An Efficient Solution to the Homography-Based Relative Pose Problem With a Common Reference Direction: Supplementary Material

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1. Implementation details

We first show the implement details for the equal and unknown focal length problem. Consider Eq.(12) in the paper

$$Q = R_2^T K_2^{-1} G K_1 R_1 - kR_y, \tag{1}$$

Let $K_2^{-1} = \mathrm{diag}(\frac{1}{f}, \frac{1}{f}, 1)$, $K_1 = \mathrm{diag}(f, f, 1)$. Then we obtain the system of polynomial equations (24) in the paper. In order to remove fraction in the equations, we obtain the following system of polynomial equations

$$\begin{aligned}
f((h_1-h_9)(h_5-k)-h_2h_4+h_6h_8)&=0,\\
f((h_3+h_7)(h_5-k)-h_2h_6-h_4h_8)&=0,\\
f^2(k^2(h_2^2+h_8^2)-(h_1h_8-h_2h_7)^2-(h_3h_8-h_2h_9)^2)&=0,\\
f^2(k^2(h_4^2+h_6^2)-(h_1h_6-h_3h_4)^2-(h_4h_9-h_6h_7)^2)&=0,\\
f^2((h_5-k)(h_7^2+h_9^2-k^2)+(h_3-h_7)h_4h_8-(h_1+h_9)h_6h_8)&=0.
\end{aligned}$$

2. Improving numerical stability

In this section, we show the implement details for the varying focal lengths problem, and the normalization. Consider Eq.(4) in the paper

$$\lambda R_2^T K_2^{-1} m_2 = H_y R_1^T K_1^{-1} m_1, \tag{2}$$

with $K_1^{-1} = \mathrm{diag}(1, 1, f_1)$, $K_2^{-1} = \mathrm{diag}(1, 1, f_2)$. By scaling the points we obtain two transformation matrices

$$T_1 = \mathrm{diag}(1, 1, \frac{1}{\sigma_1}), T_2 = \mathrm{diag}(1, 1, \frac{1}{\sigma_2}). \tag{3}$$

Applying the transformation (3) to (2) we obtain

$$\lambda R_2^T K_2^{'-1} m_2' = H_y R_1^T K_1^{'-1} m_1'. \tag{4}$$

where $K_1^{'-1} = \mathrm{diag}(1, 1, \sigma_1 f_1)$, $K_2^{'-1} = \mathrm{diag}(1, 1, \sigma_2 f_2)$, and m_1', m_2' are the normalized image points. In this case, the Eq.(10) in the paper can be written as

$$Q = R_2^T K_2^{'-1} G' K_1' R_1 - kR_y, \tag{5}$$

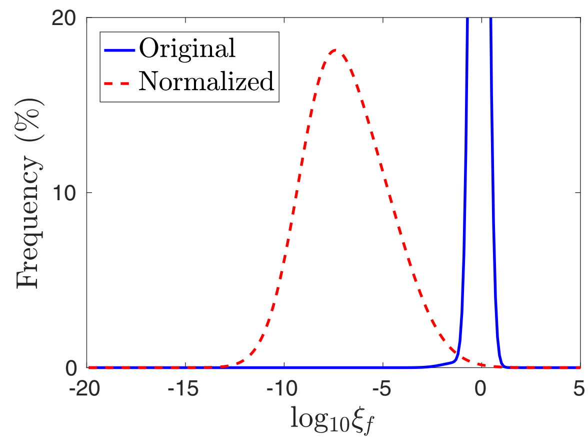

Figure 1: log_{10} relative focal length errors for the original and normalized solvers (equal and unknown focal length problem).

with

$$\lambda m_2' = G' m_1', \tag{6}$$

where G' transforms the normalized image points. In order to reduce the complexity of the equations, we can let $K_1' = \mathrm{diag}(1, 1, f_1')$ in (5), where $f_1' = \frac{1}{\sigma_1 f_1}$. Once the equations are solved, the solutions only need to be divided by σ_1, σ_2 to get the real focal lengths. For the equal focal length problem, let $\sigma_2 = \sigma_1$. This normalization is inspired by [2]. It is necessary, otherwise the solvers are unstable. Figure 1 shows the comparison for the equal and unknown focal length problem with noise-free data. We show the focal length error as an example.

3. Degenerate configurations

3.1. Pure rotation

We show that the proposed solvers are compatible with pure rotation case without the prior knowledge of the motion. Figure 2 shows the focal length errors with noise-free

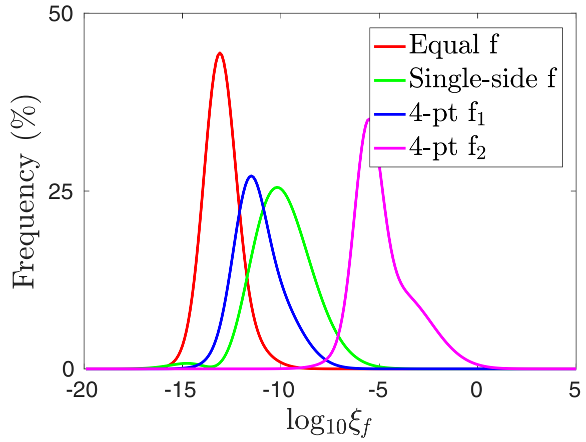

Figure 2: log_{10} relative focal length errors for the proposed solvers under pure rotation.

data. We choose the solution which is closest to the ground truth. As we can see, all of the proposed solvers are numerically stable.

3.2. Critical motions

Equal and unknown focal length

Arbitrary planar motions when the optical axes lie in the plane are critical motions for the standard 6-point solver [3]. The proposed fHf solver can deal with this case (non-zero rotation angle). See Figure 3.

One unknown focal length

In [1], the authors show that the single-side 6-point solver can deal with pure translation except for the pure forward motion. In contrast, the proposed 3.5-point Hf solver can also deal with pure translation (except for $R_1 = R_2 = I, t_y = 0$, which means that the y-axes of the cameras are coincided with the gravity direction and the translation along the gravity direction is zero). See Figure 4.

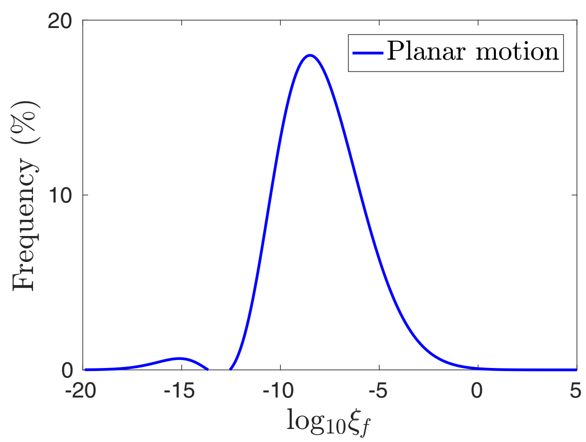

Figure 3: log_{10} relative focal length error for the fHf solver under planar motion when the optical axes lie in the plane.

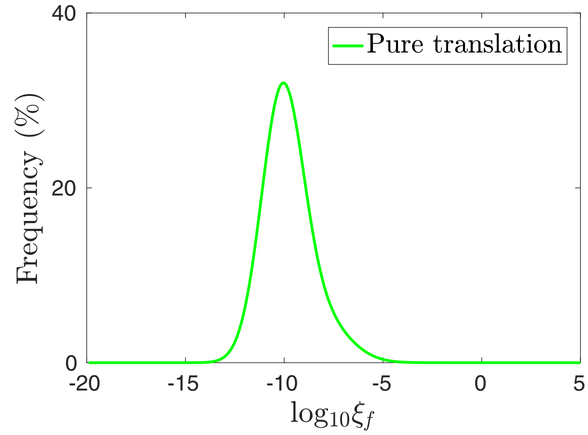

Figure 4: log_{10} relative focal length error for the Hf solver under pure translation (except for $R_1 = R_2 = I, t_y = 0$).

For the different and unknown focal lengths problem, currently we do not find special properties.

References

[1] Martin Bujnak, Zuzana Kukelova, and Tomas Pajdla. 3d reconstruction from image collections with a single known focal length. In *Computer Vision, 2009 IEEE 12th International Conference on*, pages 1803–1810. IEEE, 2009.

[2] Richard I Hartley. In defence of the 8-point algorithm. In *Computer Vision, 1995. Proceedings., Fifth International Conference on*, pages 1064–1070. IEEE, 1995.

[3] Fredrik Kahl and Bill Triggs. Critical motions in euclidean structure from motion. In *International Conference on Computer Vision & Pattern Recognition (CVPR'99)*, pages 366–372. IEEE Computer Society, 1999.