Efficient Low Rank Tensor Ring Completion Supplementary Material

Wenqi Wang, Vaneet Aggarwal
Purdue University, West Lafayette, IN
{wang2041, vaneet } @purdue.edu

Shuchin Aeron
Tufts University, Medford, MA
shuchin@ece.tufts.edu

1. Supplementary Material

1.1. Proof of Lemma 1

Proof. Based on definition of tensor permutation in Definition 6, on the left hand side, the $(j_1, ..., j_n)$ entry of the tensor is

$$\mathcal{X}^{P_i}(j_1, ..., j_n) = \mathcal{X}(j_{n-i+2}, ..., j_n, j_1, ..., j_{n-i+1}). \tag{1}$$

On the right hand side, the $(j_1, ..., j_n)$ entry of the tensor gives

$$\begin{aligned} & f(\mathcal{U}_i \cdots \mathcal{U}_{i-1})(j_1, \cdots, j_n) \\ =& \text{Trace}(\mathcal{U}_i(:, j_1, :)\mathcal{U}_{i+1}(:, j_2, :)...\mathcal{U}_n(:, j_{n-i+1}, :) \\ & \mathcal{U}_1(:, j_{n-i+2}, :) \cdots \mathcal{U}_{i-1}(:, j_n, 1)). \end{aligned} \tag{2}$$

Since trace is invariant under cyclic permutations, we have

$$\begin{aligned} & \text{Trace}(\mathcal{U}_i(:, j_1, :)\mathcal{U}_{i+1}(:, j_2, :)...\mathcal{U}_n(:, j_{n-i+1}, :) \\ & \mathcal{U}_1(:, j_{n-i+2}, :) \cdots \mathcal{U}_{i-1}(:, j_n, :)) \\ =& \text{Trace}(\mathcal{U}_1(:, j_{n-i+2}, :) \cdots \mathcal{U}_{i-1}(:, j_n, :) \\ & \mathcal{U}_i(:, j_1, :)\mathcal{U}_{i+1}(:, j_2, :)...\mathcal{U}_n(:, j_{n-i+1}, :)) \\ =& f(\mathcal{U}_1 \cdots \mathcal{U}_n)(j_{n-i+2}, \cdots, j_n, j_1, \cdots, j_{n-i+1}), \end{aligned} \tag{3}$$

which equals to the right hand side of equation (1). Since any entries in $\mathcal{X}^{P_i}$ are the same as those in $\mathcal{U}_i\mathcal{U}_{i+1} \cdots \mathcal{U}_n\mathcal{U}_1 \cdots \mathcal{U}_{i-1}$, the claim is proved. □

1.2. Proof of Lemma 2

Proof. First we note that tensor permutation does not change tensor Frobenius norm as all the entries remain the same as those before the permutation. In Lemma 2, we have

$$\mathcal{U}_i = \text{argmin}_{\mathcal{Y}} \|\mathcal{P}_\Omega \circ f(\mathcal{U}_1 \cdots \mathcal{U}_{i-1}\mathcal{Y}\mathcal{U}_{i+1} \cdots \mathcal{U}_n) - \mathcal{X}_\Omega)\|_F^2. \tag{4}$$

Since the permutation operation does not change the Frobenius norm, equivalently we have

$$\mathcal{U}_i = \text{argmin}_{\mathcal{Y}} \|\mathcal{P}_\Omega^{P_i} \circ (f(\mathcal{U}_1 \cdots \mathcal{U}_{i-1}\mathcal{Y}\mathcal{U}_{i+1} \cdots \mathcal{U}_n))^{P_i} - \mathcal{X}_\Omega^{P_i}\|_F^2. \tag{5}$$

Based on Lemma 1, we have

$$(f(\mathcal{U}_1 \cdots \mathcal{U}_{i-1}\mathcal{Y}\mathcal{U}_{i+1} \cdots \mathcal{U}_n))^{P_i} = f(\mathcal{Y}\mathcal{U}_{i+1} \cdots \mathcal{U}_n\mathcal{U}_1 \cdots \mathcal{U}_{i-1}), \tag{6}$$

thus equation (5) becomes

$$\mathcal{U}_i = \text{argmin}_{\mathcal{Y}} \|\mathcal{P}_\Omega^{P_i} \circ f(\mathcal{Y}\mathcal{U}_{i+1} \cdots \mathcal{U}_n\mathcal{U}_1 \cdots \mathcal{U}_{i-1}) - \mathcal{X}_\Omega^{P_i}\|_F^2. \tag{7}$$

Thus we prove our claim. □