Revisiting Radial Distortion Absolute Pose
Supplementary Material

This supplementary material provides technical details on the 1D radial camera model from [2] that we use in the paper (*c.f*. Sec. 3.2 in the paper).

A. The 1D Radial Camera Model

For a pinhole camera the projection equations are

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{f}{d}\begin{bmatrix} R_1 X + t_1 \\ R_2 X + t_2 \end{bmatrix} \quad , \tag{1}$$

where $d = R_3 X + t_3$. Note that the factor f/d only scales the projections along the radial lines, *i.e*., the lines going from the principal point[1] to the image points (x, y). Similarly, radial distortion enters as a scaling on either side of equation (1) (depending on if it is a undistortion or a distortion model).

The main idea in the 1D radial camera model is to only consider the equality in (1) up to scale. In this case we get

$$\begin{bmatrix} x \\ y \end{bmatrix} \simeq \begin{bmatrix} R_1 X + t_1 \\ R_2 X + t_2 \end{bmatrix} \quad . \tag{2}$$

Eliminating the unknown scale factor we can rewrite this as

$$x(R_2 X + t_2) - y(R_1 X + t_1) = 0 \quad . \tag{3}$$

Each 2D-3D point correspondences directly gives linear constraints on the rotation and translation which are independent of the focal length and radial distortion. Note however, that the constraints are also independent of the third element of the translation vector t_3. Using only these constraints it is therefore not possible to recover the forward translation t_3. See Figure A for an illustration.

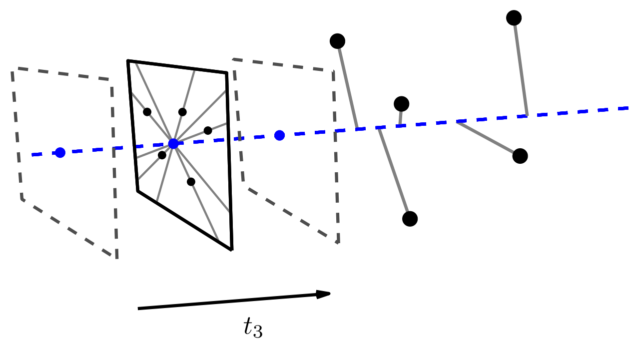


Figure A: The remaining translation parameter t_3 moves the camera along the principal axis. This moves the projection of the points along the radial lines.

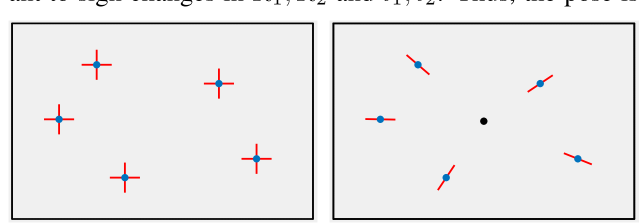
A.1. Resectioning with Radial Projections

Geometrically, the problems of pose estimation for radial cameras and pinhole cameras are very similar. In both cases the problem can be formulated as aligning backprojected lines with 3D points. To see this, compare (3) with the corresponding linear constraints for the pinhole camera model,

$$\begin{cases} f(R_1 X + t_1) - x(R_3 X + t_3) = 0 \quad , \\ f(R_2 X + t_2) - y(R_3 X + t_3) = 0 \quad . \end{cases} \tag{4}$$

These two equations ensure that the 2D projection of the 3D point lies on the two lines $\ell_x = (1, 0, -x)$ and $\ell_y = (0, 1, -y)$. These lines are parallel to the two image axes and pass through the image point (x, y). Similarly, (3) corresponds to $\ell_r = (-y, x, 0)$ which is the radial line passing through (x, y). See Figure B.

The pose of a (calibrated) radial camera can be described by a rotation matrix and two parameters for the translation, t_1 and t_2. Note that the projection equations (2) are invariant to sign changes in R_1, R_2 and t_1, t_2. Thus, the pose is



(a) Pinhole camera (b) Radial camera

Figure B: Pinhole vs. Radial. *(a)* Pinhole cameras give two measurements per point, the x and y coordinate. *(b)* Radial cameras only measures the direction of the projection.

[1] In the paper, we assume that the center of distortion is identical to the principal point.

only defined up to sign. However, in our setting this is not a problem since the *incorrect* sign simply corresponds to having a negative focal length. In this case we can simply switch the sign of f, R_1, R_2, t_1 and t_2 to recover the correct solution. The minimal pose problem with five 2D-3D point correspondences was originally solved by Kukelova et al. [1].

References

[1] Z. Kukelova, M. Bujnak, and T. Pajdla. Real-time solution to the absolute pose problem with unknown radial distortion and focal length. In *International Conference on Computer Vision (ICCV)*, 2013. 2

[2] S. Thirthala and M. Pollefeys. Radial multi-focal tensors. *International Journal of Computer Vision (IJCV)*, 96(2):195–211, 2012. 1