

# Supplementary material for “Convex Relaxations for Consensus and Non-Minimal Problems in 3D Vision”

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This document provides the proof of the Propositions, along with some qualitative results obtained using the proposed framework. In the following, we first provide an alternative proof to propositions 3.2 and 3.3. Later, we provide the qualitative results of non-rigid reconstruction, camera autocalibration, and rigid 3D motion estimation, non-rigid reconstruction.

**Proof of Proposition 3.2/3.3:** Let us define  $\epsilon_i = \delta_i^2$ ,  $\mathbf{v} = [\mathbf{x}^\top, \delta_1, \dots, \delta_m]^\top$ ,  $\Lambda_0 = \begin{bmatrix} 0 & \\ & \mathbf{I}_{m \times m} \end{bmatrix}$ ,  $\Lambda_i^+ = \begin{bmatrix} \mathbf{G}_i & \\ & \mathbf{E}^i \end{bmatrix}$  and  $\Lambda_i^- = \begin{bmatrix} -\mathbf{G}_i & \\ & \mathbf{E}^i \end{bmatrix}$  with  $e_{ii}^i = 1$  and  $(e_{ij}^i)_{j \neq i} = 0$ . For  $g_i(\mathbf{v}) = \mathbf{z}_1(\mathbf{v})^\top \Lambda_i \mathbf{z}_1(\mathbf{v})$ , the POP of polynomials  $\{g_0(\mathbf{v}), g_i^+(\mathbf{v}), g_i^-(\mathbf{v}), i = 0, \dots, m\}$  can be relaxed, similar to (2), as

$$\min_{\Upsilon \succeq 0} \left\{ \text{tr}(\Lambda_0 \Upsilon) \mid \text{tr}(\Lambda_i^+ \Upsilon), \text{tr}(\Lambda_i^- \Upsilon) \geq 0, i = 1, \dots, m \right\}, \quad (1)$$

where  $\text{tr}(\Lambda_0 \Upsilon) := g_0(\mathbf{v}) = \sum_i \epsilon_i$ . We choose  $\Upsilon = \begin{bmatrix} \mathbf{Y} & \\ & \mathbf{I}_{m \times m} \end{bmatrix}$  such that  $g_0(\mathbf{v}) = \sum_i \delta_i^2 = \sum_i \epsilon_i$  is satisfied. For polynomials  $g_i^+(\mathbf{v}) = \mathbf{z}_1(\mathbf{v})^\top \Lambda_i^+ \mathbf{z}_1(\mathbf{v}) = \text{tr}(\mathbf{G}_i \mathbf{Y}) - \delta_i^2$  and  $g_i^-(\mathbf{v}) = \mathbf{z}_1(\mathbf{v})^\top \Lambda_i^- \mathbf{z}_1(\mathbf{v}) = \delta_i^2 - \text{tr}(\mathbf{G}_i \mathbf{Y})$ , (1) becomes,

$$\min_{\mathbf{Y} \succeq 0} \left\{ \sum_i \delta_i^2 \mid -\delta_i^2 \leq \text{tr}(\mathbf{G}_i \mathbf{Y}) \leq \delta_i^2, i = 1, \dots, m \right\}. \quad (2)$$

After considering  $\epsilon_i = \delta_i^2$ , (2) becomes an SDP of (6/7). ■

## References

- [1] T.-J. Chin, Y. Heng Kee, A. Eriksson, and F. Neumann. Guaranteed outlier removal with mixed integer linear programs. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 5858–5866, 2016. 4, 5, 6

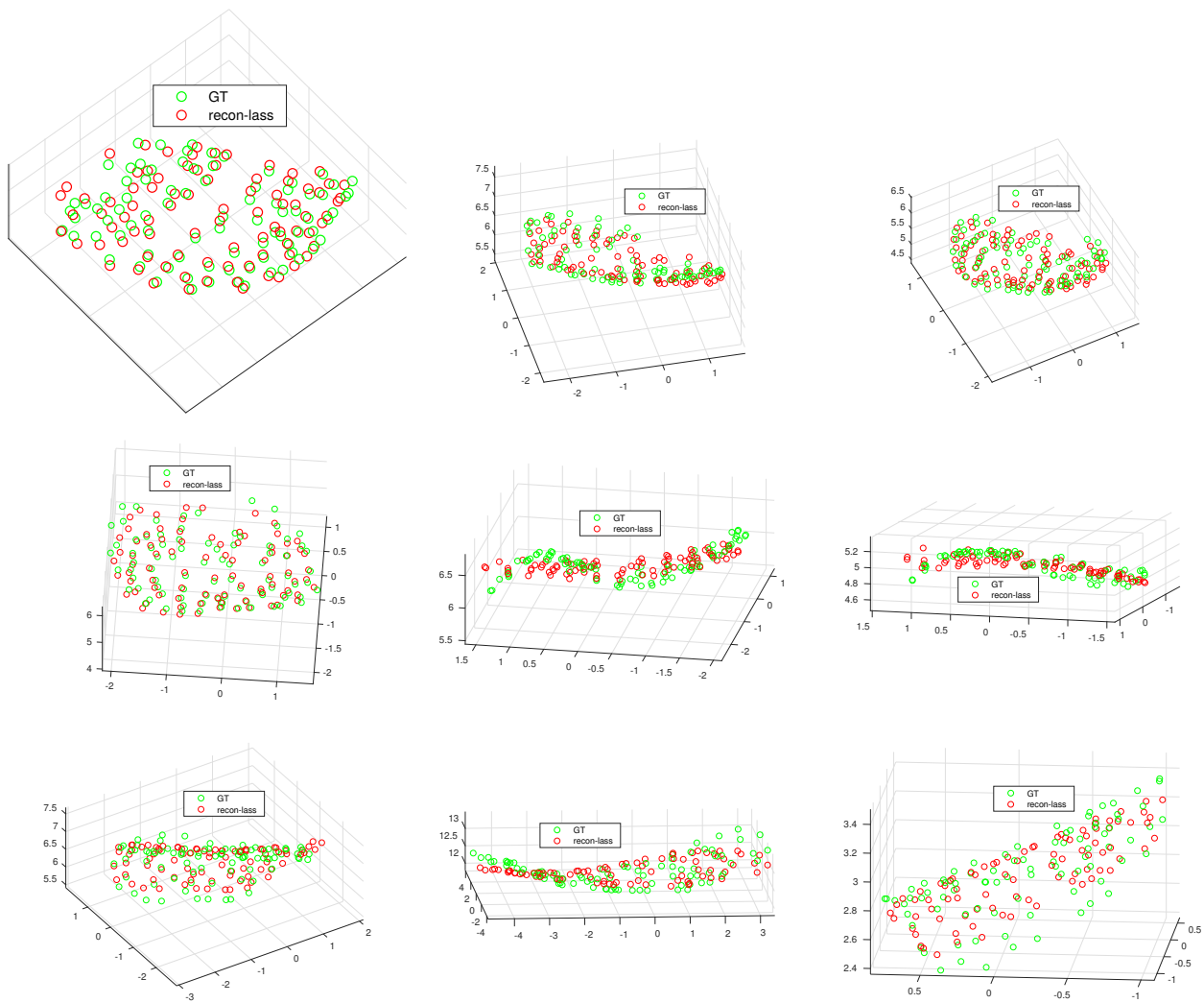


Figure 1: Nine views reconstructions of tshirt dataset using Lassare's relaxation.

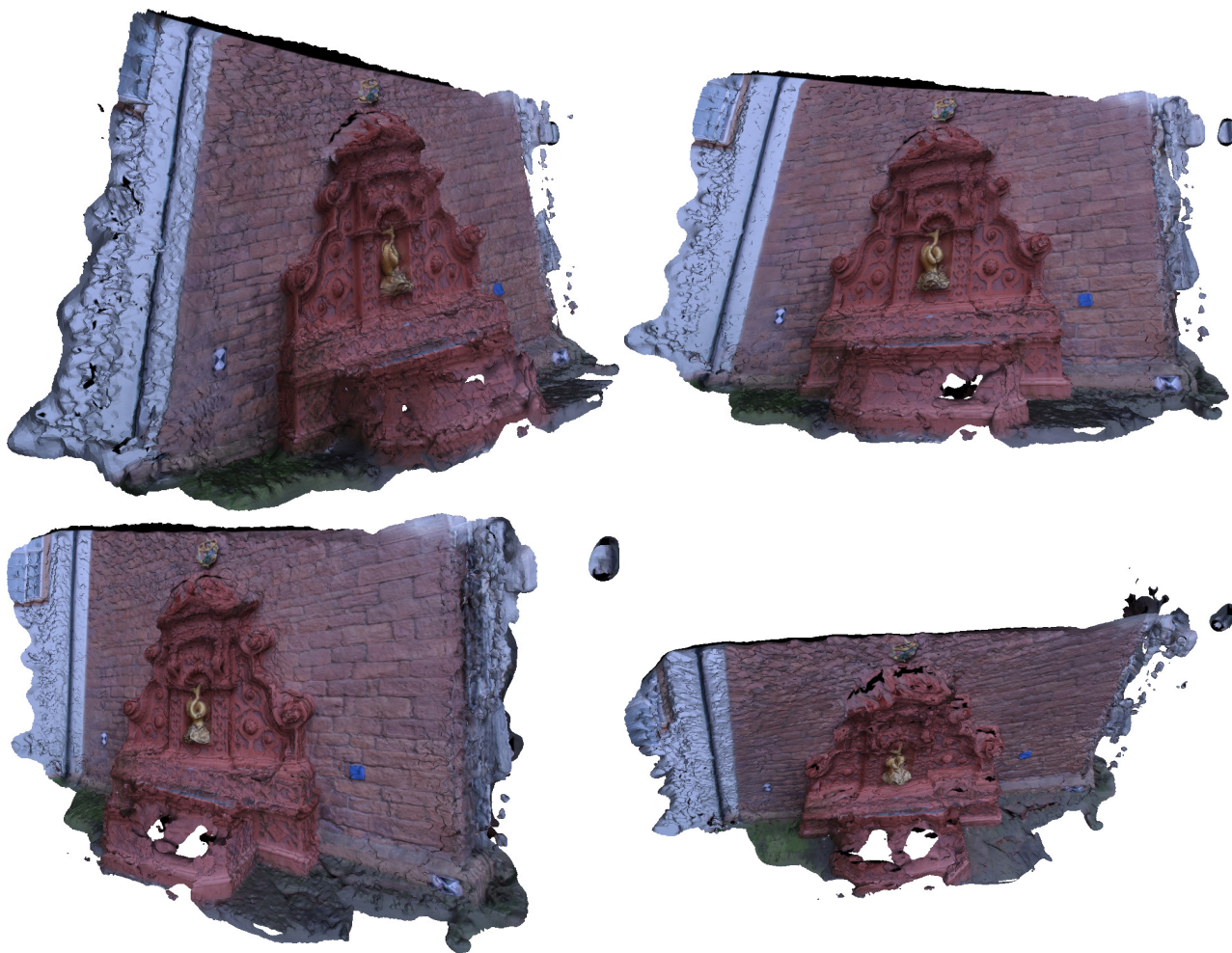


Figure 2: Rendered views for the 3D reconstruction of the Fountain sequence using the intrinsics obtained from our method.

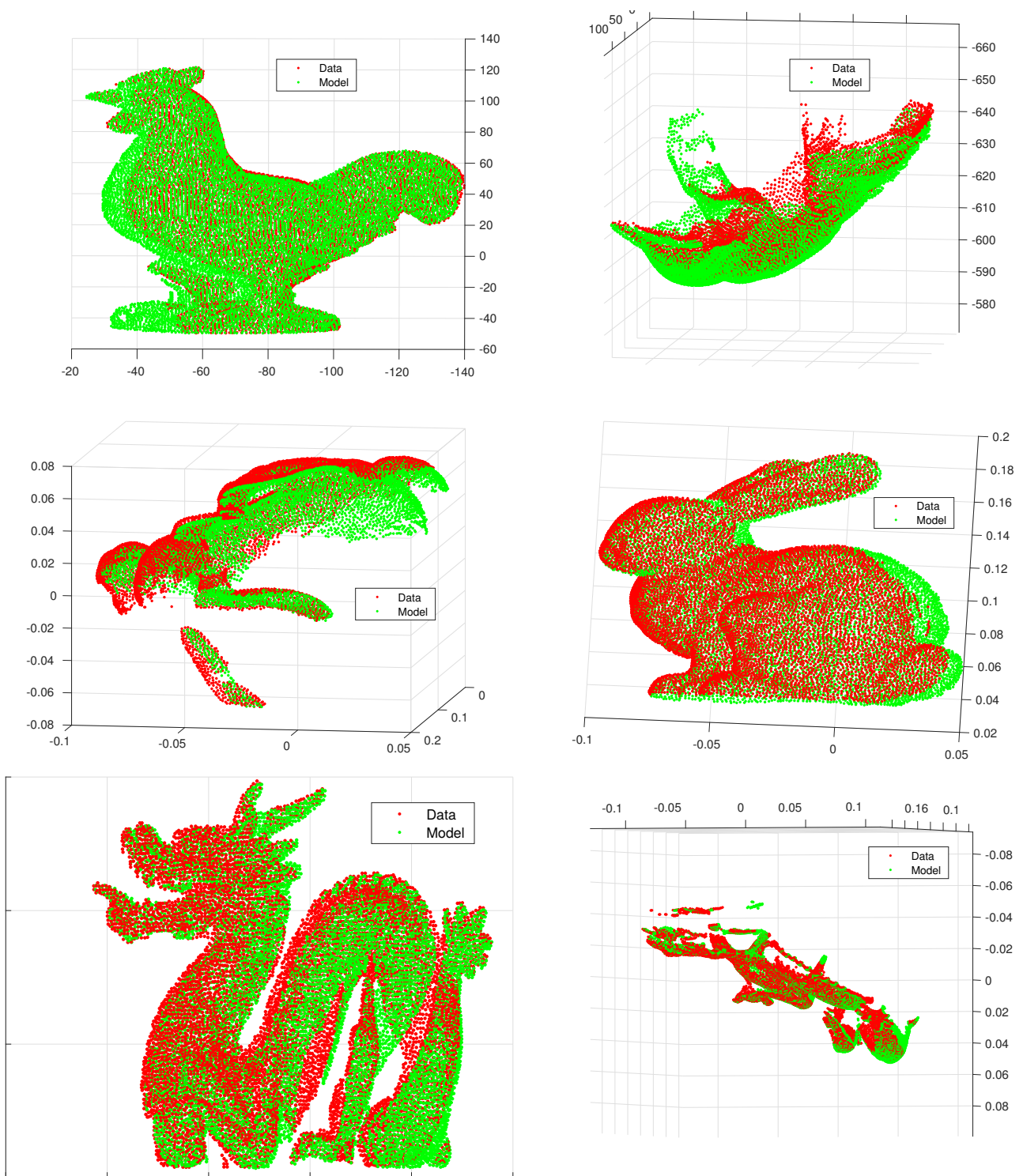


Figure 3: Qualitative results of the non-minimal registration (our method) on the publicly dataset [1]. Each row shows two views of the same registration obtained by non-minimal solver. Illustration shows the model in green and the data in red.



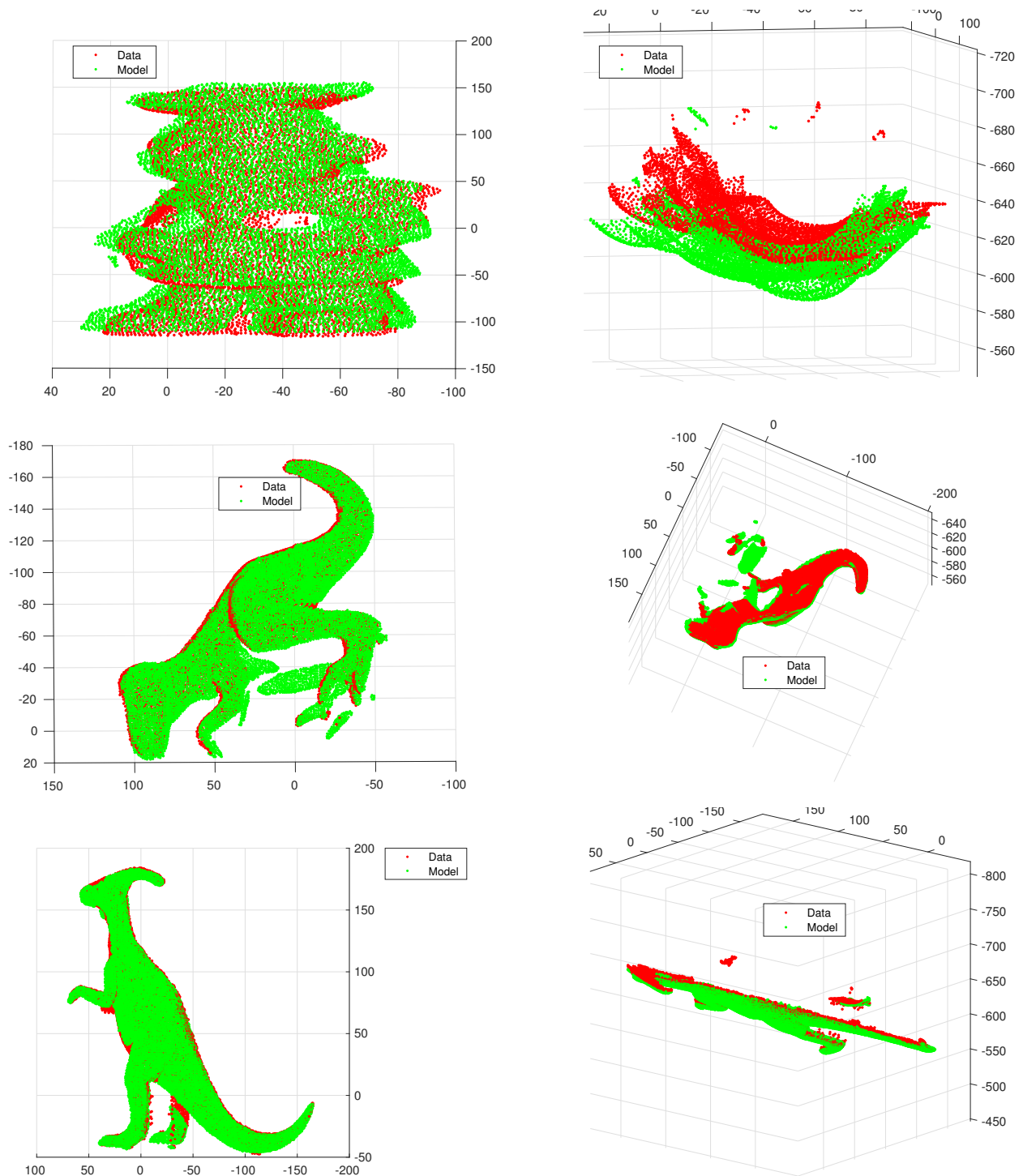


Figure 4: Qualitative results of the non-minimal registration (our method) on the publicly dataset [1]. Each row shows two views of the same registration obtaine by non-minimal solver. Illustration shows the model in green and the data in red.

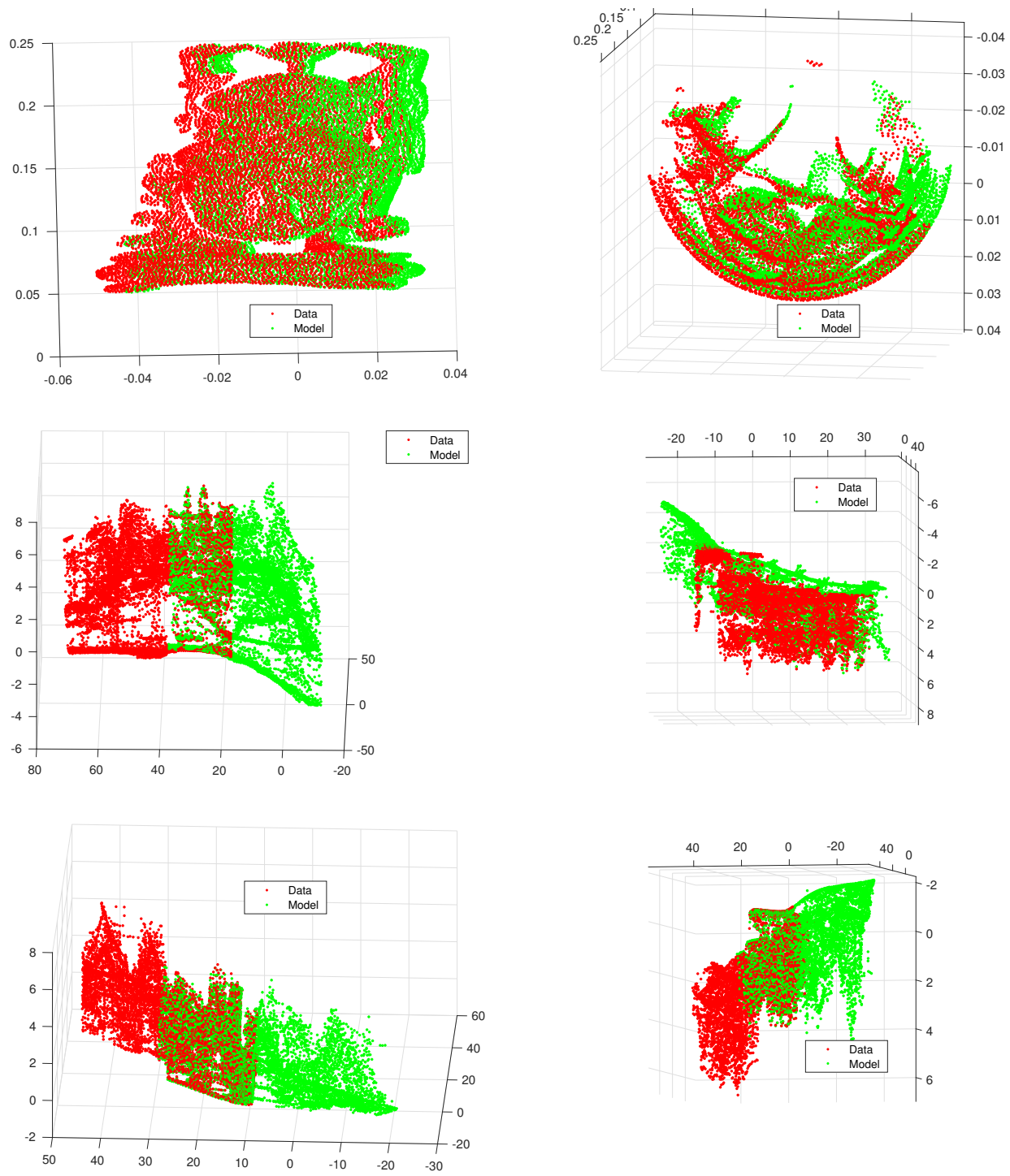


Figure 5: Qualitative results of the non-minimal registration (our method) on the publicly dataset [1]. Each row shows two views of the same registration obtained by non-minimal solver. Illustration shows the model in green and the data in red.