# Appendix

## A. Proof of the Optimal Discriminator

We derive the optimal discriminator in Euqation 5 by minimizing the $V_{\text{LSGAN}}(D)$ in Euqation 4 as follows:

$$\begin{aligned} V(D) =& \frac{1}{2}\mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}\big[(D(\boldsymbol{x})-b)^2\big] + \frac{1}{2}\mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}}\big[(D(G(\boldsymbol{z}))-a)^2\big] \\ =& \frac{1}{2}\mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}\big[(D(\boldsymbol{x})-b)^2\big] + \frac{1}{2}\mathbb{E}_{\boldsymbol{x}\sim p_{g}}\big[(D(\boldsymbol{x})-a)^2\big] \\ =& \int_{\mathcal{X}} \frac{1}{2}\big(p_{\text{data}}(\boldsymbol{x})(D(\boldsymbol{x})-b)^2 + p_g(\boldsymbol{x})(D(\boldsymbol{x})-a)^2\big)\mathrm{d}x. \end{aligned}$$

Consider the internal function:

$$\frac{1}{2}\big(p_{\text{data}}(\boldsymbol{x})(D(\boldsymbol{x})-b)^2 + p_g(\boldsymbol{x})(D(\boldsymbol{x})-a)^2\big),$$

It achieves the mimimum at $\frac{bp_{\text{data}}(\boldsymbol{x})+ap_g(\boldsymbol{x})}{p_{\text{data}}(\boldsymbol{x})+p_g(\boldsymbol{x})}$ with respect to $D(\boldsymbol{x})$, which is the definition of the optimal discriminator in Equation 5.