# Supplementary Material to

Fast Pose Graph Optimization via Krylov-Schur and Cholesky Factorization

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This document provides additional derivations and benchmarks for the paper *Fast Pose Graph Optimization via Krylov-Schur and Cholesky Factorization*.

## 1 High SNR hypothesis

The log-likelihood function we put forward,

$$f_{\mathrm{ML}} = -\frac{1}{2\sigma_t^2} \sum_{(i,j)\in E} ||\widetilde{t}_{ij} - t_i + \mathbf{R}_i \mathbf{R}_j^\top t_j||^2 + \frac{1}{\sigma_R^2} \sum_{(i,j)\in E} \mathrm{tr}\big(\widetilde{\mathbf{R}}_{ij} \mathbf{R}_j \mathbf{R}_i^\top\big), \tag{1}$$

is in general not separable in the translation and rotation terms. Notwithstanding, provided there is a high Signal-to-Noise Ratio (SNR), an approximate optimum can be obtained by maximizing first the second term on the right side of (1), which is a function of the rotations only, and then using these rotation estimates to compute the approximate optimal translations. In order to show that this is a reasonable approximation we start by rewriting the log-likelihood as a sum of two functions $\psi$ and $\zeta$. The goal is to group together in $\zeta$ all the rotations as follows.

$$\begin{aligned} f_{\mathrm{ML}} &= -\frac{1}{2\sigma_t^2} \sum_{(i,j)\in E} ||\widetilde{t}_{ij} - t_i + \mathbf{R}_i \mathbf{R}_j^\top t_j||^2 + \frac{1}{\sigma_R^2} \sum_{(i,j)\in E} \mathrm{tr}\big(\widetilde{\mathbf{R}}_{ij} \mathbf{R}_j \mathbf{R}_i^\top\big) \\ &= \psi(t) \underbrace{- \frac{1}{\sigma_t^2} \sum_{(i,j)\in E} (\widetilde{t}_{ij} - t_i)^\top \mathbf{R}_i \mathbf{R}_j^\top t_j + \frac{1}{\sigma_R^2} \sum_{(i,j)\in E} \mathrm{tr}\big(\widetilde{\mathbf{R}}_{ij} \mathbf{R}_j \mathbf{R}_i^\top\big)}_{\zeta(\mathbf{R},t)}. \end{aligned} \tag{2}$$

Let $t^*$ and $\mathbf{R}^*$ denote the set of translations and rotations, respectively, that maximize $f_{\mathrm{ML}}$. If we have relative pose measurements with a high SNR, we can expect

$$\forall\ (i,j) \in E\ :\ \widetilde{t}_{ij} \approx t_i^* - \widetilde{\mathbf{R}}_{ij} t_j^* \tag{3}$$

to be a valid approximation. From (3), we can replace $\widetilde{t}_{ij} - t_i^*$ in (2) and rearrange the rotations in the trace of the rightmost term to obtain

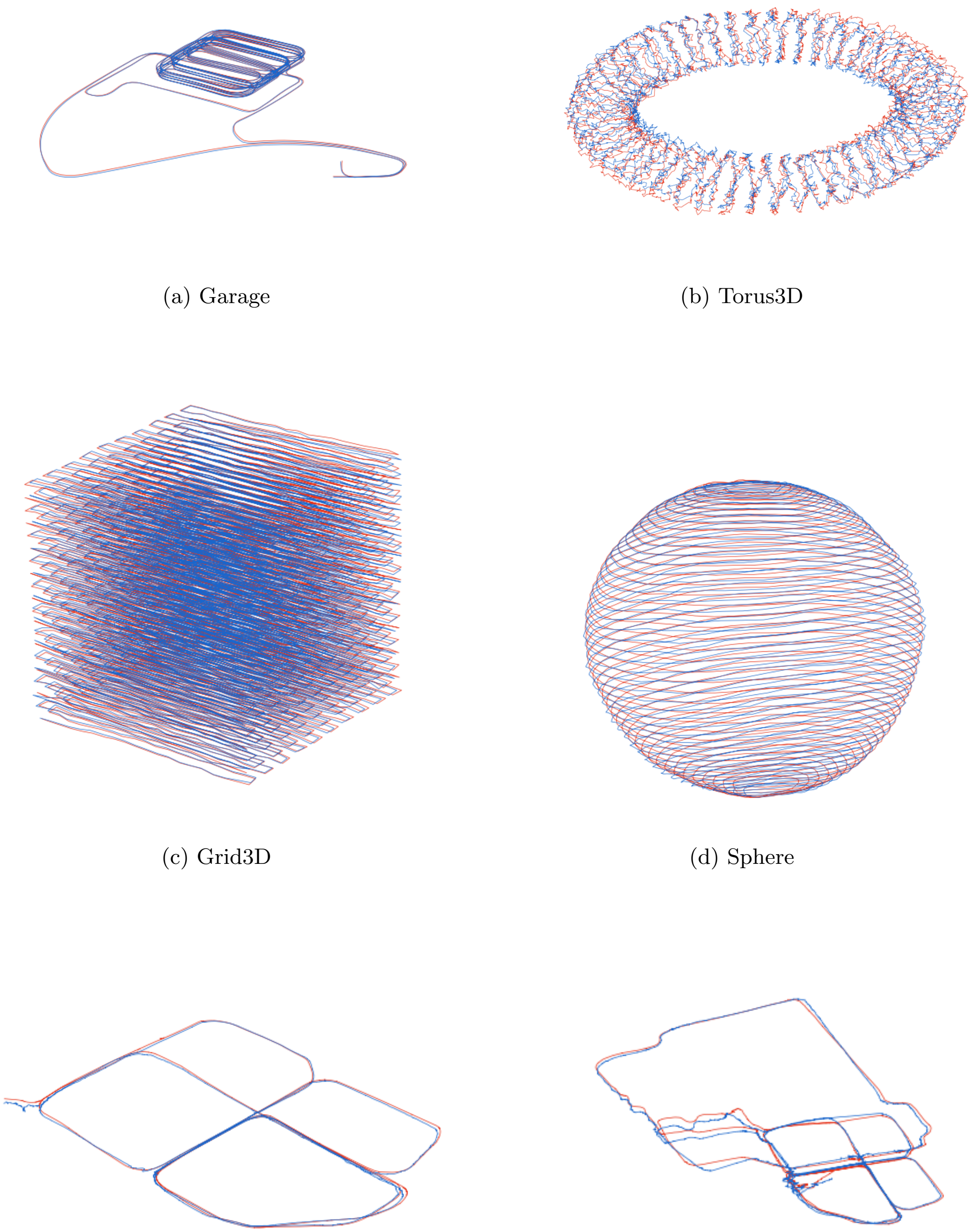
$$\zeta(\mathbf{R}, t^*) \approx \frac{1}{\sigma_t^2} \sum_{(i,j)\in E} \mathrm{tr}\big(t_j^* t_j^{*\top} \widetilde{\mathbf{R}}_{ij}^\top \mathbf{R}_i \mathbf{R}_j^\top\big) + \frac{1}{\sigma_R^2} \sum_{(i,j)\in E} \mathrm{tr}\big(\widetilde{\mathbf{R}}_{ij}^\top \mathbf{R}_i \mathbf{R}_j^\top\big). \tag{4}$$

The matrix $t_j^* t_j^{*\top}$ is symmetric and rank-one. Consequently, under the high SNR hypothesis in (3), the optimization problem

$$\underset{\mathbf{R}\in \mathrm{SO}(3)^n}{\arg\max} \sum_{(i,j)\in E} \mathrm{tr}\big(\widetilde{\mathbf{R}}_{ij} \mathbf{R}_j \mathbf{R}_i^\top\big), \tag{5}$$

will yield a good approximation of the set of rotations that maximize $f_{\mathrm{ML}}$.

## 2 Comparison between MAKS and Gauss-Newton



(a) Garage

(b) Torus3D

(c) Grid3D

(d) Sphere

(e) Cubicle

(f) Rim

Figure 1: Trajectory estimates. Comparison between MAKS (blue) and 10 Gauss-Newton iterations using g2o (red) initialized from MAKS for 6 SLAM datasets by Carlone et al.