An Efficient Solution to the Homography-Based Relative Pose Problem With a Common Reference Direction: Supplementary Material

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1. Implementation details

We first show the implement details for the equal and unknown focal length problem. Consider Eq.(12) in the paper

\[ Q = R^T \sigma K_2^{-1} G K_1 R_1 - kR_y, \]  
(1)

Let \( K_2^{-1} = \text{diag}(\frac{1}{f_1}, \frac{1}{f_2}, 1) \), \( K_1 = \text{diag}(f, f, 1) \). Then we obtain the system of polynomial equations (24) in the paper.

In order to remove fraction in the equations, we obtain the following system of polynomial equations

\[
\begin{align*}
&f((h_1-h_9)(h_5-k)-h_2h_4+h_6h_8)=0, \\
&f((h_3+h_7)(h_5-k)-h_2h_6-h_3h_8)=0, \\
&f^2(k_2(h_2^2+h_8^2)-(h_1h_8-h_2h_7)^2-(h_3h_8-h_2h_9)^2)=0, \\
&f^2(k_2(h_2^2+h_8^2)-(h_1h_6-h_3h_4)^2-(h_4h_9-h_6h_7)^2)=0, \\
&f^2((h_5-k)(h_2^2+h_8^2-k^2)+(h_3-h_7)h_4h_8-(h_1+h_9)h_6h_8)=0.
\end{align*}
\]

2. Improving numerical stability

In this section, we show the implement details for the varying focal lengths problem, and the normalization. Consider Eq.(4) in the paper

\[
\lambda R_1^T K_2^{-1} m_2 = H_y R_1^T K_1^{-1} m_1,  
\]  
(2)

with \( K_1^{-1} = \text{diag}(1, 1, f_1) \), \( K_2^{-1} = \text{diag}(1, 1, f_2) \). By scaling the points we obtain two transformation matrices

\[
T_1 = \text{diag}(1, 1, \frac{1}{\sigma_1}), T_2 = \text{diag}(1, 1, \frac{1}{\sigma_2}).  
\]  
(3)

Applying the transformation (3) to (2) we obtain

\[
\lambda R_1^T K_2^{-1} m_2 = H_y R_1^T K_1^{-1} m_1',  
\]  
(4)

where \( K_2^{-1} = \text{diag}(1, 1, \sigma_1 f_1), K_2^{-1} = \text{diag}(1, 1, \sigma_2 f_2), \) and \( m_1', m_2' \) are the normalized image points. In this case, the Eq.(10) in the paper can be written as

\[
Q = H_2^T K_2^{-1} G' K_1' R_1 - kR_y,  
\]  
(5)

with

\[
\lambda m_2' = G' m_1',  
\]  
(6)

where \( G' \) transforms the normalized image points. In order to reduce the complexity of the equations, we can let \( K_1' = \text{diag}(1, 1, f_1') \) in (5), where \( f_1' = \frac{1}{\sigma_1 f_1} \). Once the equations are solved, the solutions only need to be divided by \( \sigma_1, \sigma_2 \) to get the real focal lengths. For the equal focal length problem, let \( \sigma_2 = \sigma_1 \). This normalization is inspired by [2]. It is necessary, otherwise the solvers are unstable. Figure 1 shows the comparison for the equal and unknown focal length problem with noise-free data. We show the focal length error as an example.

3. Degenerate configurations

3.1. Pure rotation

We show that the proposed solvers are compatible with pure rotation case without the prior knowledge of the motion. Figure 2 shows the focal length errors with noise-free
data. We choose the solution which is closest to the ground truth. As we can see, all of the proposed solvers are numerically stable.

### 3.2. Critical motions

**Equal and unknown focal length**

Arbitrary planar motions when the optical axes lie in the plane are critical motions for the standard 6-point solver [3]. The proposed $fHf$ solver can deal with this case (non-zero rotation angle). See Figure 3.

**One unknown focal length**

In [1], the authors show that the single-side 6-point solver can deal with pure translation except for the pure forward motion. In contrast, the proposed 3.5-point $Hf$ solver can also deal with pure translation (except for $R_1 = R_2 = I$, $t_y = 0$), which means that the $y$-axes of the cameras are coincided with the gravity direction and the translation along the gravity direction is zero). See Figure 4.

For the different and unknown focal lengths problem, currently we do not find special properties.

### References

