6. Appendix

6.1. Proof

6.1.1 Proposition 3.1

Proof. Note that the objective can be expressed as:

$$\min_{\mathbf{M}_{(3)},\mathbf{A}^\top\mathbf{A}=\mathbf{I}} \frac{1}{2}\|(\mathbf{Y}_i)_{(3)} - \mathbf{M}_{(3)}\mathbf{A}\|_F^2,$$

which is equal to find the best K-rand approximation of $(\mathbf{Y}_i)_{(3)}$. Thus, let rank-K SVD of $(\mathbf{Y}_i)_{(3)}$ be $\mathbf{U}$, $\mathbf{S}$ and $\mathbf{V}$, the closed-form solution of (4) in the paper is given by $\mathbf{A}_i = \mathbf{V}$ and $\bar{\mathcal{M}}_i = \text{fold}_3(\mathbf{US})$. □

6.1.2 Proposition 3.2

Proof. Since $\mathcal{Y} = \mathcal{X} + \mathcal{N}$, then

$$\mathcal{Y} \times_3 \mathbf{P} = \mathcal{X} \times_3 \mathbf{P} + \mathcal{N} \times_3 \mathbf{P}, \tag{8}$$

where the noise is given by $\mathcal{N} \times_3 \mathbf{P}$. Note that

$$\text{mean}\left[\mathcal{N} \times_3 \mathbf{P}\right] = \mathbf{0}. \tag{9}$$

Thus, the mean of the noise is zero. Let $\mathbf{a}$ be a column in $\mathbf{N}_{(3)}$, then one column $\mathbf{b}$ in $(\mathcal{N} \times_3 \mathbf{P})_{(3)}$ can be expressed as

$$\mathbf{b} = \mathbf{Pa}. \tag{10}$$

Follow the definition of variance, we have

$$\begin{aligned} \text{var}\left[\mathbf{b}\right] &= \text{mean}\left[(\mathbf{b} - \text{mean}\left[\mathbf{b}\right])^2\right] \\ &= \text{mean}\left[\mathbf{bb}^\top\right] \\ &= \text{mean}\left[\mathbf{a}^\top\mathbf{P}^\top\mathbf{Pa}\right] \\ &= \text{mean}\left[\mathbf{a}^\top\mathbf{a}\right] = \sigma_0\mathbf{I}. \end{aligned}$$

Thus, we obtain the proposition. □

6.2. Extra Experiments Results

Figure 9 and 10 show the color images of PaU [24] (composed of bands 80, 34 and 9) and WDC [45] (composed of bands 190, 60 and 27) before and after denoising.

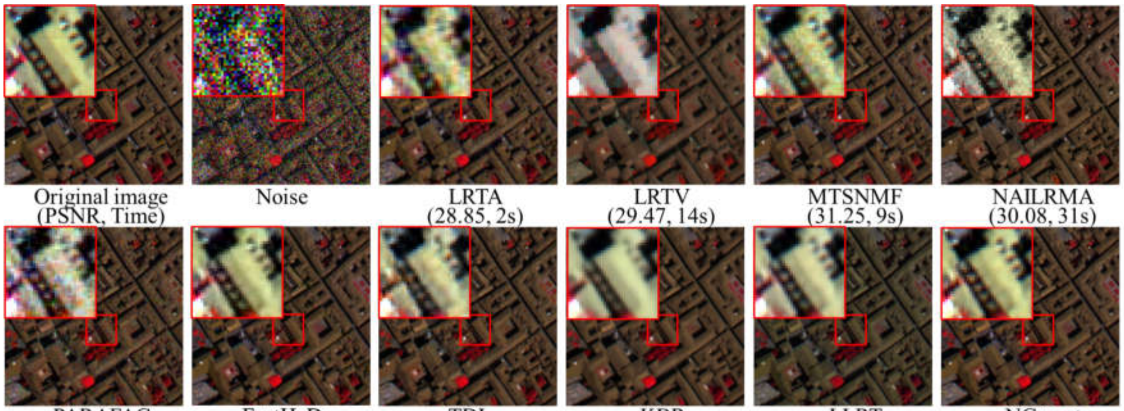

Figure 9. Denoising results on the PaU image with the noise variance 50. The color image is composed of bands 80, 34, and 9 for the red, green, and blue channels, respectively.

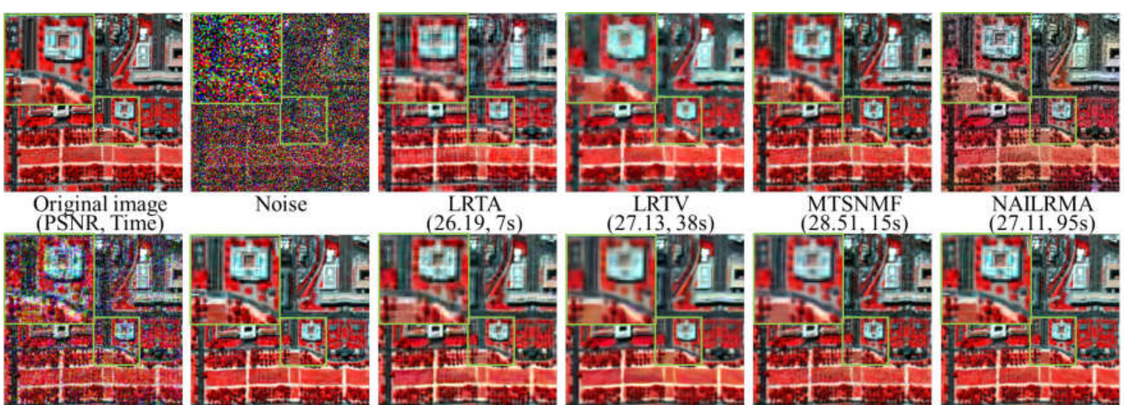

Figure 10. Denoising results on the WDC image with the noise variance 100. The color image is composed of bands 190, 60 and 27 for the red, green, and blue channels, respectively.