Unsupervised learning of object landmarks by factorized spatial embeddings

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Abstract

Learning automatically the structure of object categories remains an important open problem in computer vision. In this paper, we propose a novel unsupervised approach that can discover and learn landmarks in object categories, thus characterizing their structure. Our approach is based on factorizing image deformations, as induced by a viewpoint change or an object deformation, by learning a deep neural network that detects landmarks consistently with such visual effects. Furthermore, we show that the learned landmarks establish meaningful correspondences between different object instances in a category without having to impose this requirement explicitly. We assess the method qualitatively on a variety of object types, natural and man-made. We also show that our unsupervised landmarks are highly predictive of manually-annotated landmarks in face benchmark datasets, and can be used to regress these with a high degree of accuracy.

1. Introduction

The appearance of objects in images depends strongly not only on their intrinsic properties such as shape and material, but also on accidental factors such as viewpoint and illumination. Thus, learning from images about objects as intrinsic physical entities is extremely difficult, particularly if no supervision is provided.

Despite these difficulties, the performance of object detection algorithms has been rising steadily, and deep neural networks now achieve excellent results on benchmarks such as PASCAL VOC [17] and Microsoft COCO [39]. Still, it is unclear whether these models conceptualise objects as intrinsic entities. Early object detectors such as HOG [13] and DPMs [18] were based on 2D templates applied in a translation and scale invariant manner to images. Recent detectors such as SSD [42] make this even more extreme and learn different templates (filters) for different scales and even different aspect ratios of objects. Hence, these models are likely to capture objects as image-based phenomena, representing them as a collection of weakly-related 2D patterns.

Achieving a deeper understanding of objects requires modeling their intrinsic viewpoint-independent structure. Often this structure is defined manually by specifying entities such as landmarks, parts, and skeletons. Given sufficient manual annotations, it is possible to teach deep neural networks and other models to recognize such structures in images. However, the problem of learning such structures without manual supervision remains largely open.

In this paper, we contribute a new approach to learn viewpoint-independent representations of objects from images without manual supervision (fig. 1). We formulate this task as a factorization problem, where the effects of image deformations, for example arising from a viewpoint change, are explained by the motion of a reference frame attached to the object and independent of the viewpoint.

After describing the general principle (sec. 3.1), we in-
vestigate a particular instantiation of it. In this model, the
structure of an object is expressed as a set of landmark
points (sec. 3.2) detected by a neural network. Differently
from traditional keypoint detectors, however, the network
is learned without manual supervision. Learning considers
pairs of images related by a warp and requires the detector’s
output to be equivariant with the transformation (sec. 3.3).
Transformations could be induced by real-world viewpoint
changes or object deformations, but we show that meaning-
ful landmarks can be learned even by considering random
perturbations only.

We show that this method works for individual rigid and
deformable object instances (sec. 3.1.1) as well as for object
categories (sec. 3.1.2). This only requires learning a sin-
gle neural network to detect the same set of landmarks for
images containing different object instances of a category.
While there is no explicit constraint that forces landmarks
for different instances to align, we show that, in practice,
this tends to occur automatically.

The method is tested qualitatively on a variety of differ-
ent object types, including shoes, animals, and human faces
(sec. 4). We also show that the unsupervised landmarks are
highly predictive of manually-annotated landmarks, and as
such can be used to detect these with a high degree of ac-
curacy. In this manner, our method can also be used for
unsupervised pretraining of semantic landmark detectors.

2. Related work

Flow. Matching images up to a motion-induced deforma-
tion links back to the work of Horn and Schunck [26] on
optical flow and to deep learning approaches for its computa-
tion [21, 57, 28]. Flow can also be defined semantically
rather than geometrically [40, 32, 46, 77, 76]. While our
method also establishes geometric and (indirectly) semantic
correspondences, it goes beyond that by learning a single
set of viewpoint independent landmarks which are valid for
all images at once.

Parts. A traditional method to describe the structure of
objects is to decompose them into their constituent parts.
Several unsupervised methods to learn parts exist, from the
constellation approach used in [19, 9, 62] to the Deformable
Parts Model (DPM) [18] and many others. More recently,
AnchorNet [48] successfully learns parts that match differ-
ent object instances as well as different object categories
using only image-level supervision; furthermore, they pro-
pose a part orthogonality constraint similar to our own.
While the concepts of landmarks and parts are similar, our
training method differs substantially from these approaches:
rather than learning parts as a byproduct of learning a (de-
formable) discriminator, our landmark points are trained to
fit geometric deformations directly.

neural network that, given two images, predicts a Thin Plate
Spline (TPS [6]) that aligns them. While our landmarks
can also be seen as a representation of transformations (as
matching them between image pairs induces one), learning
such landmarks is unique to our method. The Deep De-
formation Network of [69] predicts image transformations
to refine landmarks using a “Point Transformer Network”,
but their landmarks are learned using full manual supervi-
sion, whereas our method is fully unsupervised. Very
recently [53] learn a neural network that also aligns two im-
ages by estimating the transformation between them, im-
plcitly learning feature extractors that could be similar to
keypoints; however, our work explicitly trains a network
to output keypoints that are equivariant to such transforma-
tions.

Landmark detection. There is an extensive literature
on landmark detectors, particularly for faces. Examples
include Active Appearance Models [11], along with
subsequent improvements [44, 12] and others using tem-
plates [51] or parts [80]. Other approaches directly regress
the landmark coordinates [59, 14, 10, 52]. Deep learning
methods use cascaded CNNs [56], coarse-to-fine autoen-
coders [70], auxiliary attribute prediction [73, 74], learned
deformations [69] and LSTMs [64]. Beyond faces, there is
work on humans [65, 58], birds [55, 41, 69] and furniture
[63]. More general pose estimation including the case of
landmarks is explored in [16]. Our method can build on any
such detector architecture and can be used as a pretraining
strategy to learn landmarks with less or no supervision.

Equivariance constraint. A variant of the equivariance
constraint used by our method was proposed by [37] to learn
feature point detectors for image matching. We build on a
similar principle, but use it to learn intrinsic landmarks for
object categories instead of generic SIFT-like features with
a robust learning objective and learn to detect a set of com-
plementary landmarks rather than a single one at a time.

Unsupervised pretraining. Unsupervised pretraining has
received significant interest with the popularization of data-
hungry deep networks [5, 24, 23]. Unsupervised learn-
ing is based on training a network to solve auxiliary tasks,
for which supervision can be obtained without manual an-
notations. The most common of such tasks is to gener-
ate the data (autoencoders [7, 4, 25]); or one can remove
some information in images and train a network to recon-
struct it (denoising [60], ordering patches [15, 47], inpaint-
ing [50], analyzing motion [1, 49, 61, 20, 45], and coloriz-
ing [71, 35]). Our method can be seen in this light as trying
to undo a synthetic deformation applied to an image.

Our method is also related to unsupervised learning for
faces, such as alignment based on a face model [78], learn-
ing meaningful descriptors [67, 22], and learning a part
model [38]. Huang et al. [27] learn joint alignment of faces
using deep features, and Jaiswal et al. [29] use clustering
to discover head modes in order to refine manually-defined
landmarks in an unsupervised manner, both using genera-
The method developed above extends essentially with no modification to deformable objects. Suppose that the surface \( S \) deforms between images according to isomorphisms \( w : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \). We tie the shape variants \( wS = \{w(p) : p \in S\} \) together by introducing a common reference space \( S_0 \), which we call an object frame. Barring topological changes, we can establish isomorphisms \( \pi_S \) mapping reference points \( r \in S_0 \) to fixed surface points \( \pi_S(r) \in S \), in the sense that \( \forall w : w(\pi_S(r)) = \pi_w(\pi_S(r)) \). Then, by using the substitution \( \Phi(r; x) = \Phi_S(\pi_S(r); x) \), we can rewrite the equivariance constraint (2) as

\[
\forall r \in S_0 : \Phi(r; x \circ g) = g(\Phi(r; x)). \tag{3}
\]

This simply states that one expects surface points to be detected equivariantly with viewpoint-induced deformations as well as with deformations of the object surface.

3.1.2 Object categories

In addition to deformable objects, our formulation can easily account for shape variations between object instances in the same category. To do this, one simply makes the assumption that all object surfaces \( S \) are isomorphic to the same reference shape \( S_0 \) (fig. 2).

Differently from the case of deformable objects, geometry alone does not force the mappings \( \pi_S \) for different object indices. None of these methods learns landmarks from scratch.

### 3. Method

Sec. 3.1 introduces the method of viewpoint factorization for learning an intrinsic reference frame for object instances and categories. Then, sec. 3.2 applies it to learn object landmarks and sec. 3.3 discusses the details of the learning formulation.

#### 3.1. Structure from viewpoint factorization

Let \( S \subset \mathbb{R}^3 \) be the surface of a physical object, say a bird, and let \( x : \Lambda \rightarrow \mathbb{R} \) be an image of the object, where \( \Lambda \subset \mathbb{R}^2 \) is the image domain (fig. 2). The surface \( S \) is an intrinsic property of the object, independent of the particular image \( x \) and of the corresponding viewpoint. We consider the problem of learning a function \( q = \Phi_S(p; x) \) that maps object points \( p \in S \) to the corresponding pixels \( q \in \Lambda \) in the image.

We propose a new method to learn \( \Phi_S \) automatically through a process of viewpoint factorization. To this end, consider a second image \( x' \) of the object seen from a different viewpoint. Occlusion notwithstanding, one can write \( x' \approx x \circ g \) where \( g : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) is the image warp induced by the viewpoint change. Using the map \( \Phi_S \), the warp \( g \) can be factorised as follows:

\[
g = \Phi_S(\cdot; x') \circ \Phi_S(\cdot; x)^{-1}. \tag{1}
\]

In other words, we can decompose the warp \( q : q \mapsto q' \) as first finding the intrinsic object point \( p = \Phi_S^{-1}(q; x) \) corresponding to pixel \( q \) in image \( x \) and then finding the corresponding pixel \( q' = \Phi_S(p; x') \) in image \( x' \).

The factorization eq. (1) is more conveniently expressed as the following equivariance constraint:

\[
\forall p \in S : \Phi_S(p; x \circ g) = g(\Phi_S(p; x)). \tag{2}
\]

This constraint simply states that the points \( p \) must be detected in a manner which is consistent with a viewpoint change.

In order to learn the map \( \Phi_S \), we express the latter as a deep neural network and train it to satisfy constraint (2) in a Siamese configuration, supplying triplets \((x, x', g)\) to the learning process. Note that, if we are given two views \( x \) and \( x' \) of the same object, the viewpoint transformation \( g \) is often unknown. Instead of trying to recover \( g \), inspired by [30], we propose to synthesize transformations \( g \) at random and use them to generate \( x' \) from \( x \). While this approach only uses unannotated images of the object, it can still learn meaningful landmarks (sec. 4).

### Discussion

While learning only considers deformations of the same image, the model still learns to bridge automatically across moderately different viewpoints (see fig. 5). However we leave very large out-of-plane rotations, which would require to handle partial occlusions of the landmarks, to future work.

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1. If \( x \) and \( x' \) are given but \( g \) is unknown, one can rewrite eq. (2) by expressing the warp \( g \) as a function of the predicted landmarks (as the solution of the equation \( \forall p : \Phi_S(p; x') = g\Phi_S(p; x) \)), and then by measuring the alignment quality in appearance space as \( \|x' - xo g\| \). However, this approach provides a weaker supervisory signal and is somewhat more complex to implement.
instances $S$ to be related. Nevertheless, we would like to choose such mappings to be semantically consistent; for example, if $\pi_S(r)$ is the right eye of face $S$, then we would like $\pi_{S'}(r)$ to be the right eye of face $S'$. An important contribution of this work is to show that semantically-meaningful correspondences emerge automatically by simply sharing the same learned mapping $\Phi$ between all object instances in a given category. The idea is that, by learning a single rule that detects object points consistently with deformations, these points tend to align between different object instances as this is the smoothest solution.

3.2. Landmark detection networks

In this section we instantiate concretely the method of sec. 3.1. First, one needs to decide how to represent the maps $\Phi(x) : S_0 \rightarrow \Lambda$ as the output of a neural network or other computational model. Our approach is to sample this function at a set of $K$ discrete reference locations $\Phi(x) = (\Phi(r_1; x), \ldots, \Phi(r_K; x))$. In this manner, the function $\Phi(x)$ can be thought of as detecting the location $p_k = \Phi(r_k; x)$ of $K$ object landmarks. We do not attach particular constraints to the set of landmarks, which can be thought of as an index set $r_k = k, k = 1, 2, \ldots, K$.

If $\Phi$ is implemented as a neural network, one can use any of the existing architectures for keypoint detection (sec. 2). Most such architectures are based on estimating score maps $\Psi(x) \in \mathbb{R}^{H \times W \times K}$, associating a score $\Psi(x)_{uk}$ to each landmark $r_k$ and image location $u \in \{1, \ldots, H\} \times \{1, \ldots, W\} \subset \mathbb{R}^2$. The score maps can be transformed into probability maps by using the softmax operator $\sigma$:

$$p(u|x, r) = \sigma[\Psi(x)]_{ur} = \frac{e^{\Psi(x)_{ur}}}{\sum_v e^{\Psi(x)_{vr}}}.$$ 

Following [56], it is then possible to extract a landmark location by using the soft argmax operator, which computes the expected value of this density:

$$u^*_r = \sigma[\Psi(x)]_r = \sum_u p(u|x, r) = \frac{\sum_u u e^{\Psi(x)_{ur}}}{\sum_v e^{\Psi(x)_{vr}}}.$$ 

The overall network, computing the location of the $K$ landmarks, can then be expressed as

$$\Phi(x) = \sigma[\Psi(x)].$$ 

3.3. Learning formulation

In this section, we show how the equivariance constraint (3) can be used to learn $\Phi$ from examples. The idea is to setup the learning problem as a Siamese configuration, in which the output of $\Phi$ on two images $x$ and $x'$ is assessed for compatibility with respect to the deformation $g$ and the equivariance constraint (3). We can express this condition as the loss term:

$$L_{\text{align}} = \frac{1}{K} \sum_{r=1}^{K} ||\Phi(x \circ g)_r - g(\Phi(x)_r)||^2.$$ 

In the rest of the section, we discuss two extensions to eq. (5) that allow the system to train better landmarks: formulating the loss directly in terms of the keypoint probabilities and adding a diversity term.

**Probability maps loss.** Equation (5) uses the soft argmax operator in order to localise and then compare landmarks. We show here that one can skip this step by writing a loss directly in terms of the probability maps, which provides a more direct and stable gradient signal. The idea is to replace eq. (5) with the loss term

$$L'_{\text{align}} = \frac{1}{K} \sum_{r=1}^{K} \sum_{uv} ||u - g(v)||^2 p(u|x, r)p(v|x', r)$$ 

where $p(u|x, r) = \sigma[\Psi(x)]_{ur}$ and $p(v|x', r) = \sigma[\Psi(x')]_{vr}$ are the landmark probability maps extracted from images $x$ and $x'$.

Minimizing loss (6) has two desirable effects. First, it encourages the two probability maps to overlap and, second, it encourages them to be highly concentrated. In fact, the loss is zero if, and only if, both $p$ and $q$ are delta functions and if the corresponding landmark locations match up to $g$.

While a naive implementation of (6) requires to visit all pairs of pixels $u$ and $v$ in both images, with a quadratic complexity, a linear-time implementation is possible by decomposing the loss as:

$$\sum_u ||u||^2 p(u|x, r) + \sum_v ||g(v)||^2 p(v|x', r)$$ 

$$- 2 \left( \sum_u p(u|x, r) \right)^\top \cdot \left( \sum_v g(v)p(v|x', r) \right).$$
Diversity loss. The equivariance constraint eq. (3) and its corresponding losses eqs. (5) and (6) ensure that the network learns at least one landmark aligned with image deformations. However, there is nothing to prevent the network from learning $K$ identical copies of the same landmark.

In order to avoid this degenerate solution, we add a diversity loss that requires probability maps of different landmarks to fire in different parts of the image. The most obvious approach is to penalize the mutual overlap between maps for different landmarks $r$ and $r'$:

$$
\mathcal{L}_{\text{div}}(x) = \frac{1}{K^2} \sum_{r=1}^{K} \sum_{r'=1}^{K} \sum_{u} p(u|x, r)p(u|x, r').
$$

(7)

This term is zero only if, and only if, the support of the different probability maps is disjoint.

The disadvantage of this approach is that it is quadratic in the number of landmarks. An alternative and more efficient diversity loss is:

$$
\mathcal{L}'_{\text{div}}(x) = \sum_{u} \left( \sum_{r=1}^{K} p(u|x, r) - \max_{r=1, \ldots, K} p(u|x, r) \right).
$$

(8)

Just like eq. (7), this loss is zero only if the support of the distributions is disjoint. In fact the sum of probability values at a given point $u$ is always greater than the max unless all but one probability are zero. Note that we can rewrite (8) more compactly as:

$$
\mathcal{L}'_{\text{div}}(x) = K - \sum_{u} \max_{r=1, \ldots, K} p(u|x, r).
$$

In practice, we found it beneficial to apply the diversity loss after downsampling (by $m \times m$ sum pooling) the probability maps as this encourages landmarks to be extracted farther apart. Thus we consider:

$$
\mathcal{L}''_{\text{div}}(x) = \sum_{u} \max_{r=1, \ldots, K} \sum_{\delta_u} p(mu + \delta_u, x, r).
$$

where $\delta_u \in \{0, \ldots, m - 1\}^2$.

Learning objective. The learning objective considers triplets $(x_i, x'_i, g_i)$ of images $x_i$ and $x'_i$ related by a viewpoint warp $g_i$ and optimizes:

$$
\min_{\psi} \lambda \mathcal{R}(\psi) + \frac{1}{N} \sum_{i=1}^{N} \left( \mathcal{L}'_{\text{align}}(x_i, x'_i, g_i; \psi) + \gamma \mathcal{L}''_{\text{div}}(x_i; \psi) + \gamma \mathcal{L}''_{\text{div}}(x'_i; \psi) \right),
$$

(9)

where $\mathcal{R}$ is a regulariser (weight shrinkage for a neural network). As noted before, if triplets are not available, they can be synthesized by applying a random transformation $g_i$ to an image $x_i$ to obtain $x'_i = x_i \circ g$. Note that all functions are easily differentiable for backpropagation.

4. Experiments

In this section, we first describe the implementation details (sec. 4.1) and then report both qualitative (sec. 4.2) and quantitative (sec. 4.3) results demonstrating the power of our unsupervised landmark learning method.

4.1. Implementation details

In all the experiments, the detector $\Phi$ contains six convolutional layers with 20, 48, 64, 80, 256, $K$ filters respectively, where $K$ is the number of object landmarks. Each convolutional layer is followed by a batch normalization and a ReLU layer. This network is proposed in [74] for supervised facial keypoint estimation. Differently, instead of downsampling the feature map after each convolutional layer, we use only one $2 \times 2$ max pooling layer with a stride of 2 after the first convolutional layer (conv1). Thus, given an input size of $H \times W \times 3$, the network outputs an $H/2 \times W/2 \times K$ feature map. We apply a spatial softmax operator to the output of the last convolutional layer to obtain $K$ probability maps, one for each landmark.

During training, we supply a set of triplets of $(x_i, x'_i, g_i)$ as input to the network. In order to generate them, given an example image $I$, one can naively sample a random TPS and warp the image accordingly. However, as the input images are typically centered and at most very slightly rotated, the learned weights can be biased towards such a setting. Instead, we randomly sample two TPS transformations $(g_1, g_2)$ and consecutively warp the given image to generate an image pair i.e. $x = I \circ g_1$ and $x' = x \circ g_2$ (computed using inverse image warping as $x \circ (g_2 \circ g_1)$). The TPS warps are parametrized as in [6] which can be decomposed into affine and deformation parts. To render realistic and diverse warps, we randomly sample scale, rotation angle and translation parameters within the pre-determined ranges. Examples of the transformations are shown in figs. 3 to 5.

We initialize the weights of convolutions with random gaussian noise and optimize the objective function (eq. (9)) (weight decay $\lambda = 5 \cdot 10^{-4}$, $\gamma = 500$) by using Adam [33] with an initial learning rate $10^{-4}$ until convergence, then reduce it by one tenth until no further improvement is seen.

4.2. Qualitative results

We train our unsupervised landmarks from scratch on three different domains: shoes (fig. 3), cat faces (fig. 4), and faces (fig. 5), and assess them qualitatively. We train landmark detectors on 49525 shoes from the UT Zappos50k dataset of [68] and 8609 images from the cat heads dataset of [72] and keep the rest for validation. Facial landmarks are learned on the CelebA dataset [43] which contains more than 200k celebrity images for 10k identities with 5 annotated landmarks. We use the provided cropped face images, which are roughly centered and scaled to the same size.

We train an 8 or 10 landmark network for each of the tasks to allow for clearer visualization. In addition, we show
Figure 3. Unsupervised landmarks on shoes (8 landmark network). Top: synthetic TPS deformations (original image leftmost). Bottom: different instances. Note that landmarks are consistently detected despite the significant variation in pose, shape, materials, etc.

Figure 4. Unsupervised landmarks on cat faces (10 landmark network). Top-left quintuple: synthetic deformations (original image leftmost) transformed by rotation (images 2,3) and TPS warps (images 4,5). Remaining examples: different instances.

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Table 1. Results on MAFL test set in terms of the inter-ocular distance as in [74, 52]. For each setting, \( n \) unsupervised landmarks, that is learned on the CelebA training set, are regressed into 5 manually-defined landmarks. The regressor is learnt on CelebA or MAFL training set.

<table>
<thead>
<tr>
<th>( n ) landmarks</th>
<th>Regressor training</th>
<th>Mean error</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>MAFL</td>
<td>7.95</td>
</tr>
<tr>
<td>30</td>
<td>MAFL</td>
<td>7.15</td>
</tr>
<tr>
<td>50</td>
<td>MAFL</td>
<td>6.67</td>
</tr>
<tr>
<td>10</td>
<td>CelebA</td>
<td>6.32</td>
</tr>
<tr>
<td>30</td>
<td>CelebA</td>
<td>5.76</td>
</tr>
<tr>
<td>50</td>
<td>CelebA</td>
<td>5.33</td>
</tr>
</tbody>
</table>

Table 2. Comparison to state-of-the-art supervised landmark detectors on MAFL.

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>TCDCN [74]</td>
<td>7.95</td>
</tr>
<tr>
<td>Cascaded CNN [56]</td>
<td>9.73</td>
</tr>
<tr>
<td>CFAN [70]</td>
<td>15.84</td>
</tr>
<tr>
<td>Our Method (50 points)</td>
<td>6.67</td>
</tr>
</tbody>
</table>

Table 3. Comparison to state-of-the-art supervised landmark detectors on AFLW (5 pts) in terms of inter-ocular distance.

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCPR [8]</td>
<td>11.6</td>
</tr>
<tr>
<td>Cascaded CNN [56]</td>
<td>8.97</td>
</tr>
<tr>
<td>CFAN [70]</td>
<td>10.94</td>
</tr>
<tr>
<td>TCDCN [74]</td>
<td>7.65</td>
</tr>
<tr>
<td>RAR [64]</td>
<td>7.23</td>
</tr>
<tr>
<td>Our Method (51 points)</td>
<td>10.53</td>
</tr>
</tbody>
</table>

4.3. Quantitative results

In this section we evaluate the performance of our unsupervised landmarks quantitatively by testing how well they correlate with and predict manually-labelled landmarks. To do this, we consider standard facial landmark benchmarks containing manual annotations for semantic landmarks (e.g. eyes, corner of the mouth, etc). We first learn a detector for \( K \) landmarks without supervision, freeze its weights, and
then use the supervised training data in the benchmark to learn a linear regressor mapping the unsupervised landmark to the manually defined ones. The regressor takes as input the $2K$ coordinates of the unsupervised landmarks, stacks them in a vector $x \in \mathbb{R}^{2K}$, and maps the latter to the corresponding coordinates of the manually-defined landmarks as $y = W x$. Learning $W$ can be seen as a fully connected layer with no bias, and is trained similarly to the unsupervised network, using our warps as data augmentation. Note that there is no backpropagation to the unsupervised weights, which remain fixed. $W$ is visualized in fig. 7.

**Benchmark data.** We first report results on the MAFL dataset [74], a subset of CelebA with 19k training images and 1k test images annotated with 5 facial landmarks (corners of mouth, eyes and nose). We follow the standard evaluation procedure in [74] and report errors in inter-ocular distance (IOD) in table 1. Since the MAFL test set and the CelebA training set overlap partially, we remove the MAFL test images from CelebA when the latter is used for training.

We also consider the more challenging 300-W dataset [54] containing 68 landmarks, obtained by merging and re-annotating other benchmarks. We follow [52] and use 3148 images from AFW [80], LFPW-train [3] and Helen-train [75] as training set, and 689 images from IBUG, LFPW-test and Helen-test as test set.

Finally we use the AFLW [34] dataset, which contains
24,386 faces from Flickr. Although it contains up to 21 annotated landmarks, we follow [74, 64] in only evaluating five and testing on the same 2995 faces cropped and distributed in the MFL set of [73]. For training we use 10,122 faces that have all five points labelled and whose images are not in the test set.

**MAFL results.** First, we train the unsupervised landmarks on the CelebA training set and learn a corresponding regressor on the MAFL training set. The accuracy of the regressor on the MAFL test data is reported in Table 1 and qualitative results are shown in fig. 6.

Regressing from \( K = 10, 30, 50 \) unsupervised landmarks improves the results. This can be explained by the fact that more unsupervised landmarks means a higher chance of finding some highly correlated with the five manually-labelled ones and thus a more robust mapping (fig. 7). This can also increase accuracy since our landmarks are detected with a resolution of two pixels (due to the downsampling in the network). Table 2 compares these results to state-of-the-art fully supervised landmark localization methods. Encouragingly, our best regressor outperforms the supervised methods (6.67 error rate vs 7.95 of TCDCN [74]). This shows that our unsupervised training method is indeed able to find meaningful landmarks.

Next, in Table 4 we assess how many manual landmark annotations are required to learn the regressor. We consider the problem of regressing from \( K = 30 \) unsupervised landmarks and we observe that the regressor performs well even if only 10 or 20 images are considered (errors 8.5 and 8.06). By comparison, using all 19,000 training samples reduces the error to 7.15, which shows that most of the required information is contained in the unsupervised landmarks from the outset. This indicates that our method is very effective for unsupervised pretraining of manually annotated landmarks as well, and can be used to learn good semantic landmarks with few annotations.

**300-W results.** We use our best performing model, the 50 point network, trained unsupervised on CelebA, and report results in Table 5 for two settings. In the first one, the unsupervised landmarks are learned on CelebA and only the regressor is learned on the 300-W training set; we obtain an error of 9.30. In the second setting, the unsupervised detector is fine-tuned (also without supervision) on the 300-W data to adapt the features to the target dataset. The fine-tuning lowers the error to 7.97 and yields a comparable result with the state-of-the-art supervised methods. This shows another strength of our method: our unsupervised learner can be used to adapt an existing network to new datasets, also without using labels.

**AFLW results.** Due to tighter face crops, we adapt our 50-landmark CelebA network, fine-tuning it first on similarly cropped CelebA images and then on the AFLW training set. The adapted network has 51 landmarks. We compare against other methods in Table 3. Once more, landmarks linearly-regressed from the unsupervised ones are competitive with fully supervised detectors (10.53 vs 7.23). The regressor can be trained with as low as 1 or 5 labelled images almost saturating performance (errors 14.79 and 12.94 respectively). By comparison, the same architecture trained supervised from scratch using 5 and 10 labelled images with TPS data augmentation but no unsupervised pretraining has substantially higher 23.85 and 22.31 errors (achieved essentially by predicting the average landmark locations which has error 24.40).

We also visualize what the regressor learns and which of the source (discovered) landmarks contribute to the target (semantic) ones in fig. 7. To do so, for each target landmark, we take the corresponding column of the regressor, compute the absolute value of its coefficients, \( \ell^1 \) normalize it, remove the entries smaller than 0.2. We show this mapping as a directional graph with arrows between the target landmarks (green circular nodes) and the source ones (colored crosses). We observe that the contributions are proportional to the distance between source and target points. In addition, the landmark on the forehead, not in the convex hull of the target points is ignored, as expected.

**5. Conclusions**

In this paper we have presented a novel approach to learn the structure of objects in an unsupervised manner. Our key contribution is to reduce this problem to the one of learning landmark detectors that are equivariant, i.e. compatible, with image deformations. This can be seen as a particular instantiation of the more general idea of factorizing deformations by learning an intrinsic reference frame for the object. We have shown that this technique works for rigid and deformable objects as well as object categories, it results in landmarks highly-predictive of manually annotated ones, and can be used effectively for pretraining.

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