Supplementary Material for "AdaBits: Neural Network Quantization with Adaptive Bit-Widths"

1. Proof of Theorem 1

Theorem For any x in $[0, 1]$ and any two positive integers $a > b$,

$$\lfloor 2^a x \rfloor >> (a - b) = \lfloor 2^b x \rfloor \tag{S1}$$

Proof We need to prove that

$$\left\lfloor \frac{\lfloor 2^a x \rfloor}{2^{a-b}} \right\rfloor = \lfloor 2^b x \rfloor \tag{S2}$$

We first notice that

$$\frac{\lfloor 2^a x \rfloor}{2^{a-b}} \leq \frac{2^a x}{2^{a-b}} = 2^b x \tag{S3}$$

so we have

$$\left\lfloor \frac{\lfloor 2^a x \rfloor}{2^{a-b}} \right\rfloor \leq \lfloor 2^b x \rfloor \tag{S4}$$

due to the non-decreasing monotonicity of the floor function. At the same time, we have

$$\begin{aligned} \frac{\lfloor 2^a x \rfloor}{2^{a-b}} &= \frac{\lfloor 2^{a-b} \cdot 2^b x \rfloor}{2^{a-b}} && \text{(S5a)} \\ &\geq \frac{\lfloor 2^{a-b} \lfloor 2^b x \rfloor \rfloor}{2^{a-b}} && \text{(S5b)} \\ &= \frac{2^{a-b} \lfloor 2^b x \rfloor}{2^{a-b}} && \text{(S5c)} \\ &= \lfloor 2^b x \rfloor && \text{(S5d)} \end{aligned}$$

where in the penultimate equality we have used the fact that $a > b$. Thus we have

$$\left\lfloor \frac{\lfloor 2^a x \rfloor}{2^{a-b}} \right\rfloor \geq \lfloor 2^b x \rfloor \tag{S6}$$

From (S4) and (S6), we can get the desired result (S1).