## Low-level Vision by Consensus in a Spatial Hierarchy of Regions: Supplementary Material

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## **Appendix A: Inference Algorithm Listing**

We present here a summary of the inference algorithm for reference. It takes as input the following elements:

- 1. Sets of regions  $P_k, k \in \{1, ... K\}$  at K different scales.
- 2. For each region  $p \in P_k, k > 1$ , a set  $H_p$  consisting of non-overlapping child regions that partition p, and are from scales smaller than k (i.e.  $H_p \subset \bigcup_{k'=1}^{k-1} P_{k'}$ ).
- 3. The data cost functions  $D_p(\cdot)$  and scalar outlier costs  $\tau_p$  for every region p.
- 4. The value of the consistency weight  $\lambda$ . Additionally, its value  $\lambda_0$  to be used at the beginning of the iterations, the factor  $\lambda_f > 1$  by which it is to be increased at every  $T_{\lambda}$  iterations.
- 5. An initial estimate  $Z_0(n)$  of the scene value map.

Given these elements, the algorithm to minimize the cost function L is reproduced below:

```
# Initialization
Set \lambda^* = \lambda_0, Z(n) = Z_0(n) for all n.
In parallel, for all p \in P_1:
   Set Q_p = \sum_{n \in p} U(n)^T U(n).
for k = 2 \dots K
   In parallel, for all p \in P_k:
      Set Q_p = \sum_{c \in H_p} Q_c.
   end;
end;
# Main Iterations
for iters = 1 \dots MAXITERS
   # Upsweep
   In parallel, for all p \in P_1:
      Set \phi_p = \sum_{n \in p} U(n)^T Z(n), e_p = \sum_{n \in p} ||Z(n)||^2.
   for k = 2 \dots K
      In parallel, for all p \in P_k:
        Set \phi_p, e_p as per (10).
      end;
   end;
   # Minimize
   In parallel, for all p \in P:
      Set \theta_p, I_p as per (8) and (9).
   end;
   # Downsweep
   for k = K, K - 1, \dots 1
      In parallel, for all p \in P_k:
         Set \theta_p^+, I_p^+ as per (11).
      end;
   end;
   In parallel, for all n:
      Set Z(n) as per (12).
   end;
   # Update \lambda^*
   Set \lambda^* = MIN(\lambda^* \times \lambda_f, \lambda) if mod(iters, T_{\lambda}) = 0.
```

## **Appendix B: Evolution of Consensus Objective during Optimization**

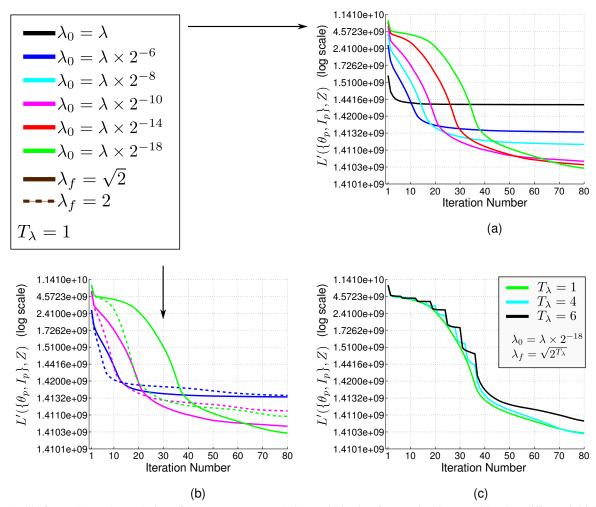


Figure 5. This figure shows the evolution of the consensus cost during optimization for a typical image with using different initial values  $\lambda_0$  and update schedules  $\lambda_f$ ,  $T_\lambda$  (see Appendix A) for  $\lambda'$ . The consensus cost shown is computed with the true value of the consistency weight  $\lambda$  (even for the iterations when minimization is done with lower values  $\lambda'$ ), and the occlusion-based correction step is omitted.

As described in Sec. 4, to avoid poor local minima and promote convergence to a good solution with a low cost, we use a lower value  $\lambda'$  of the consistency weight in the early iterations of the alternating minimization method, and increase it slowly to the desired weight  $\lambda$ . Figure 5 illustrates the effect of different schedules for  $\lambda'$  on convergence for a typical example. In Fig. 5 (a), we show the evolution of the objective starting with different values of  $\lambda' = \lambda_0$ , and increasing it by a constant factor of  $\lambda_f = \sqrt{2}$  at every iteration, and keeping it fixed after it reaches  $\lambda' = \lambda$ . We see that the direct alternating minimization case ( $\lambda_0 = \lambda$ ) decreases the consensus cost sharply in the first few iterations, but then stagnates at a local minima with a relatively high cost. As we lower the starting value of  $\lambda_0$ , the cost has higher values and decreases more gradually in the initial iterations, but continues to decrease over a larger number of iterations and eventually converges to a better solution with a lower cost. Figure 5 (b) explores the effect of a higher rate  $\lambda_f$  of increasing  $\lambda'$ . We see that like with a lower starting value for  $\lambda_0$ , a slower rate  $\lambda_f$  leads to convergence to a better solution, albeit more gradually.

In addition to requiring more iterations to converge, another computational penalty of changing  $\lambda'$  across iterations is that it requires re-doing any pre-computations that depend on the consistency weight. For our stereo algorithm, minimizing the sum of the data and consistency costs involves solving a  $3\times 3$  linear system for each region, and changing the value of  $\lambda'$  requires re-doing the LDL decompositions of the system matrices. Since this is expensive, it is desirable to avoid changing the value of  $\lambda'$  at every iteration. In Fig. 5 (c), we consider different cases with the same value of  $\lambda_0$  while jointly setting the increase factor  $\lambda_f$ , and the interval  $T_\lambda$  at which it is applied, so that the total number of iterations taken for  $\lambda'$  to reach its final value  $\lambda$  remains the same (i.e., by applying a higher rate at larger intervals). We see that choosing higher intervals leads to a "stair-casing" effect in the evolution of the objective, but the solution it converges to is only worse by a relatively small margin. We find this to be an acceptable trade-off between convergence to a low-cost solution and limiting computational expense, and use the parameters  $\lambda_0 = \lambda/2^{18}$ ,  $\lambda_f = \sqrt{2^{T_\lambda}}$ ,  $T_\lambda = 6$  in our stereo implementation.