A. The 1D Radial Camera Model

For a pinhole camera the projection equations are

\[
\begin{bmatrix}
x \\
y \\
\end{bmatrix} = \frac{f}{d} \begin{bmatrix}
R_1 X + t_1 \\
R_2 X + t_2 \\
\end{bmatrix},
\]

(1)

where \( d = R_3 X + t_3 \). Note that the factor \( f/d \) only scales the projections along the radial lines, i.e., the lines going from the principal point\(^1\) to the image points \((x, y)\). Similarly, radial distortion enters as a scaling on either side of equation (1) (depending on if it is a undistortion or a distortion model).

The main idea in the 1D radial camera model is to only consider the equality in (1) up to scale. In this case we get

\[
\begin{bmatrix}
x \\
y \\
\end{bmatrix} \approx \begin{bmatrix}
R_1 X + t_1 \\
R_2 X + t_2 \\
\end{bmatrix}.
\]

(2)

Eliminating the unknown scale factor we can rewrite this as

\[
x(R_2 X + t_2) - y(R_1 X + t_1) = 0.
\]

(3)

Each 2D-3D point correspondences directly gives linear constraints on the rotation and translation which are independent of the focal length and radial distortion. Note however, that the constraints are also independent of the third element of the translation vector \( t_3 \). Using only these constraints it is therefore not possible to recover the forward translation \( t_3 \). See Figure A for an illustration.

A.1. Resectioning with Radial Projections

Geometrically, the problems of pose estimation for radial cameras and pinhole cameras are very similar. In both cases the problem can be formulated as aligning backprojected lines with 3D points. To see this, compare (3) with the corresponding linear constraints for the pinhole camera

\[
\begin{cases}
f(R_1 X + t_1) - x(R_3 X + t_3) = 0, \\
f(R_2 X + t_2) - y(R_3 X + t_3) = 0.
\end{cases}
\]

(4)

These two equations ensure that the 2D projection of the 3D point lies on the two lines \( \ell_x = (1, 0, -x) \) and \( \ell_y = (0, 1, -y) \). These lines are parallel to the two image axes and pass through the image point \((x, y)\). Similarly, (3) corresponds to \( \ell_r = (-y, x, 0) \) which is the radial line passing through \((x, y)\). See Figure B.

The pose of a (calibrated) radial camera can be described by a rotation matrix and two parameters for the translation, \( t_1 \) and \( t_2 \). Note that the projection equations (2) are invariant to sign changes in \( R_1, R_2 \) and \( t_1, t_2 \). Thus, the pose is

\[
\begin{cases}
f(R_1 X + t_1) - x(R_3 X + t_3) = 0, \\
f(R_2 X + t_2) - y(R_3 X + t_3) = 0.
\end{cases}
\]

(4)

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\(^1\)In the paper, we assume that the center of distortion is identical to the principal point.
only defined up to sign. However, in our setting this is not a problem since the incorrect sign simply corresponds to having a negative focal length. In this case we can simply switch the sign of \( f, R_1, R_2, t_1 \) and \( t_2 \) to recover the correct solution. The minimal pose problem with five 2D-3D point correspondences was originally solved by Kukelova et al. [1].

References
