SparseMask: Differentiable Connectivity Learning for Dense Image Prediction Supplementary Material

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1. Theorems

We present two theorems in Section 3.1.2 (main text), of which the proofs are given in this section.

Theorem 1. *Concatenating the features and then applying convolution is equal to applying convolution to each feature and then take a summation.*

Proof. Given M input features F_{in}^m with shape $N \times C_{in}^m \times H \times W$, the concatenated feature is noted as F_{in} with shape $N \times C_{in} \times H \times W$, where $C_{in} = \sum_{m=0}^{M-1} C_{in}^m$. The corresponding convolution kernel is noted as W with shape $C_{out} \times C_{in} \times KH \times KW$, which can be split into M weights W^m with shape $C_{out} \times C_{in}^m \times KH \times KW$. The output feature F_{out} is represented as following:

$$\begin{aligned} F_{out}[n, c_{out}, h, w] &= conv(F_{in}, W)[n, c_{out}, h, w] \\ &= \sum_{kh,kw} \sum_{c_{in}=0}^{C_{in}-1} W[c_{out}, c_{in}, kh, kw] F_{in}[n, c_{in}, h+kh, w+kw] \\ &= \sum_{kh,kw} \sum_{m=0}^{M-1} \sum_{c_{in}=0}^{C_{in}^m-1} W^m[c_{out}, c_{in}, kh, kw] F_{in}^m[n, c_{in}, h+kh, w+kw] \\ &= \sum_{m=0}^{M-1} \sum_{kh,kw} \sum_{c_{in}=0}^{C_{in}^m-1} W^m[c_{out}, c_{in}, kh, kw] F_{in}^m[n, c_{in}, h+kh, w+kw] \\ &= \sum_{m=0}^{M-1} conv(F_{in}^m, W^m)[n, c_{out}, h, w]. \end{aligned} \tag{1}$$

□

Theorem 2. *The order of bilinear upsampling and point-wise convolution is changeable.*

Proof. The input feature is F_{in} with shape $N \times C_{in} \times H_{in} \times W_{in}$, while the corresponding convolution kernel is W with

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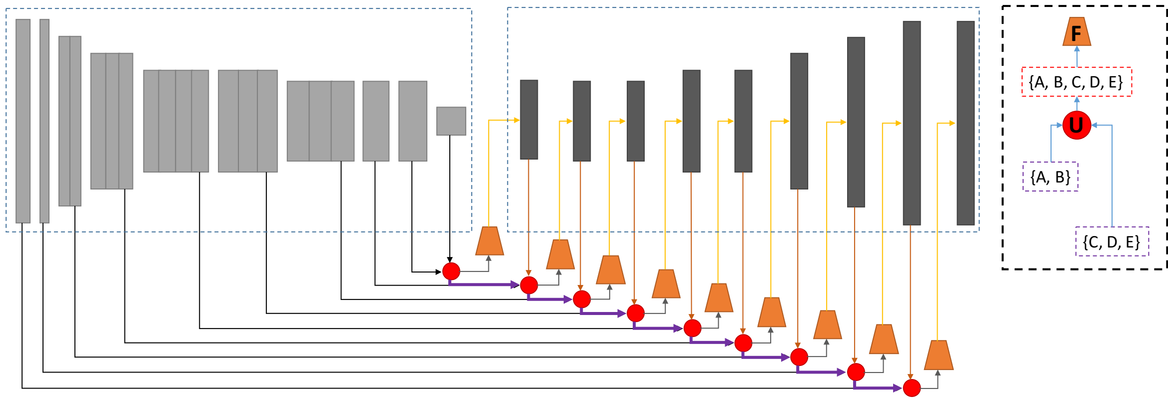


Figure 1: **Fully Dense Network based on MobileNet-V2 [2].** The inputs to the red circle (**U**) are multiple feature sets, while the output is the union of all the sets. **F** is the decoder stage, which takes a feature set as the input. Best viewed in color.

shape $C_{out} \times C_{in} \times 1 \times 1$. The output features F_{out} is then represented as following:

$$\begin{aligned} F_{out}[n, c_{out}, h_{out}, w_{out}] &= conv(f_{\uparrow}(F_{in}), W)[n, c_{out}, h_{out}, w_{out}] \\ &= \sum_{c_{in}} W[c_{out}, c_{in}, 0, 0] f_{\uparrow}(F_{in})[n, c_{in}, h_{out}, w_{out}] \\ &= \sum_{c_{in}} W[c_{out}, c_{in}, 0, 0] \sum_{i=0}^{3} |h_{in} - h_{in}^i||w_{in} - w_{in}^i| F_{in}[n, c_{in}, h_{in}^i, w_{in}^i] \\ &= \sum_{i=0}^{3} \sum_{c_{in}} W[c_{out}, c_{in}, 0, 0] |h_{in} - h_{in}^i||w_{in} - w_{in}^i| F_{in}[n, c_{in}, h_{in}^i, w_{in}^i] \\ &= \sum_{i=0}^{3} |h_{in} - h_{in}^i||w_{in} - w_{in}^i| \sum_{c_{in}} W[c_{out}, c_{in}, 0, 0] F_{in}[n, c_{in}, h_{in}^i, w_{in}^i] \\ &= \sum_{i=0}^{3} |h_{in} - h_{in}^i||w_{in} - w_{in}^i| conv(F_{in}, W)[n, c_{out}, h_{in}^i, w_{in}^i] \\ &= f_{\uparrow}(conv(F_{in}, W))[n, c_{out}, h_{out}, w_{out}], \end{aligned} \tag{2}$$

where $f_{\uparrow}(\cdot)$ is bilinear upsampling, $h_{in} = h_{out}/H_{out} \times H_{in}$ and $w_{in} = w_{out}/W_{out} \times W_{in}$. h_{in}^i and w_{in}^i is calculated as follows:

$$\begin{aligned} h_{in}^0 &= \lfloor h_{in} \rfloor, w_{in}^0 = \lfloor w_{in} \rfloor; h_{in}^1 = \lceil h_{in} \rceil, w_{in}^1 = \lfloor w_{in} \rfloor \\ h_{in}^2 &= \lfloor h_{in} \rfloor, w_{in}^2 = \lceil w_{in} \rceil; h_{in}^3 = \lceil h_{in} \rceil, w_{in}^3 = \lceil w_{in} \rceil. \end{aligned} \tag{3}$$

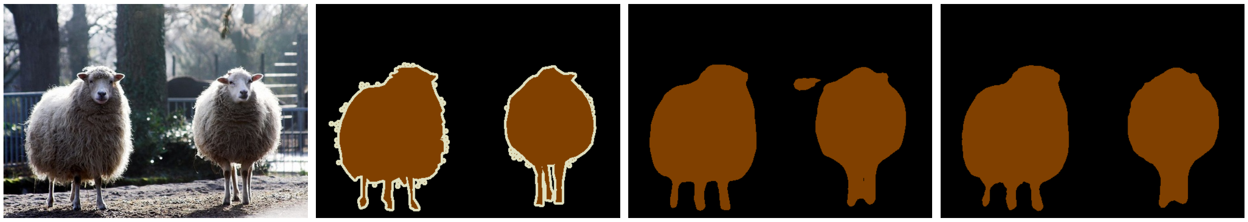
□

2. Fully Dense Network based on MobileNet-V2

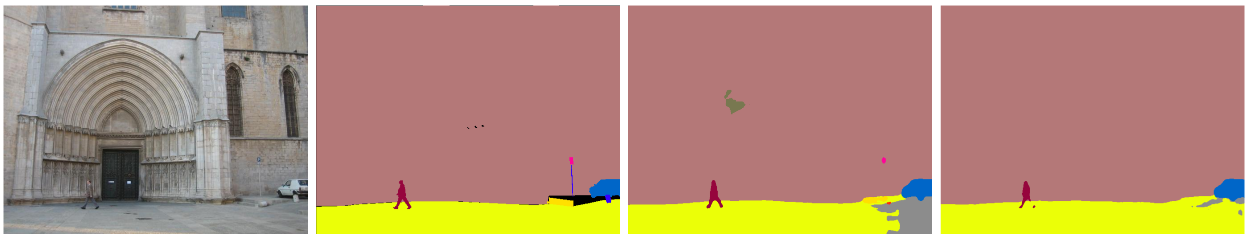
Figure 1 presents the Fully Dense Network based on MobileNet-V2. The inputs to the red circle (**U**) are multiple feature sets, while the output is the union of all the sets. **F** is the decoder stage, which takes a feature set as the input.

3. Visual Results

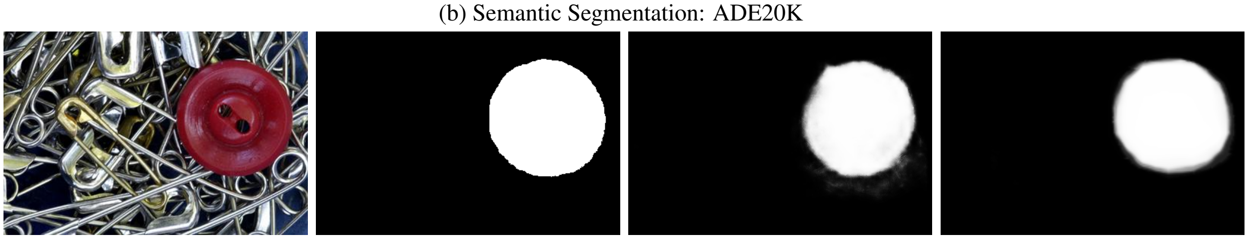
The visual results for experiments in Section 5 (main text) are shown in Figure 2.



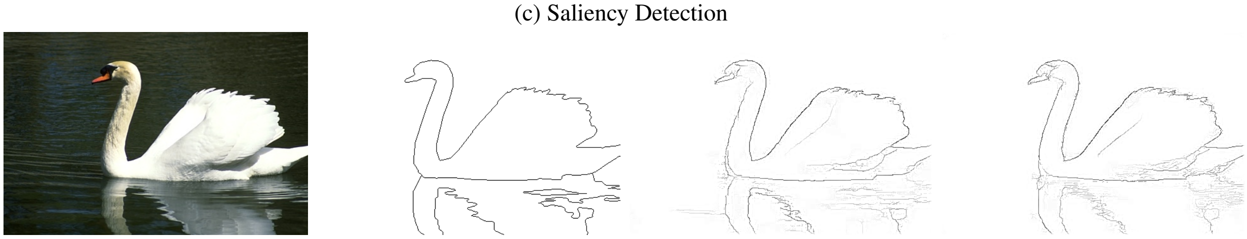
(a) Semantic Segmentation: PASCAL VOC 2012



(b) Semantic Segmentation: ADE20K



(c) Saliency Detection



(d) Edge Detection

Figure 2: **Qualitative Results.** Our method is not only quantitively but also qualitatively comparable to the baseline method.

References

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