Learning an event sequence embedding for dense event-based deep stereo

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Abstract

Today, a frame-based camera is the sensor of choice for machine vision applications. However, these cameras, originally developed for acquisition of static images rather than for sensing of dynamic uncontrolled visual environments, suffer from high power consumption, data rate, latency and low dynamic range.

An event-based image sensor addresses these drawbacks by mimicking a biological retina. Instead of measuring the intensity of every pixel in a fixed time interval, it reports events of significant pixel intensity changes. Every such event is represented by its position, sign of change, and timestamp, accurate to the microsecond.

Asynchronous event sequences require special handling, since traditional algorithms work only with synchronous, spatially gridded data. To address this problem we introduce a new module for event sequence embedding, for use in different applications. The module builds a representation of an event sequence by firstly aggregating information locally across time, using a novel fully-connected layer for an irregularly sampled continuous domain, and then across discrete spatial domain. Based on this module, we design a deep learning-based stereo method for event-based cameras. The proposed method is the first learning-based stereo method for an event-based camera and the only method that produces dense results. We show large performance increases on the Multi Vehicle Stereo Event Camera Dataset (MVSEC), which became the standard set for the benchmarking of event-based stereo methods.

1. Introduction

Stereo matching is the problem of finding for every point in an image taken from one viewpoint its physically corresponding one in an image taken from another viewpoint. Given the parameters of a stereo camera setup, the matching results allow to compute the 3d structure of a scene, with many applications, *e.g.* in robotics [29], medical imaging [32], remote sensing [47], or computational photography [55, 2].

1.1. Deep stereo for frame-based cameras

Currently, most successful stereo matching methods are based on deep learning. First successes of deep learning in stereo matching were achieved by replacing individual algorithmic elements in legacy methods (often [28, 16]) with neural networks, *e.g.* similarity metric [62, 24, 51, 61, 8], smoothness penalty [45, 21], matching confidence [46] and disparity post-processing [13].

Current works solve the stereo matching by training a neural network end-to-end, which combines embedding, matching, regularization, and sometimes refinement modules in a single model [10, 27, 20, 63, 36, 19, 23, 7, 52, 59, 50]. An embedding module computes image descriptors for left and right images, a matching module performs a correlation [10, 27, 36, 19, 23, 59, 50], computes matching signatures [52] or simply concatenates [20, 7, 63] left and shifted right descriptors for every disparity. The regularization module, implemented as an hourglass network with shortcut connections between the contracting and the expanding parts and 2d [27, 10, 36, 23] or 3d [20, 52, 63, 19, 7] convolutions, enforces stereo matching constraints and computes disparities or a distribution over disparities. Finally, some methods [36, 23, 19] have a refinement module, that improves the initial low-resolution disparity relying on left-right warping error.

Best results are obtained with fully-supervised training on large synthetic datasets with ground truth [27] and an L^1 or cross-entropy [52] loss, while some methods use weakly supervised settings [63, 39, 59], relying on geometric constraints of the task. Most recent methods [59, 50] further

Table 1. Comparison of event-based image sensor, such as [3] to a frame-based sensor. The numbers show orders of magnitude for every characteristic, rather than precise values. Advantages of event-based sensors are highlighted.

Characteristic	Frame-based	Event-based
Dynamic range, [dB]	50	130
Power consumption, [W]	1	0.01
Data rate, [Mb/s]	100	0.1
Latency, [ms]	10	0.001
Resolution, [MP]	1	0.01
Intensity information	✓	✗

improve results using multi-task learning.

Many stereo methods exist for frame-base cameras, however, stereo matching with novel bio-inspired event-based sensors is a relatively new area of research, with many interesting research challenges.

1.2. Event-based sensors

Conventional frame-based camera sensors capture stroboscopic sequences of still pictures at a fixed time interval or frame rate. In contrast, the retina in the human eye operates on completely different principles. Nobel prize wining experiments [18] showed that the retina is most sensitive to temporal brightness gradients, and is blind to static scenes in absence of eye movements [41]. These principles inspired the development of event-based image sensors [49, 3].

In an event-based image sensor, pixels are sensitive to temporal brightness contrast and trigger binary events with a rate proportional to the temporal gradient of photocurrent. Events can have positive or negative polarity depending on the sign of the gradient (*i.e.* "dark to bright" or "bright to dark"). Triggered events are recorded from the sensor in an asynchronous manner with information about their spatial positions, polarities, and timestamps accurate to microseconds.

Event-based image sensors have several advantages over frame-based sensors. First, they have higher dynamic range and thus do not saturate in extreme lighting conditions, such as bright daylight and night with minimum illumination, thanks to pixel-wise gain and integration time control. Secondly, they have lower power consumption and data rates, since they only transmit information about significant brightness changes. This enables use in power-constrained systems. Finally, due to immediate transmission of every triggered event with a microsecond-accurate timestamp, event-based sensors have lower latency and can be used in time-critical applications. However, these advantages come at a cost. Event-based sensors have lower resolution, because their pixels are more complex, and do not provide rich intensity information. A comparison between frame-based and event-based sensors is shown in Table 1.

Unique properties of event-based image sensors make them attractive for low-latency dynamic vision applications in environments with uncontrolled illumination, such as tracking [12], robot control [25], or object recognition [48]. Depth estimation, that we investigate in this work, could drastically improve performance in these applications, and open the way to novel use cases, such as augmented reality.

1.3. Event-based stereo

Due to the novelty of event-based image sensors, only a few event-based stereo methods have been proposed, none of which is learning-based.

One line of research investigates how to represent and compare events. This is a hard problem because events have few spatio-temporal neighbors and only one binary feature. Early methods [22, 42] compared events using only their timestamps which led to matching ambiguities, due to the noise, variable cameras sensitivity, and imperfect camera synchronization. Therefore, later methods relied on hand-crafted descriptors [4, 68, 69] and similarity measures [44, 67, 38]. In [4] descriptors are computed as a bank of orientation-sensitive spatial filter responses, in [68] as a vector of distances to closest events in several spatial directions, in [69] as a histogram of orientations of vectors pointing to the closest events in a spatial window. As for similarity measures, spatial windows with event are compared in [44] using average distances to the closest events, in [38] using average inverse timestamp difference between corresponding events, and in [67] using intersection-over-union of events.

The second research direction explores how to apply regularization in stereo matching. This is a challenging problem due to the sparsity of data in both time and space, which consequently cannot be represented with conventional Markov Random Field (MRF) models. Some works [37, 11, 38] adopt heuristic cooperative regularization from [26] by defining a spatio-temporal inhibitory and excitatory neighborhood for each event, while others [56, 57] use belief propagation and semi-global matching on sparse MRF models, were nodes are active only during a fixed interval after receiving an event.

The third line of research explores how to accumulate events over time to cope with the fact of individual events being noisy and not very informative, though leading to the undesirable effect of blurring object boundaries when using long accumulation times. Most methods use fixed accumulation intervals [22, 42, 4, 44], while [69] sets accumulation time equal to the average of the inverse of the event rate, and [67] warps event positions as if they were all triggered at the same time using depth hypothesis and known camera motion.

Another challenging problem is to perform dense stereo matching using sparse event data. While most of the methods produce sparse disparity, the work [64] reconstructs semi-dense disparities by fusing information from several view-points using a known camera motion. In [69] a disparity is computed at every location without an event by fitting a plane to its neighbor disparities.

Finally, a variety of works is dedicated to implement and perform stereo matching on neuromorphic chips and field-programmable gate arrays (FPGA) [1, 9].

All existing methods use hand-crafted event representations and grid-based image models, which support only simplistic priors, such as smoothness. Meanwhile, most successful frame-based stereo methods use learning-based representations [61] and regularization based on deep networks, which are able to perform area-based regularization [20] and even use monocular depth cues [14]. This motivates our work: an end-to-end deep learning model for event-based stereo matching.

1.4. Deep learning with event sequences

Event sequences can be processed using convolutional neural networks (CNNs), recurrent neural networks (RNNs) and specialized networks for event data.

One option is to use off-the-shelf CNNs, that are successful in frame-based image processing. However, one problem is that convolutional modules work with dense images, where each pixel lies in a 2d or 3d (in case of video) discrete space, with an intensity value assigned to it. In contrast, an event sequence is a sparse number of 3d points, with two discrete spatial dimensions, one continuous temporal dimension, and with a binary variable (polarity) as a feature. Therefore, such a sequence needs to be transformed to a frame-based representation before it can be input to a standard CNN. Note, however, that naive binning of the time dimension is problematic since it would produce tensors with a prohibitively large temporal dimensions.

Existing methods [34, 25, 30, 53, 66, 60] use hand-crafted transformations to convert event sequences to frame-based representations, that we call *event images*. For example, [34] saves the polarity of the last event, [30] sums event polarities in every location during a predetermined time interval, and [25] counts the number of positive and negative events to avoid information loss due to polarity cancellation. To preserve time information, in addition to positive and negative event counts, [66] stores the timestamp of the last positive and negative events at every location, while [60] saves the average timestamps of the updates. To capture time dynamic, [53, 60] stack several event images described earlier, for consecutive time intervals. The main drawback of all these representations is that they loose precise timing information about events.

Another seemingly natural choice, given the sequential nature of the data, is to use RNNs. An application to even-based recognition with RNNs and long-term memory is [33]. That model has the drawback that it does not preserve the spatial information, which is a crucial ingredient for example to stereo matching. Note, that convolutional RNNs, such as [58], preserve spatial information, but are not applicable for the same reason as CNNs.

Finally, one can use asynchronous networks, where every neuron has an internal state that is updated by events, such as Spiked Neural Networks (SNN) [6] or specially designed convolutional neural networks [5, 35]. Unfortunately, it is hard to build and train such networks, because they are not easily differentiable, and additionally difficult to implement in a standard framework that enable the use of available computational back-ends such as GPUs.

1.5. Contribution

The contributions of this paper are the following:

1. We propose a learnable representation (embedding) for event sequences that, explicitly treats sequences as stream of sparse 3d points with two discrete spatial and one continuous temporal coordinate. It takes into account both spatial positions and accurate timeing information of all recorded events.

2. We use this embedding to design the first deep learning-based method for event-based stereo reconstruction. The method is based on an architecture with large receptive field that uses large context and allows stereo reconstruction for locations without events. We demonstrate that this method significantly outperforms other competing approaches on the Multi Vehicle Stereo Event Camera Dataset (MVSEC).

2. Method

Let the left and right event sequences be E^l, E^r, each consisting of n events sorted by the time of arrival $E = ((x_i, y_i, t_i, p_i) \mid t_{i+1} > t_i)_{i=1\dots n}$. Each event is a point in a three dimensional space with two discrete spatial coordinates and one continuous temporal coordinate $(x, y, t) \in [0 \dots w) \times [0 \dots h) \times \mathbb{R}$ and one polarity feature $p \in \{-1, 1\}$, where w and h correspond to the width and height of an image sensor. Given both E^l, E^r, the network computes an *estimated disparity* tensor $\widehat{\mathbf{D}}$ as

$$\widehat{\mathbf{D}} = \text{Net}(E^l, E^r \mid \Theta, d_{max}) \in [0, d_{max}]^{h \times w}, \quad (1)$$

where Θ is the tensor of network parameters and d_{max} is a maximum disparity. An element of the disparity tensor $\widehat{D}_{y,x}$ specifies that a pixel with coordinates (x, y) from the left camera matches the pixel with coordinates $(x - \widehat{D}_{y,x}, y)$ from the right camera. In §2.1 we describe the proposed network architecture for event-based stereo matching, and in §2.2 explain the novel embedding for event sequences.

2.1. Network architecture

Our architecture is inspired by recent stereo networks for frame-based cameras, which use large image contexts and produce disparities with sub-pixel accuracy [52, 20]. It consists of *embedding*, *matching*, *regularization* modules, followed by an *estimator*. The embedding module takes as input an event sequence E and computes its descriptor $\mathbf{F}$ of size $c \times \frac{h}{4} \times \frac{w}{4}$. The same module is applied to the left and right event sequence independently. This approach will be described in detail in §2.2. The matching module for each disparity then takes the left descriptor and shifted right descriptor and computes a *matching signature* of size $\frac{c}{8} \times \frac{h}{4} \times \frac{w}{4}$. All matching signatures for all disparities are concatenated to a 4d tensor of size $\frac{c}{8} \times \frac{d_{max}}{4} \times \frac{h}{4} \times \frac{w}{4}$, which are then passed to the regularization module. The regularization module is an hourglass neural network with 3d convolutions and shortcut connections between the contracting and the expanding parts. It produces a *matching cost* tensor $\mathbf{C}$ of size $\frac{d_{max}}{2} \times h \times w$ (matching costs are computed only for even disparities to save space), passed to the sub-pixel estimator [52] which produces an estimated disparity tensor $\widehat{\mathbf{D}}$ as

$$\widehat{D}_{y,x} = \sum_j d(j) \cdot \underset{j:|\hat{j}-j|\leq\delta}{\text{softmin}} \left(C_{j,y,x}\right)$$
$$\text{with } \hat{j} = \arg\min_j \left(C_{j,y,x}\right), \quad (2)$$

where $\Delta = 2$ is an *estimator support* and $d(j) = 2 \cdot j$ is a disparity, corresponding to index j in the matching cost tensor. More details about the network architecture are described in the supplementary material.

Network training uses a sub-pixel cross entropy loss [52]

$$L(\Theta) = \frac{1}{wh} \sum_{y,x} \sum_j \text{Laplace}(d(j) \mid \mu = D^{GT}_{y,x}, b) \times$$
$$\log(\underset{j}{\text{softmin}}(C_{j,y,x})), \quad (3)$$

where $\text{Laplace}(d \mid \mu = D^{GT}_{y,x}, b)$ is a discretized and normalized Laplace probability density function over disparities with mean equal to the ground truth disparity $\mu = D^{GT}_{y,x}$ and diversity $b = 2$.

2.2. Events sequence embedding

In this work, we focus on a special family of embedding functions that can be represented as a composition $f_S(f_\tau(\cdot))$ of two functions: *temporal aggregation* $f_\tau(\cdot)$ and *spatial aggregation* $f_S(\cdot)$. The temporal aggregation function is defined per-location and it takes a *local event sequence* $E(x,y) = ((x_i, y_i, t_i, p_i) \in E \mid (x_i, y_i) = (x,y))$ as input, and produces a *event image* $\mathbf{I}$ of size

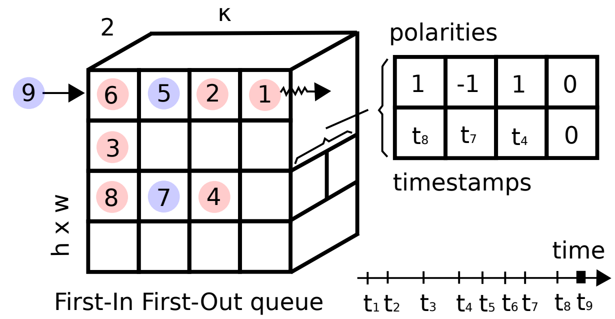


Figure 1. The **event queue** is a 4d tensor of size $2 \times \kappa \times h \times w$, where κ is a *queue capacity*. The figure depicts the queue with width and height fused to a single dimension. It stores polarities and timestamps of the κ most recent events at each location in order of arrival. When a new event, here (9), arrives, it pushes older events (6, 5, 2) to the end of the queue and occupies the first position and the oldest event (1) is pushed out of the queue.

$c \times h \times w$, as $I_{y,x} = f_\tau(E(x,y))$. The spatial aggregation is a translation-invariant function that is applied to subwindows of the event image and produces a event descriptor $\mathbf{F}$ such that $\forall y, x,\ F_{y,x} = f_S\left(\mathbf{I}_{y-\Delta:y+\Delta, x-\Delta:x+\Delta}\right)$.

The spatial grid structure of the event image allows the use of standard 2d convolutions. Therefore, throughout our experiments we use two convolutional residual blocks [15] and focus on different temporal aggregation methods.

To implement different temporal aggregation methods we need a way to efficiently accumulate events in each location. For that we propose to use a First-In First-Out (FIFO) queue shown in Figure 1. It saves the κ most recent events at each location sorted by time of their arrival. This queue could be efficiently implemented using linked lists or simpler circular buffers. Note also, that this queue works well regardless of the amount of motion: in presence of fast motion, when events are frequent, it stores only the recent ones, while in presence of slow motion, when events are rare, it preserves old ones. We prune events that arrived more than τ seconds ago from the queue and replace them with zeros before applying the temporal aggregation. We call κ the *capacity* and τ the *time horizon* of the queue.

Hand-crafted. In §1.4 we reviewed existing methods for converting event sequences to event images. All of them can be thought of as hand-crafted temporal aggregations. One of these methods produces an event image by counting the number of positive and negative events, and recording timestamps of the most recent positive and negative events at every location. Since similar methods [34, 25, 30, 53, 66, 60] worked well in many applications, we use this solution as our baseline.

Temporal convolutional network. Temporal convolutional network seems like a natural choice for temporal aggregation. However, a convolutional network usually applies to regularly sampled data, whereas in our case event timestamps are sampled irregularly and the temporal dimen-

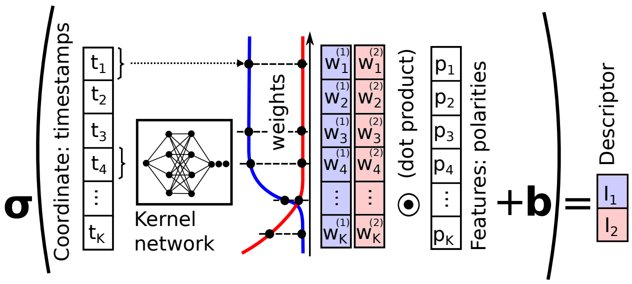
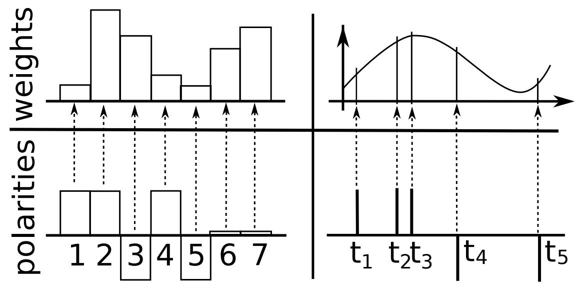


Figure 2. In the **continuous fully-connected layer** depicted here, for every timestamp the kernel network computes two weights, and for all timestamps two vectors, each corresponding to a continuous kernel. To get an event sequence descriptor we multiply each of these vectors by polarity vector using dot product, add bias and apply non-linearity. The corresponding weight vector, descriptor element and continuous kernel are shown in same color.



(a) Conventional FC (b) Proposed CFC

Figure 3. Comparison of (a) conventional fully-connected (FC) layer to (b) proposed continuous fully-connected (CFC) layer. In contrast to FC, CFC allows to embed event sequences with irregularly spaced time intervals between events.

sion in the queue only reflects the order of event arrival. Actual timestamp difference between nearby events in the queue might be different and arbitrary. To compensate for that, we feed the timestamp of each event to the network along with its polarity as a feature. Details of the network with temporal convolutions can be found in our supplementary material.

Continuous fully-connected layer. Ideally, the fact that event timestamps are continuous and sampled irregularly is taken into account. To do so, we use a *continuous fully-connected layer (CFC)*, where continuous kernels are themselves approximated by a multi-layer perceptron (MLP), that we call a *kernel network*. This network allows to model arbitrary complex kernels by modulating their capacity, and can be trained end-to-end along with the rest of the architecture. The overall idea is illustrated by Figure 2. Details about kernel networks can be found in our supplementary material.

Let us compare the proposed layer to a standard fully-connected (FC) layer, to appreciate the differences. Given event polarities $\mathbf{p} = [p_1, p_2, p_3, p_4, p_5, 0, 0]$ for some location stored in the event queue, a single output of the conventional FC layer is computed as $I = \sigma(\sum_i^7 w_i p_i + b)$, where $\mathbf{w}$ is a weights vector, b is a bias and $\sigma(\cdot)$ is a non-linearity. In contrast, a single output of the proposed CFC layer is computed as $I = \sigma(\frac{1}{5}\sum_i^5 w(t_i) \cdot p_i + b)$. Note that as shown in Figure 3, for a standard FC layer, the weight of each polarity simply depends on the events order i, while for the proposed CFC, the weight is a continuous parametric function $w(t_i) = \text{KernelNet}(t_i)$ (MLP), of real-valued event timestamp t_i. This allows to embed event sequences with irregularly spaced time intervals between events.

A similar construction was used in [54] but with the use of continuous convolutional layers. Here, we propose continuous fully-connected layer. Another difference is that in [54] the input are 3d LIDAR points in a Euclidean space.

3. Experiments

All experiments are done using the PyTorch framework [40]. Network learning uses RMSprop with standard settings. In all experiments we normalize event polarities in the queue to $\mathcal{N}(0, 1)$ and subtract the timestamp of the most recent event from all other event timestamps.

All experiments are done on publicly available datasets, and our code is available on GitHub [1].

3.1. Dataset and evaluation protocol

We use the Multi Vehicle Stereo Event Camera Dataset (MVSEC) [65] which is available online [31]. MVSEC is the only large publicly available dataset captured with a real event-based stereo system, and over recent years it has became the de-facto standard for comparing event-based stereo methods [67, 64]. It is collected by a system composed of a LIDAR and two event-based cameras with a resolution of 346×260 pixels mounted on various vehicles, such as a drone, a car and a motorcycle. LIDAR records frames with sparse depth measurements at 20Hz, while the event-based cameras acquire continuous streams of events and gray-scale video frames, which we use for visualization purposes only.

We unpack the original data in ROS bag format [43]. The depths are converted to left-view sub-pixel disparities and saved as images (sub-pixel precision is preserved by scaling the disparities). All pixels with disparities > 36 are assumed to have unknown disparities. Then, for each depth, we find the closest gray-scale image in time, and events preceding the depth by 0.05 seconds in the left and right view. We correct their optical distortions, rectify and save them. The script for data conversion is available online with the rest of the source code.

We use the *Indoor Flying* dataset from MVSEC, which is captured from a drone flying in a room with various ob-

[1] https://github.com/tlkvstepan/event_stereo_ICCV2019

Table 2. Summary of Indoor Flying splits. For each split we specify which sequences and frames are used for training and test. For example, $S^1_{140,..,1200}$ means that from sequence one only the frames 140 to 1200 are used. We use the same test intervals as in [67] to allow a fair comparison.

#	Set	Sequence and frames	Size
1	Training	$S^2_{160,..,1580} \cup S^3_{125,..,1815}$	3110
	Validation	$A \in S^1_{140,..,1200}$	200
	Test	$B \in S^1_{140,..,1200} \mid A \cap B = \emptyset$	861
2	Training	$S^1_{80,..,1260} \cup S^3_{125,..,1815}$	2870
	Validation	$A \in S^2_{120,..,1420}$	200
	Test	$B \in S^2_{120,..,1420} \mid A \cap B = \emptyset$	1101
3	Training	$S^1_{80,..,1260} \cup S^2_{160,..,1580}$	2600
	Validation	$A \in S^3_{73,..,1615}$	200
	Test	$B \in S^3_{73,..,1615} \mid A \cap B = \emptyset$	1343

jects. We compare our method to existing methods using the protocol from [67] and report results on the three sequences. The results are summarized in Table 2. Following [67], take-off and landing frames are removed. The test sequences are the same as in [67].

Similar to [67], we compute and report the *mean depth error* (MDE) and *one-pixel-accuracy* (1PA) computed in sparse locations corresponding to 15'000 events preceding each depth measurement. The one-pixel-accuracy is the percentage of locations for which the predicted disparity is off by less than one pixel.

3.2. Comparison of temporal aggregation methods

In this section, we compare performance of the temporal aggregation variants described in §2.2. We train the network three times for each method using different random initializations. For every trial we select the network that achieves the highest 1PA on the validation set over all epochs. The selected networks are then used to compute the performance on the test set. In Table 3 we report average test results along with standard deviations.

For each variant we use the architecture that was found during the grid-search experiments with the shallow stereo network from [61] on validation set. During training, we consider only ground truth at locations corresponding to the most recent 15'000 events.

All networks are initialized using the default PyTorch initialization, except the kernel network, for which we developed a custom initialization that ensures that the outputs of the network follow a normal distribution. More details can be found in the supplementary materials.

As shown in Table 3, the proposed learning-based methods for temporal aggregation outperform the hand-crafted method, probably due to the fact that they utilize timestamps of individual events. Among the learning-based

Table 3. Empirical results on the first split test set of the Indoor Flying dataset. Shown is the average test set results over three trials with the best performing method highlighted. Note, that all proposed learning-based methods outperform the hand-crafted method.

Method	MDE, [cm]	1PA, [%]
Hand-crafted	16.5 ± 0.5	87.3 ± 0.2
Temporal convolutional network	13.8 ± 0.1	90.7 ± 0.1
Continuous fully-connected layer	13.6 ± 0.2	91.3 ± 0.9

methods, the network with the continuous fully-connected layer shows the best performance as it explicitly handles events which are irregularly sampled from the continuous time domain. In all following sections we use the latter method, and call the resulting stereo matching method *Deep Dense Event Stereo* (DDES).

3.3. Empirical results

Next, we compare the proposed stereo method to the state-of-the-art event-based methods [67, 64, 38], and to two traditional methods [16, 17] which were adopted to work on event images in [64].

For quantitative comparison we use the protocol from [67] described in §3.1. According to this protocol, results are evaluated in sparse locations corresponding to 15'000 most recent events. We use the same parameters and experiments settings as in §3.2. During the experiments we noticed that for the second split there is a significant difference between test and training set. The test set has more abrupt motions, triggering a larger number of events compared to the training set (for details please refer the supple-

Table 4. Results on the Indoor Flying dataset using sparse ground truth, following the protocol from [66] described in §3.1. Results for TSES [67] and CopNet [38] are from [67] and results for Semi-Dense 3D [64], SGM* [16, 64] and FCVF* [17, 64] are from [64]. SGM* and FCVF* methods implemented in [64] are similar to the original frame-based methods but operate on event images. For Semi-Dense 3D, SGM* and FCVF* results for the second split are not available. We report average test set errors including standard deviations over the three randomized training trials. For other methods the standard deviation are not available. All methods are sorted in ascending order according to their test error. Our proposed method dubbed Deep Dense Event Stereo (DDES) is highlighted. Note, that it outperforms other single viewpoint methods, such as TSES, CopNet, SGM* and FCVF*, and even performs on-par with Semi-Dense 3D method that fuses depths from several viewpoints using known camera motion.

Method	Mean depth error, [cm]		
	Split 1	Split 2	Split 3
Semi-Dense 3D [64]	13	–	33
DDES (proposed)	13.6 ± 0.2	18.0 ± 0.2	18.4 ± 0.5
TSES [67]	36	44	36
CopNet [38]	61	100	64
SGM* [16, 64]	93	–	119
FCVF* [17, 64]	99	–	103

Table 5. Performance on the Indoor Flying dataset evaluated using dense ground truth. We train our method using the full ground truth disparity, taking into account all locations, including those without events. We select the network with the highest validation 1PA during a single training pass and report its results on the test set. Note that the results are only slightly worse than results obtained using sparse ground truth.

Mean depth error, [cm]			One pixel accuracy, [%]		
Split 1	**Split 2**	**Split 3**	**Split 1**	**Split 2**	**Split 3**
16.7	29.4	27.8	89.8	61.0	74.8

mentary materials). As a partial remedy, for the second split we trained the network using a fixed number of 130'000 events instead of a fixed time horizon and show the results in the tables. However, we believe that due to the domain shift this split has limited significance and should not be used.

The results are summarized in Table 4. Our proposed Dense Deep Event Stereo (DDES) method performs better than other single viewpoint methods, such as TSES [67], CopNet [38], SGM* [16, 64] and FCVF* [17, 64] and even performs on-par with the Semi-Dense 3D method [64] that fuses depth from several viewpoints using known camera motion.

We also train and test our method using the entire ground truth, taking into account all locations, including those without events. We select the network with the highest validation 1PA during a single training pass and report its results on the test set. The results are summarized in Table 5. Note, that the results are only slightly worse than results using the sparse ground truth. To our knowledge, this is the first successful attempt to compute dense stereo results for event-based cameras.

For qualitative comparison we estimate disparity using DDES trained on the full ground truth for example cases similar to the ones used in [67, 64]. Figure 5 contains a visual comparison of our results with those of TSES [67] and Semi-Dense 3D [64] borrowed from the respective papers. Unlike previous techniques, DDES computes truly dense and sub-pixel accurate disparity.

Our implementation of DDES runs at about 10 frames per second on a desktop PC with a GeForce GTX TITAN X GPU.

3.4. Weights of continuous fully-connected layer.

In this section, we visualize the output of the kernel network. To this end, we input uniformly sampled timestamps $\in [-0.5, 0]$ to the kernel network and plot every row of the CFC weights tensor as a smooth curve, which we call *weight kernel*.

Resulting kernels before and after the training are shown in Figure 4. At the start of training, the output of the kernel network is (by design) normally distributed, due to the initialization. After training, the weight kernels become

Table 6. Impact of the event queue capacity on performance. The table shows validation errors for split # 1 of the Indoor Flying set averaged over 2 trials.

Queue capacity κ	**1**	**3**	**7**	**15**
Mean depth error, [cm]	13.3	13.4	13.5	13.3

smooth in time and converge to one of two shapes: bell-shaped (kernels 2 and 3) or derivative (kernel 1). The bell-shaped kernels detect events with particular timestamps, while the derivative kernels compute event count changes (time-derivative) at varying time scales. Most of the kernels assign close to zero weights to old events.

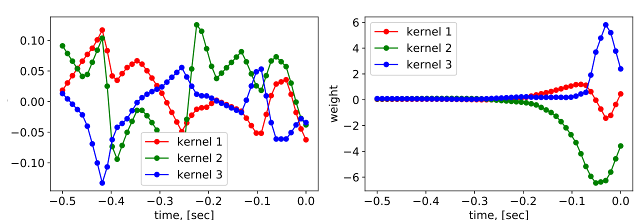
3.5. Importance of spatial and temporal context.

During our initial experiments with the temporal embedding we used the shallow stereo network with a small receptive field of size 9×9 from [61]. The shallow networks had no access to a large spatial context and larger event queue capacity and thus larger temporal context clearly helped to achieve better results. For example, with an event queue capacity $\kappa = 1$ the MDE validation error was 80.4 cm, while with $\kappa = 7$ it was 67.9 cm (the error was computed for Split 1 and averaged over two trials).

For the deep architecture from § 2.1, we noticed that the performance became very similar for different event queue capacities κ as shown in Table 6. This indicates that a network with access to a larger spatial context tends to ignore temporal context. We hypothesise, that spatial context is more reliable than temporal context, particular in dynamic sequences, such as drone videos.

4. Conclusion

In this work, we proposed a novel learning-based method for embedding event sequences as recorded by event-based vision sensors. It allows to model events as a stream of sparse 3d data points, each with two discrete spatial coordinates and one continuous temporal coordinate, and is able to use timestamps and spatial positions of all events in a time interval. We demonstrated state-of-the-art performance for



(a) Before training (b) After training

Figure 4. Visualization of kernel network output. Before training (a), the kernel network output is (by design) normally distributed. After training (b), the weight kernels have one of two shapes: bell-shaped (kernels 2 and 3) and derivative (kernel 1). Details are in the text. For clarity, we show 3 kernels out of 64.

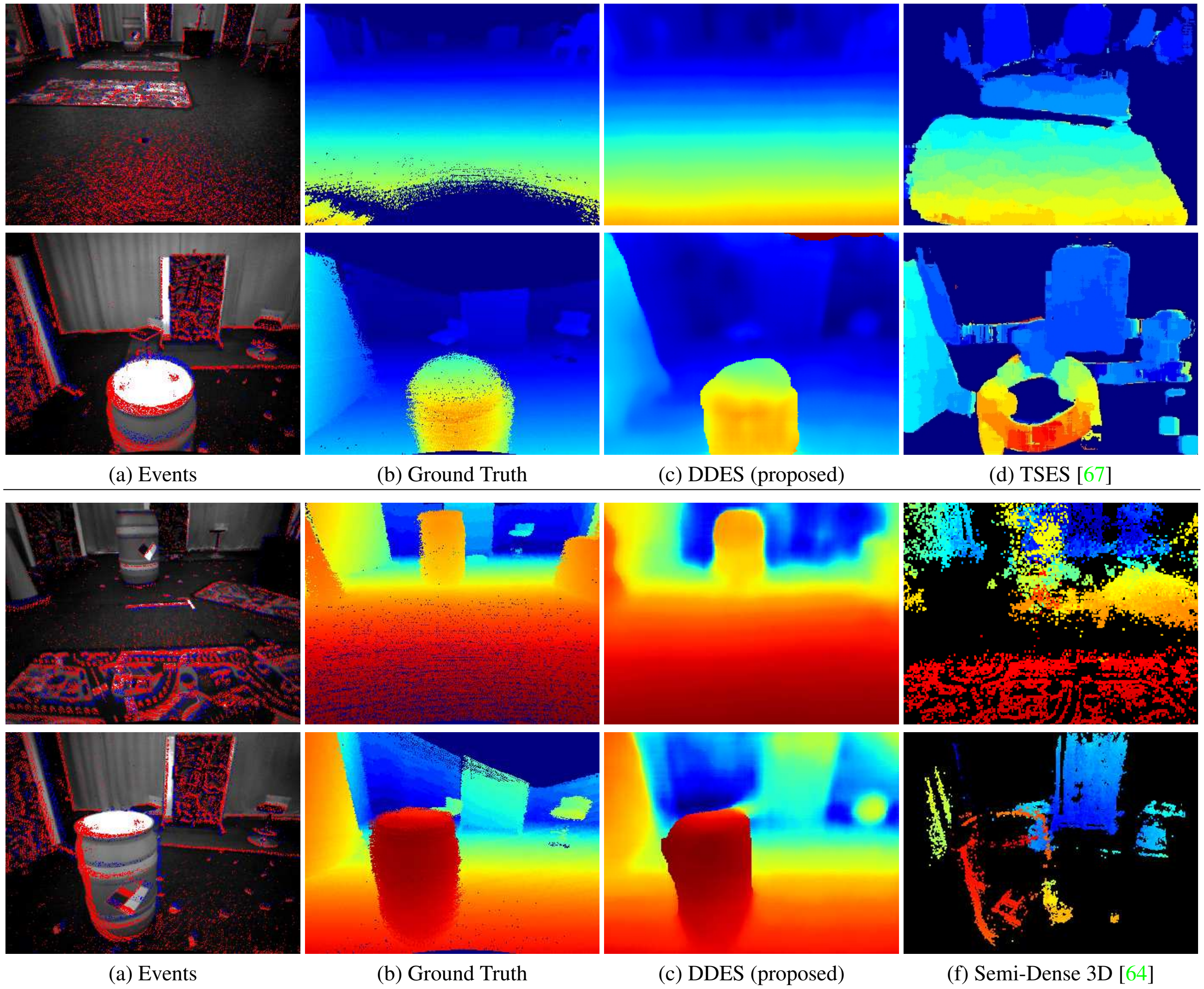


Figure 5. Qualitative comparison with recent event-based methods on the Indoor Flying dataset. For comparison, we select frames similar to the ones used in [67] and in [64]. Results for TSES [67] and Semi-Dense 3D [64] are borrowed from the respective papers. Note, that, unlike our method, Semi-Dense 3D fuses depth from several viewpoints using known camera motion. The rows correspond to frame #100 from sequence 1, frame #340 from sequence 1, frame #1700 from sequence 3 and frame #980 from sequence 1 correspondingly. To get the results for one sequence we trained the network using the remaining two. We tried to match the color-coding of the different outputs. In all figures warmer colors correspond to closer objects. In (a) we visualize the 15'000 most recent events from the left camera, overlaid with a gray-scale image, which is not used by the methods. Positive events are shown in red and negative events are shown in blue. In (b,c,d) locations without disparities are shown in dark blue and in (f) in black. Note, that our proposed method (c) computes dense disparities, while in TSES (d) some disparities are invalidated by outlier rejection and in Semi-Dense 3D (f) disparities are computed only for locations with events. Similarly to Semi-Dense 3D method, our proposed method (c) estimates sub-pixel disparities, while TSES (d) estimates integer disparities.

the task of stereo matching. Empirical results are better than the best hand-crafted as well as a learning-based embedding that uses on temporal convolutions in a discretized time domain. Using the proposed embedding we developed DDES, a deep neural network for stereo matching. This is the first deep learning-based stereo matching method for event-based cameras. We demonstrated that DDES perfoms better than prior state-of-the-art on the standard MVSEC dataset by a large margin.

Event-based cameras offer advantages such as higher dynamic range and temporal resolution over traditional frame-based cameras but require specialized handling of their event streams. We hope that the proposed embedding finds applications to more imaging algorithms beyond stereo matching.

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