A Proof of Corollary 1

Proof. A sample $\langle \mathbf{x}^0, \mathbf{y}^0 \rangle$ follows A2, and we have its corresponding \mathbf{x}_l^0 . Since $\epsilon \ge d(\mathbf{x}^0, \mathbf{x}_l^0)$ (as required by the Corollary), \mathbf{x}_l^0 is one of the perturbed examples that will be tested by Equation 2. Therefore, for model θ :

- If $\mathbf{y}^0 \neq f(\mathbf{x}^0; \theta)$, Equation 1 will be evaluated as 0, thus not accurate.
- If $\mathbf{y}^0 = f(\mathbf{x}^0; \theta)$, then $\mathbf{y}^0 \neq f(\mathbf{x}^0_l; \theta)$ (because of A2). Equation 2 will be evaluated as 0 (because \mathbf{x}^0_l is one of the perturbed examples, as mentioned above), thus not robust.

B Extra Empirical Results

B.1 Colorful Images

We also experimented with a subset of ImageNet, which was constructed by merging related images into nine major classes, leading to a nine-class classification dataset with roughly 1350 images per class. We used a ResNet-50 for the experiments.

We repeated all the experiments with the the above experiment set-up and show the results. Overall, the results demonstrate a similar result.

B.1.1 Rethinking Data before Rethinking Generalization

LFC			HFC			
r	train acc.	test acc.	r	train acc.	test acc.	
25	0.9315	0.5477	25	0.8730	0.1908	
50	0.9681	0.6477	50	0.7655	0.1515	
100	0.9759	0.8300	100	0.6476	0.1531	
150	0.9717	0.8415	150	0.2416	0.1123	

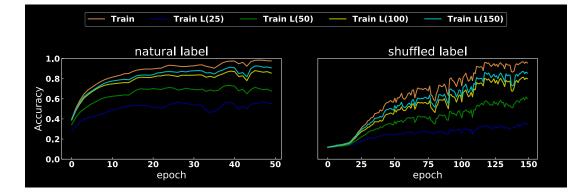


Table 3: table1

Figure 10: Training curves of the original label case (50 epoches) and shuffled label case (150 epoches), together plotted with the low-frequent counterpart of the images. All curves in this figure are from train samples.

B.1.2 Heuristics

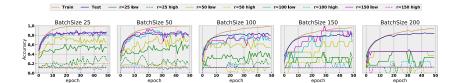


Figure 11: Plots of accuracy of different batch sizes along the epoches for train

— Train — Test —	- r=25 low r=25 high	r=50 low r=50 high r=	100 low r=100 high r=	=150 low r=150 high		
BatchSize 100	Dropout	Mix-up	BatchNorm	Adversarial Training		
	K MANA MA	Antomation				
	RIMMAN	Min				
0.2 A. A. V. V. V. V.	m Magazza V	a stranger		¥		
0.0 0 10 20 30 40 50 epoch						

Figure 12: Plots of accuracy of different heuristics along the epoches for train, test data, as well as LFC and HFC with different radii.

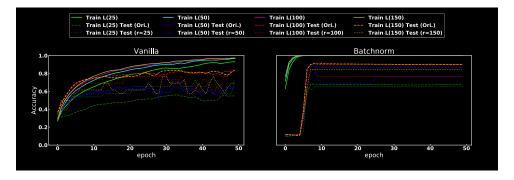
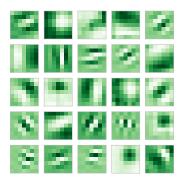


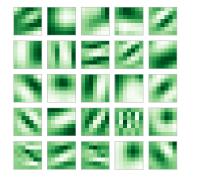
Figure 13: Comparison of models with vs. without BatchNorm trained with LFC data.

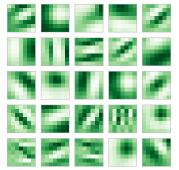
2		1		4
	N.	ų	*	
	-		-82	
۴	-	4	99	L.
8	÷	÷	٠	ð



(a) convoluational kernels of M_{natural}

(b) convoluational kernels of Madversarial





(c) convoluational kernels of $M_{natural}(r=1.0)$ (d) convoluational kernels of $M_{adversarial}(r=1.0)$

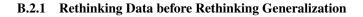
Figure 14: Visualization of convolutional kernels (64 kernels each channel \times 3 channels at the first layer) of models.

	Clean	FGSM		PGD			
	Clean	$\epsilon = 0.03$	$\epsilon = 0.06$	$\epsilon = 0.09$	$\epsilon = 0.03$	$\epsilon = 0.06$	$\epsilon = 0.09$
M _{natural}	0.821	0.004	0.002	0.002	0.010	0.011	0.013
$M_{natural}(\rho = 0.10)$	0.813	0.009	0.005	0.003	0.015	0.010	0.007
$M_{natural}(\rho = 0.25)$	0.786	0.016	0.010	0.008	0.020	0.016	0.012
$M_{natural}(\rho = 0.50)$	0.767	0.119	0.119	0.118	0.127	0.127	0.125
$M_{natural}(\rho = 1.0)$	0.760	0.370	0.368	0.368	0.382	0.382	0.378
Madversarial	0.684	0.122	0.033	0.012	0.034	0.038	0.029
$M_{adversarial}(\rho = 0.10)$	0.651	0.134	0.050	0.030	0.043	0.037	0.033
$M_{adversarial}(\rho = 0.25)$	0.613	0.15	0.08	0.07	0.032	0.031	0.019
$M_{adversarial}(\rho = 0.50)$	0.603	0.189	0.126	0.114	0.060	0.047	0.036
$M_{adversarial}(\rho = 1.0)$	0.590	0.241	0.184	0.164	0.086	0.084	0.081

Table 4: Prediction performance of models against different adversarial attacks with different ϵ .

B.2 Greyscale Images

We also repeat the major results with the greycale images (MNIST and FashionMNIST).



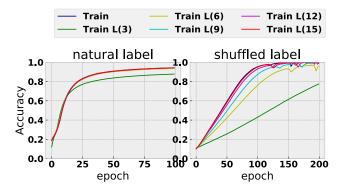


Figure 15: Experiments of natural label and shuffled labeled over MNIST data set

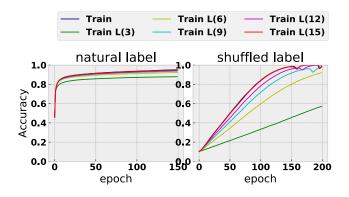


Figure 16: Experiments of natural label and shuffled labeled over FashionMNIST data set

B.2.2 Heuristics

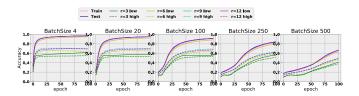


Figure 17: Experiments of batchsize over MNIST data set

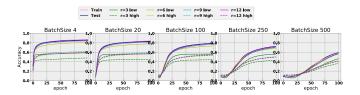


Figure 18: Experiments of batchsize over FashionMNIST data set

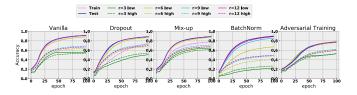


Figure 19: Experiments of heuristics for MNIST data set.

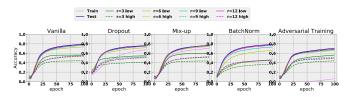


Figure 20: Experiments of heuristics for FashionMNIST data set.

B.2.3 Adversarial Attack & Defense



Figure 21: Convolutional Kernel of the models trained over MNSIT

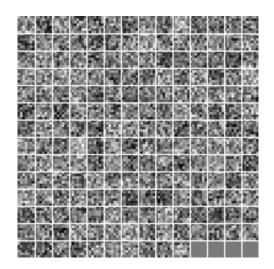


Figure 22: Convolutional Kernel of the models trained over FashionMNSIT