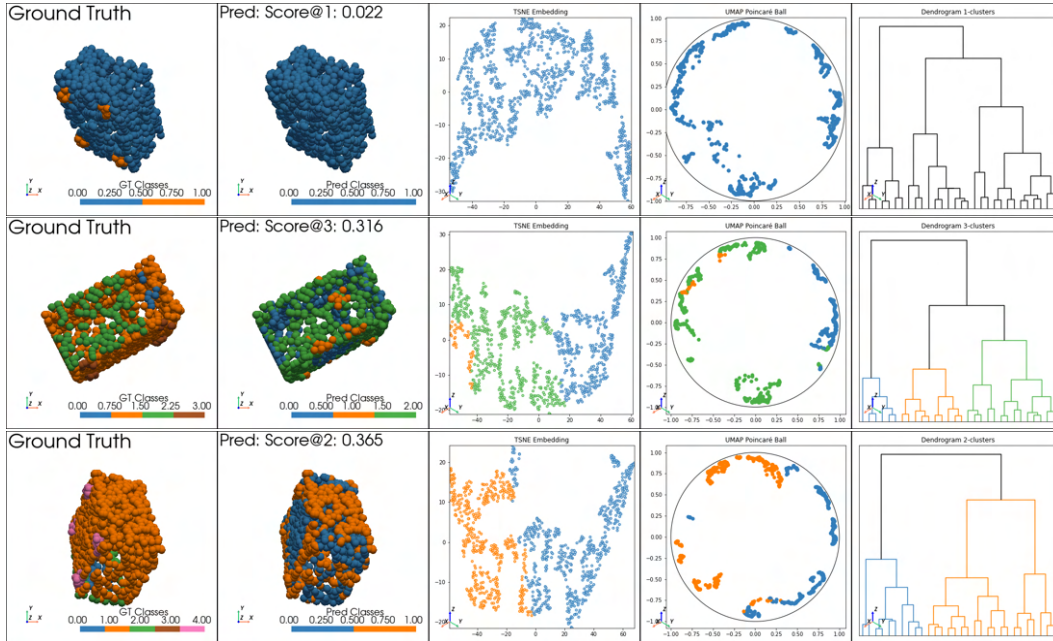


Figure 61. Hierarchical segmentation of 1st microwave example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

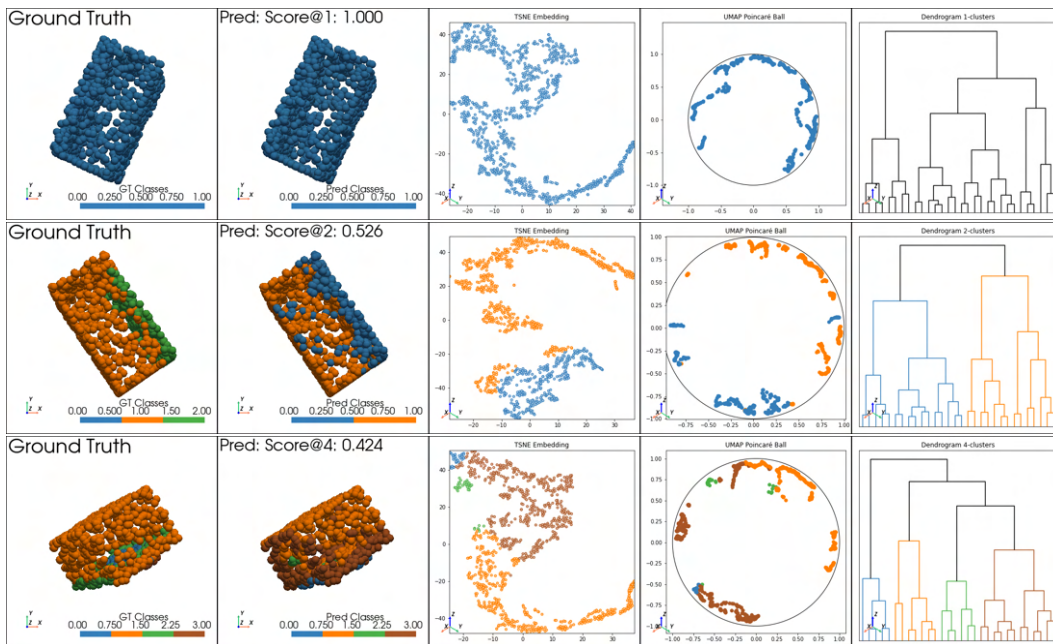


Figure 62. Hierarchical segmentation of 2nd microwave example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

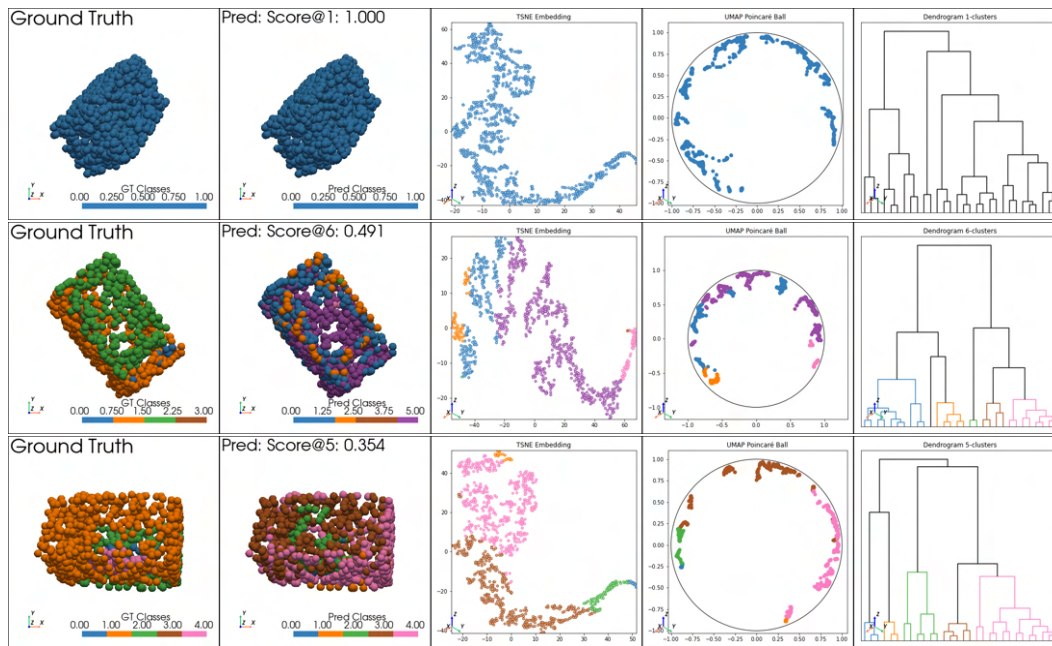


Figure 63. Hierarchical segmentation of 3rd microwave example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

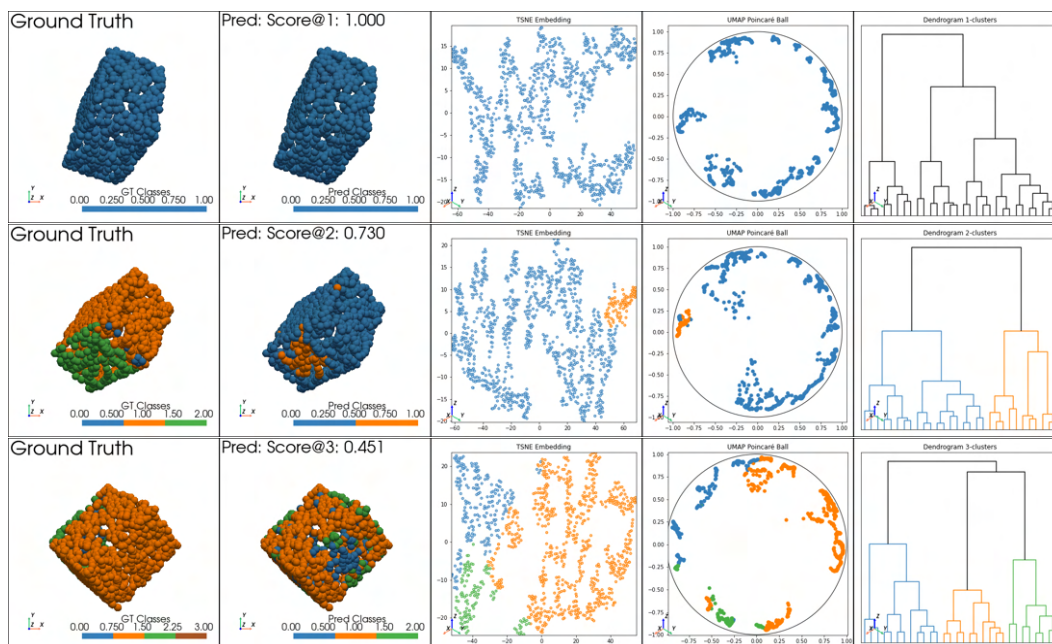


Figure 64. Hierarchical segmentation of 4th microwave example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

Refrigerator

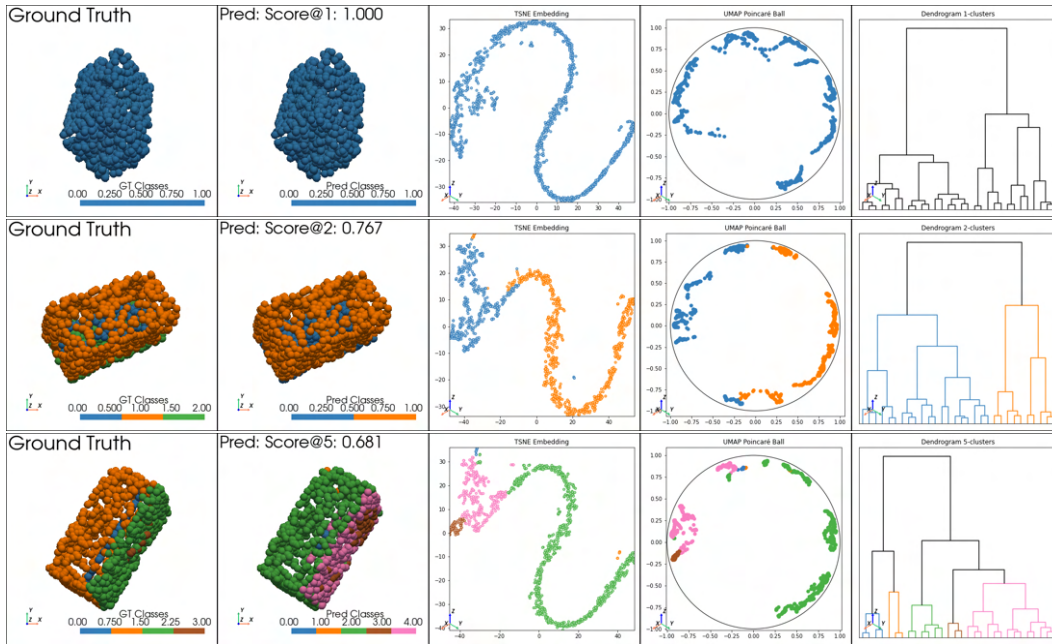


Figure 65. Hierarchical segmentation of 1st refrigerator example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

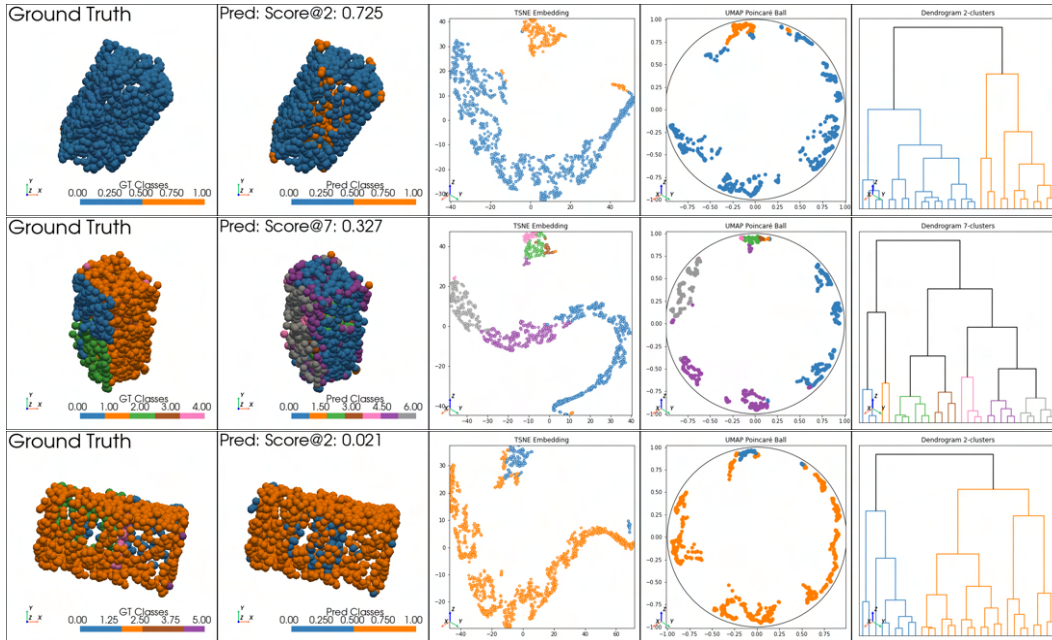


Figure 66. Hierarchical segmentation of 2nd refrigerator example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

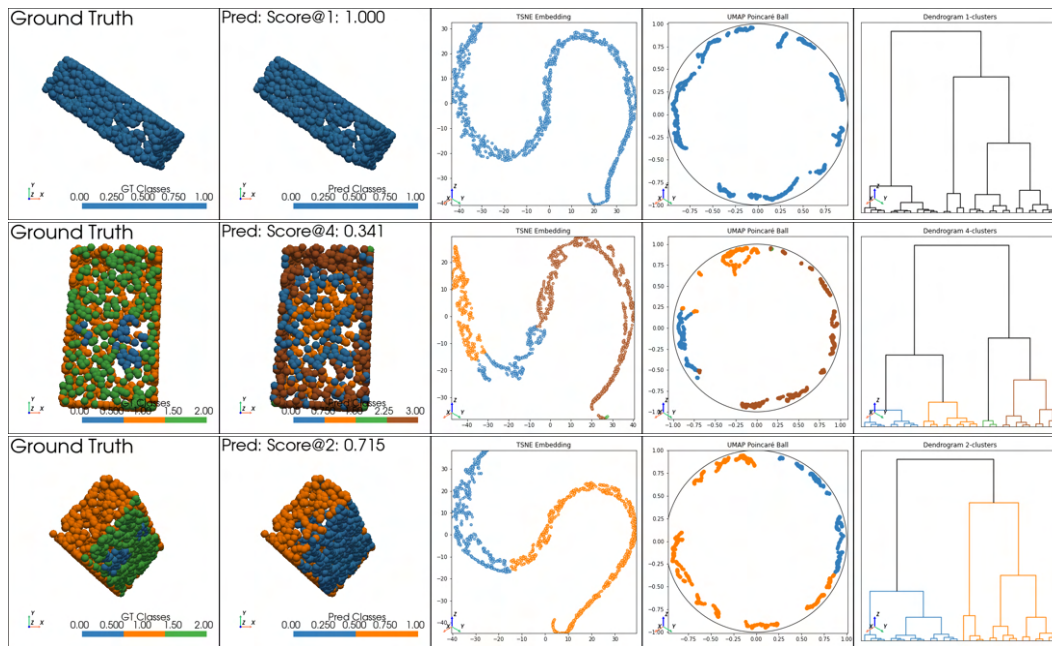


Figure 67. Hierarchical segmentation of 3rd refrigerator example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

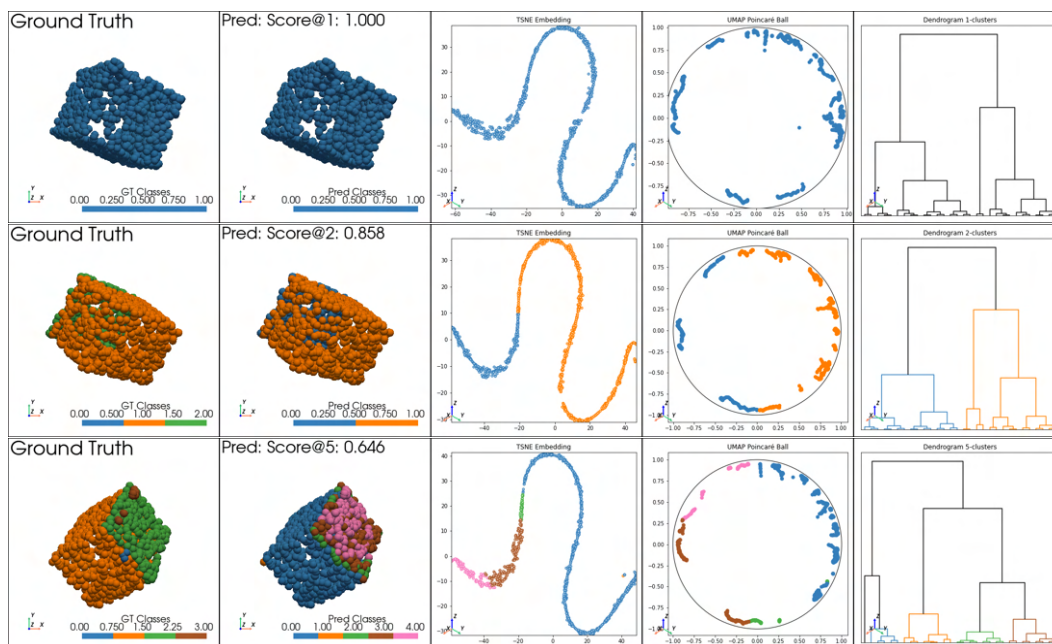


Figure 68. Hierarchical segmentation of 4th refrigerator example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

StorageFurniture

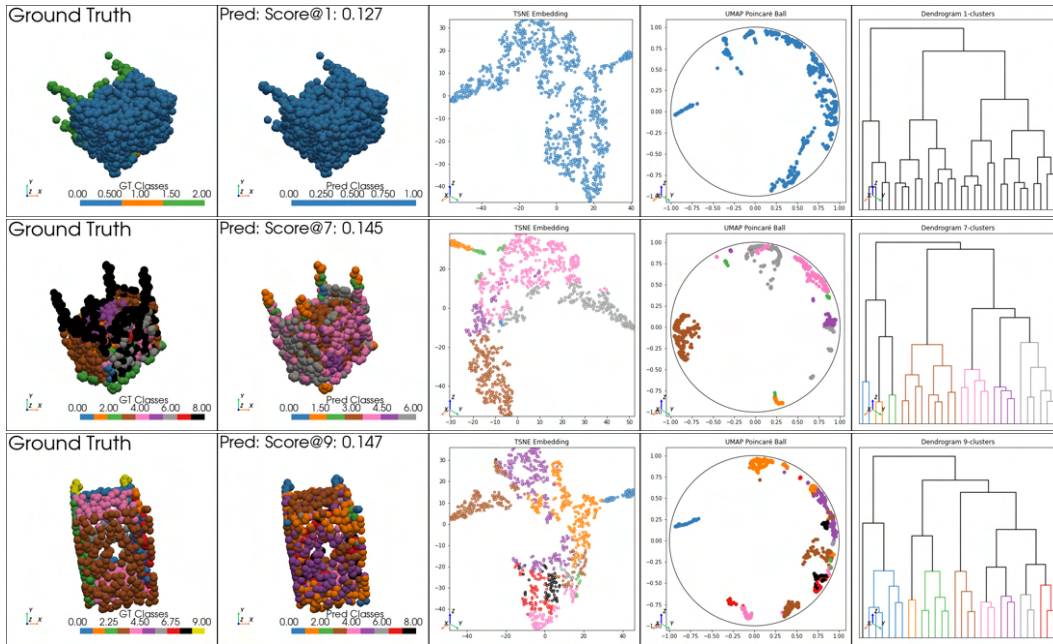


Figure 69. Hierarchical segmentation of 1st storagefurniture example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

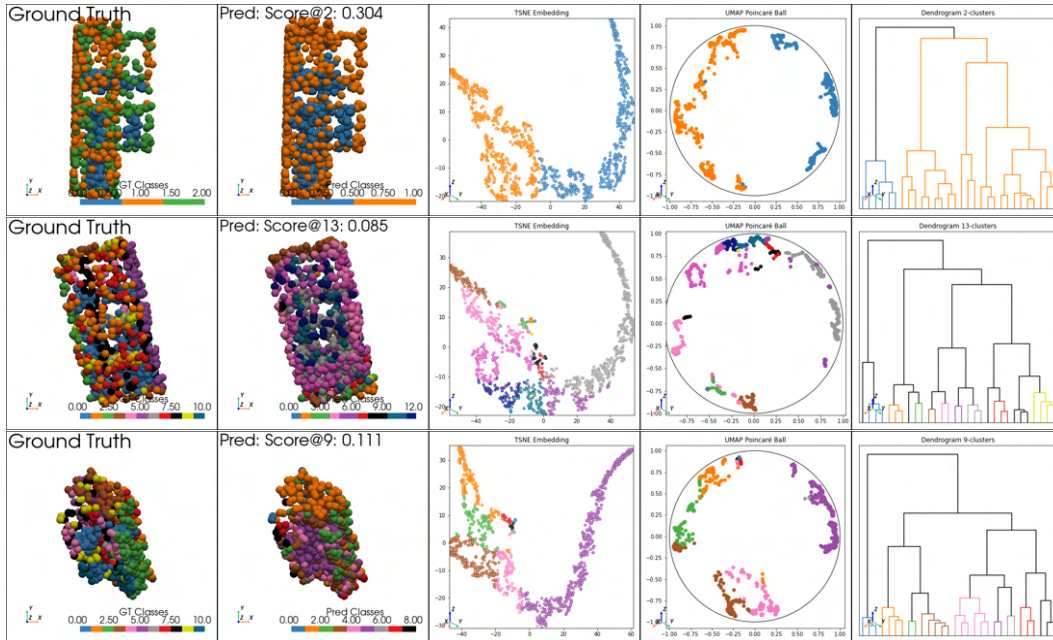


Figure 70. Hierarchical segmentation of 2nd storagefurniture example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

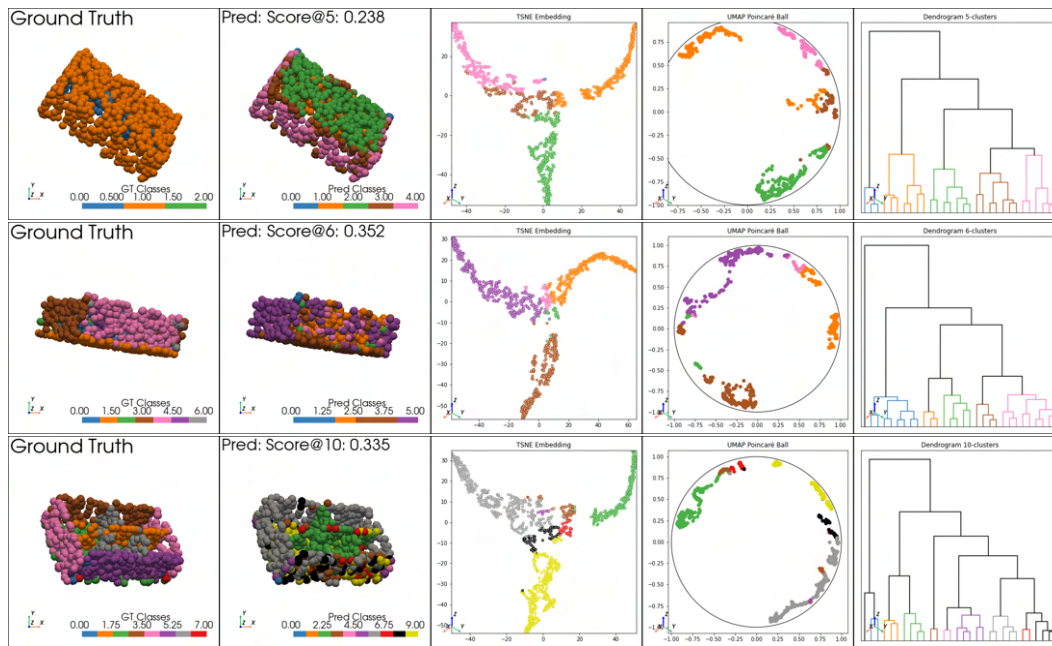


Figure 71. Hierarchical segmentation of 3rd storagefurniture example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

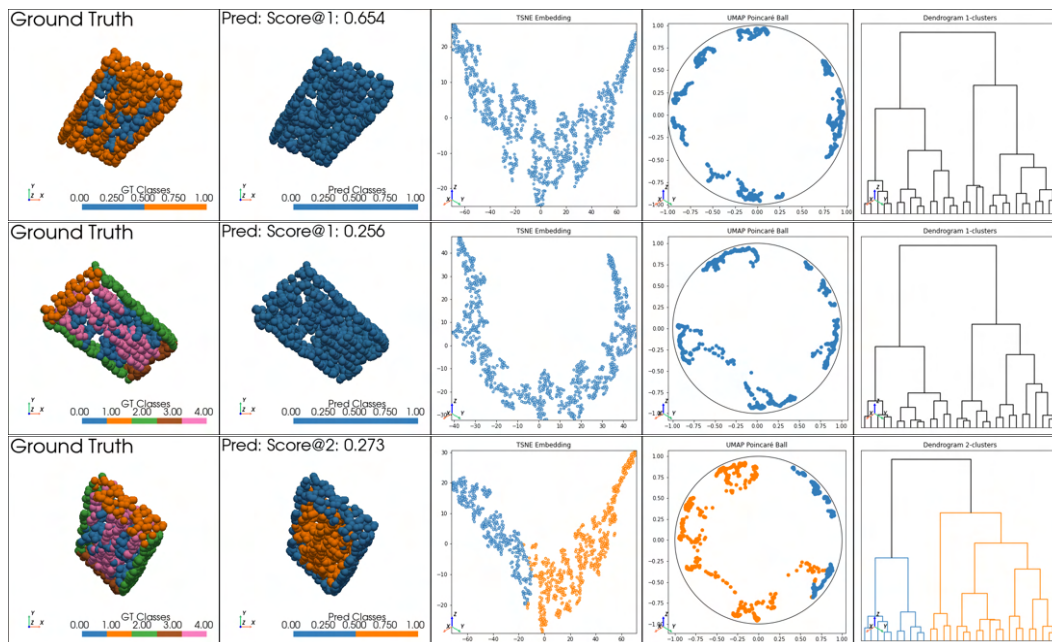


Figure 72. Hierarchical segmentation of 4th storagefurniture example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

Table

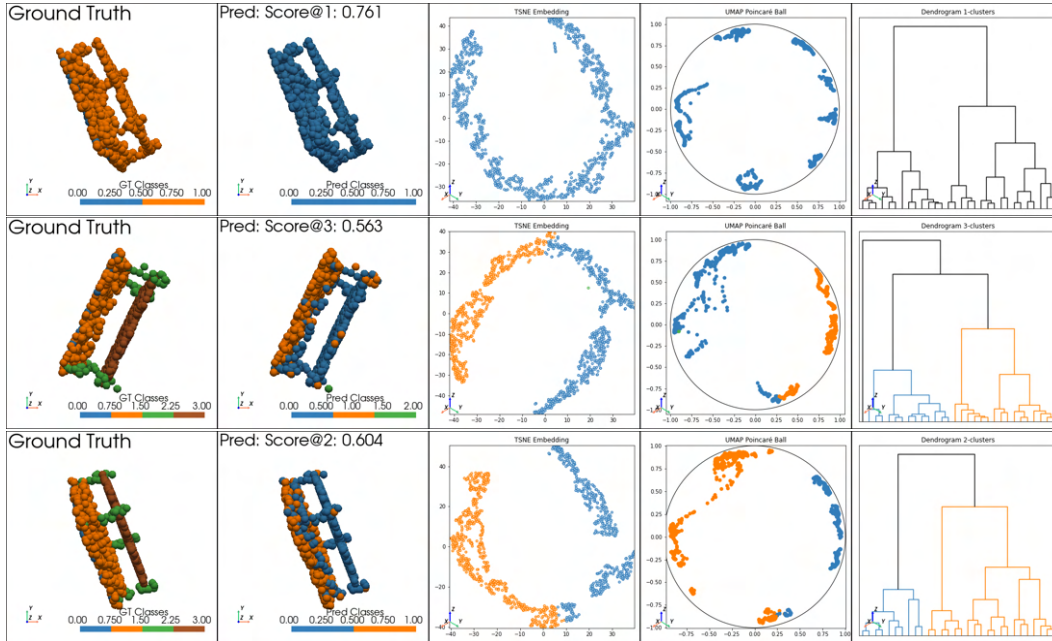


Figure 73. Hierarchical segmentation of 1st table example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

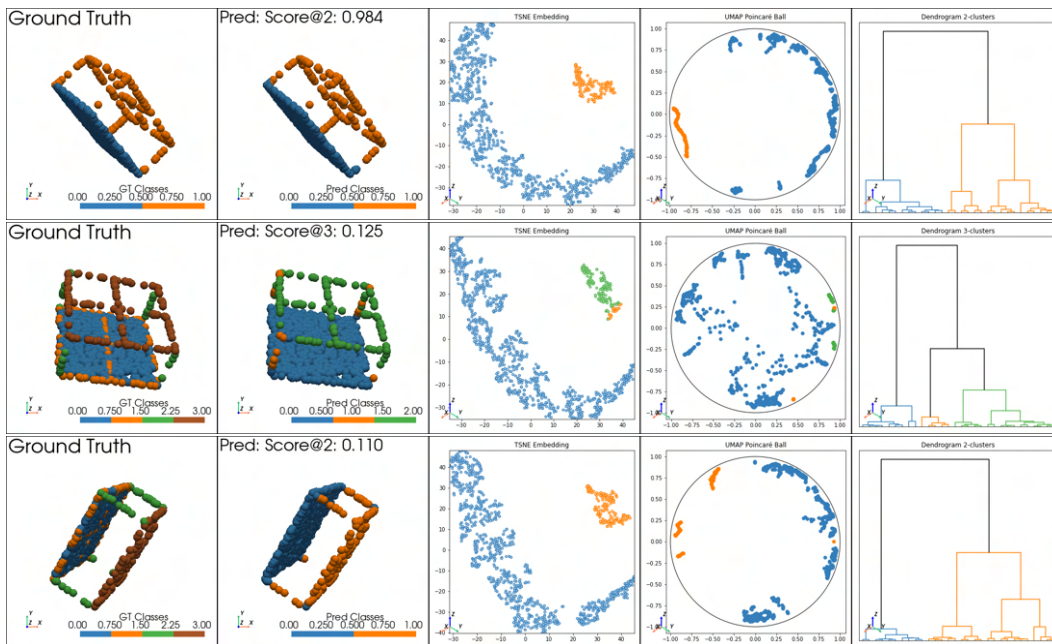


Figure 74. Hierarchical segmentation of 2nd table example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

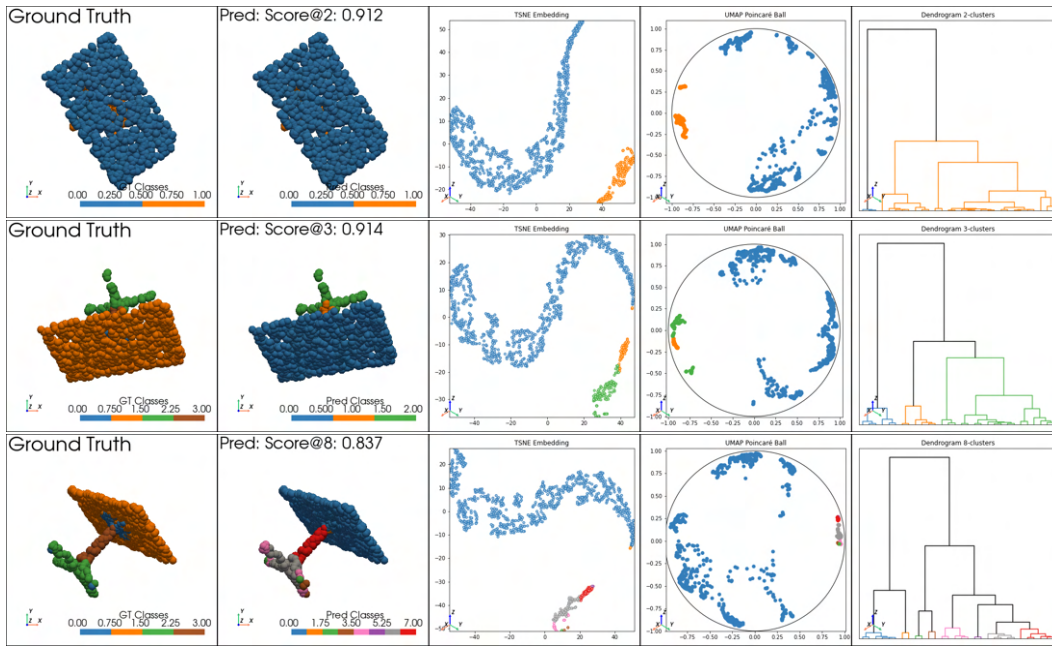


Figure 75. Hierarchical segmentation of 3rd table example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

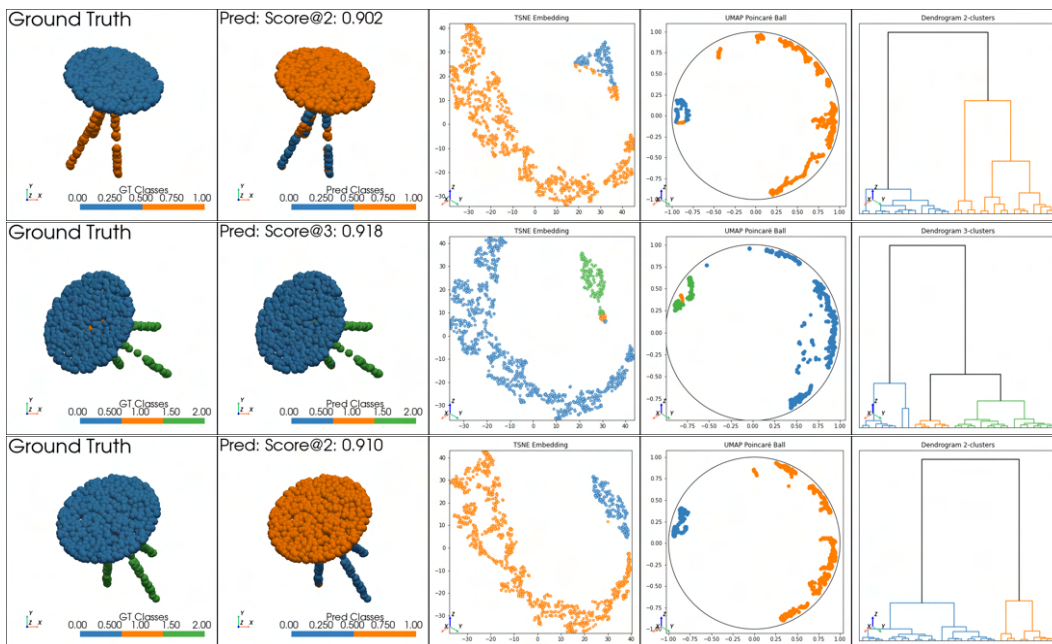
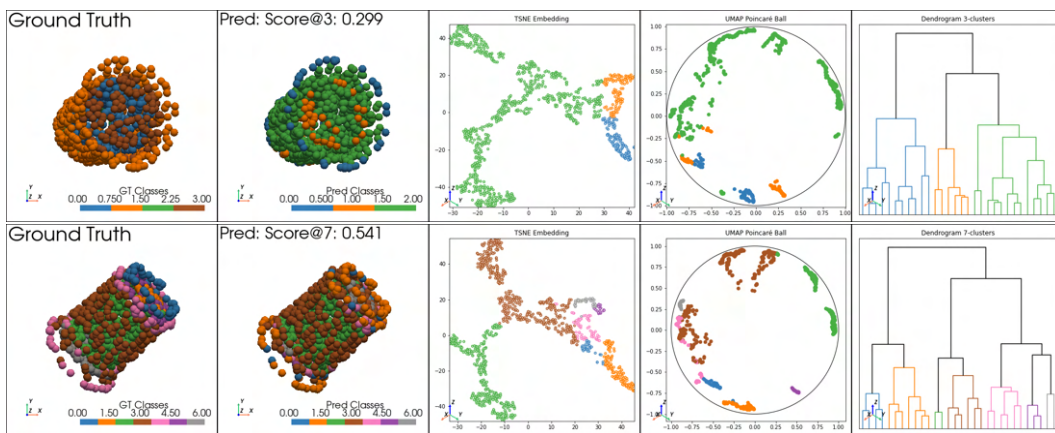
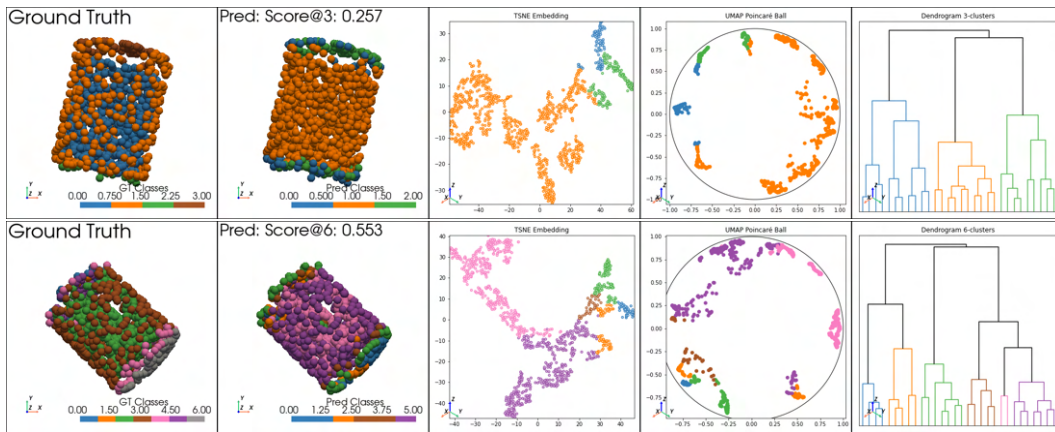
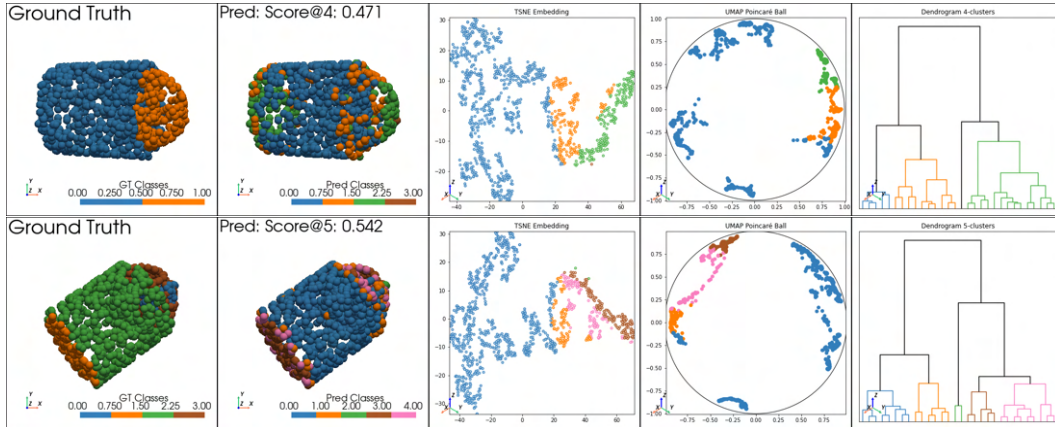


Figure 76. Hierarchical segmentation of 4th table example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

TrashCan



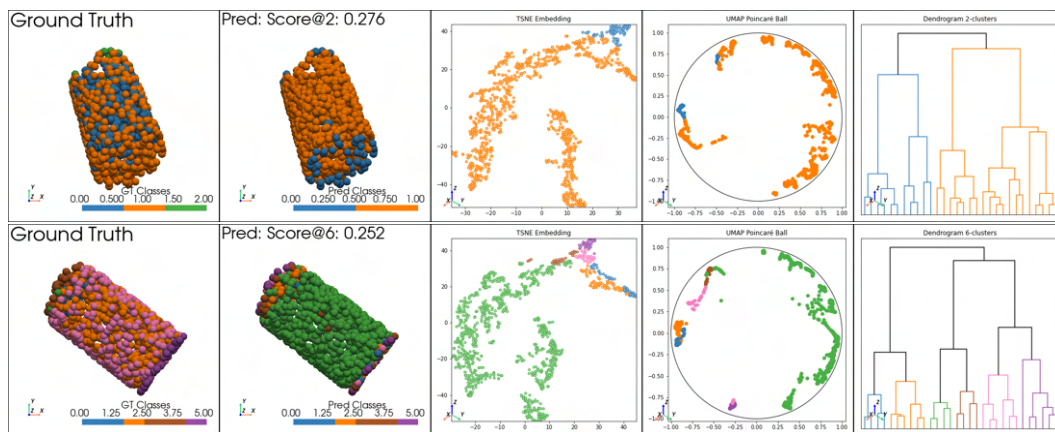


Figure 80. Hierarchical segmentation of 4th trashcan example in PartNet dataset. From top to bottom: coarse- and fine-grained segmentation.

Vase

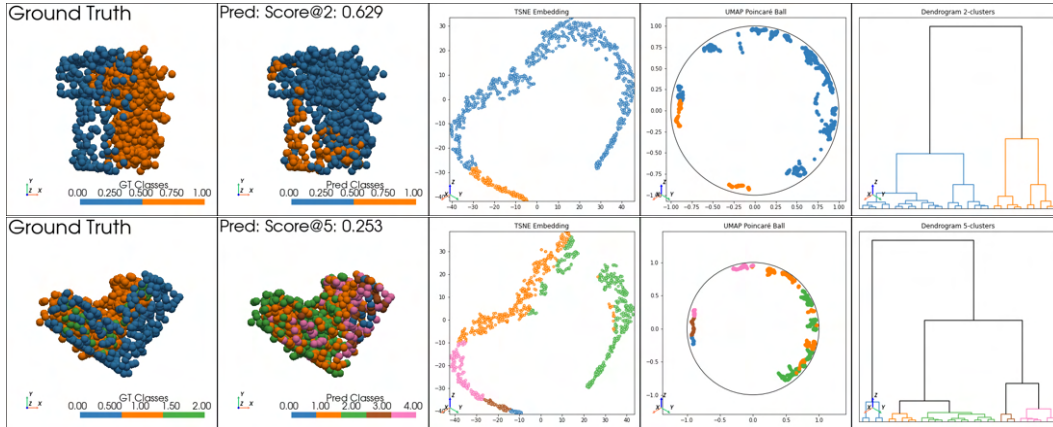


Figure 81. Hierarchical segmentation of 1st vase example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

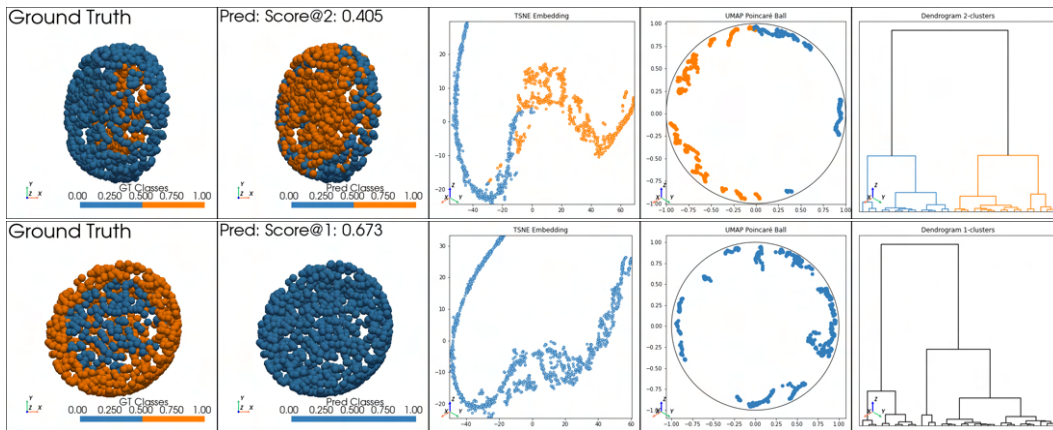


Figure 82. Hierarchical segmentation of 2nd vase example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

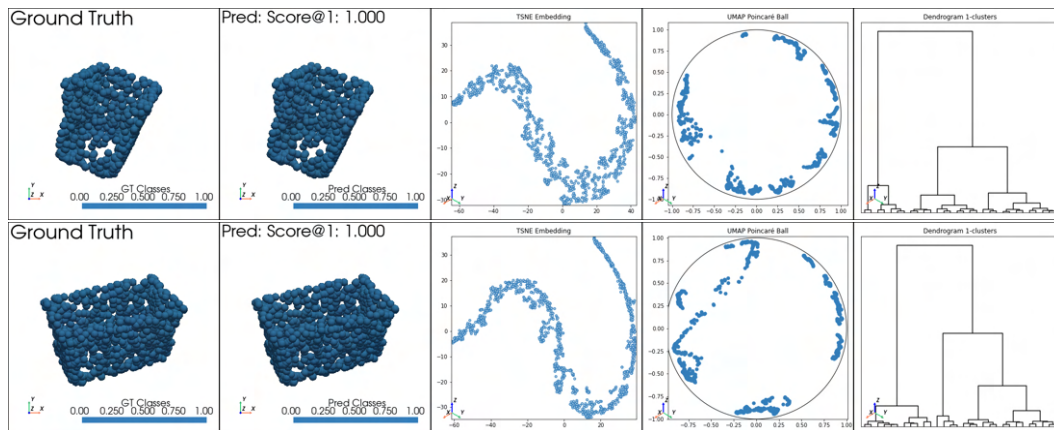


Figure 83. Hierarchical segmentation of 3rd vase example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.

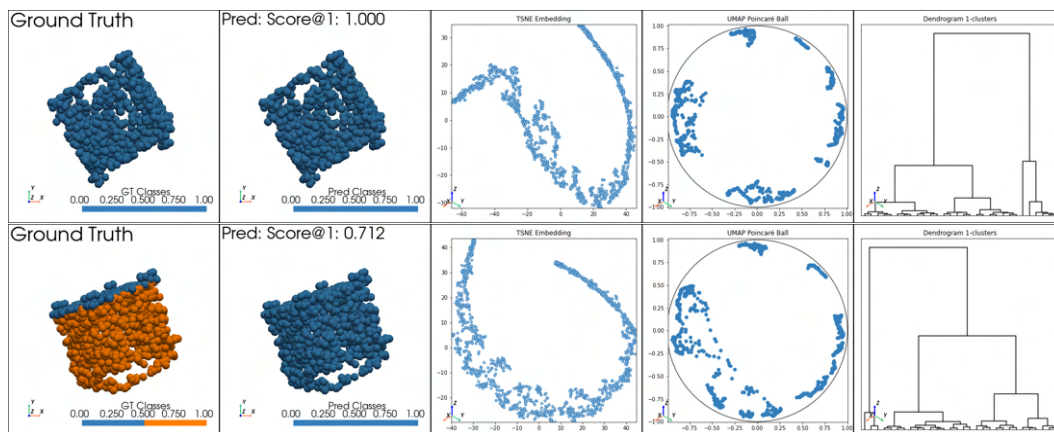


Figure 84. Hierarchical segmentation of 4th vase example in PartNet dataset. From top to bottom: coarse-, middle- and fine-grained segmentation.