Line Drawing Interpretation in a Multi-View Context
(Supplementary Material)

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Details on the computation of $\mathcal{M}(l)$

We detail here the resolution of the quadratic minimization problem under linear constraints formulated in Section 4.3. We want to minimize the function ϵ

$$\epsilon = \sum_{i \in \mathcal{V}} \|RP_i - SP_i p_i\|^2 \tag{1}$$

under the following linear constraints

$$(\forall i \in \mathcal{F}) \sum_{j \in \mathcal{E}} c_{ij} \lambda_j v_j = 0 \tag{2}$$

Each vertex P_i is defined in homogeneous coordinates using

$$P_i = P_0 + \sum_{j \in \mathcal{E}} \delta_{ij} \lambda_j v_j \tag{3}$$

where P_0 is the reference vertex expressed in the camera frame as

$$P_0 = \begin{pmatrix} \alpha\widehat{n} + \beta\widehat{u} + \gamma\widehat{v} \\ 0 \end{pmatrix} + \begin{pmatrix} 0_3 \\ 1 \end{pmatrix} \tag{4}$$

where $\widehat{n}$ is the camera direction. P_0 is rewritten in a more compact form as

$$P_0 = \omega + \sum_{k=1}^{2} \eta_k \widehat{n}_k \quad \text{with} \begin{cases} \eta_1 \widehat{n}_1 = \begin{pmatrix} \beta\widehat{u} \\ 0 \end{pmatrix} \\ \eta_2 \widehat{n}_2 = \begin{pmatrix} \gamma\widehat{v} \\ 0 \end{pmatrix} \\ \omega = \begin{pmatrix} \alpha\widehat{n} \\ 1 \end{pmatrix} \end{cases} \tag{5}$$

By first replacing the expression of P_0 from Eq. 5 into Eq. 3, and then replacing the expression of P_i into Eq. 1, we can formulate ϵ as

$$\epsilon = \sum_{i \in \mathcal{V}} \|R\omega - S\omega p_i + \sum_{k=1}^{2} \eta_k (R\widehat{n}_k - S\widehat{n}_k p_i) + \sum_{j \in \mathcal{E}} \delta_{ij} \lambda_j (Rv_j - Sv_j p_i)\|^2 \tag{6}$$

This can also be formulated as

$$\epsilon = \lambda^T A_0 \lambda + \lambda^T B_2 \eta + \eta^T J \eta + K^T \eta + C + B_1^T \lambda \tag{7}$$

where A_0, B_1, B_2, J, K and C are matrices defined by

$$\begin{cases} (\forall\,(j,k) \in [\![1, |\mathcal{E}|]\!]^2)\; A_0^{jk} = \sum_{i\in\mathcal{V}} < \delta_{ij}(Rv_j - Sv_j p_i),\; \delta_{ik}(Rv_k - Sv_k p_i) > \\ (\forall\,(j,k) \in [\![1, |\mathcal{E}|]\!] \times [\![1,2]\!])\; B_2^{jk} = \sum_{i\in\mathcal{V}} 2\delta_{ij} < Rv_j - Sv_j p_i, R\widehat{n}_k - S\widehat{n}_k p_i > \\ (\forall\,(j,k) \in [\![1,2]\!]^2)\; J_{jk} = \sum_{i\in\mathcal{V}} < R\widehat{n}_j - S\widehat{n}_j p_i, R\widehat{n}_k - S\widehat{n}_k p_i > \\ (\forall\, j \in [\![1,2]\!])\; K_j = \sum_{i\in\mathcal{V}} 2 < R\widehat{n}_j - S\widehat{n}_j p_i, R\omega - S\omega p_i > \\ C = \sum_{i\in\mathcal{V}} < R\omega - S\omega p_i, R\omega - S\omega p_i > \\ (\forall\, j \in [\![1, |\mathcal{E}|]\!])\; B_1^j = \sum_{i\in\mathcal{V}} 2\delta_{ij} < Rv_j - Sv_j p_i, R\omega - S\omega p_i > \end{cases} \tag{8}$$

Eq. 7 can be rewritten as

$$\epsilon = \begin{pmatrix} \lambda^T & \eta^T \end{pmatrix} \begin{pmatrix} A_0 & B_3 \\ B_3^T & J \end{pmatrix} \begin{pmatrix} \lambda \\ \eta \end{pmatrix} + \begin{pmatrix} B_1^T & K^T \end{pmatrix} \begin{pmatrix} \lambda \\ \eta \end{pmatrix} + C \tag{9}$$

where $B_3 = \frac{1}{2}B_2$. We have indeed $\lambda^T B_2 \eta = \lambda^T B_3 \eta + \eta^T B_3^T \lambda$.

This is also equivalent to

$$\epsilon = X^T A X + B X + C \tag{10}$$

where X, A and B are matrices give by

$$X = \begin{pmatrix} \lambda \\ \eta \end{pmatrix}, A = \begin{pmatrix} A_0 & B_3 \\ B_3^T & J \end{pmatrix}, B = \begin{pmatrix} B_1^T & K^T \end{pmatrix} \tag{11}$$

The equation (12) is the constraints defined in (2) rewritting in matrix mode using (13) and (14).

$$DX = 0 \tag{12}$$

$$D = \begin{pmatrix} D_1 & 0 \\ D_2 & 0 \\ D_3 & 0 \end{pmatrix} \tag{13}$$

$$\begin{cases} (\forall\,(i,j) \in [\![1, |\mathcal{F}|]\!] \times [\![1, |\mathcal{E}|]\!])\; D_1^{ij} = c_{ij} v_j^x \\ (\forall\,(i,j) \in [\![1, |\mathcal{F}|]\!] \times [\![1, |\mathcal{E}|]\!])\; D_2^{ij} = c_{ij} v_j^y \\ (\forall\,(i,j) \in [\![1, |\mathcal{F}|]\!] \times [\![1, |\mathcal{E}|]\!])\; D_3^{ij} = c_{ij} v_j^z \end{cases} \tag{14}$$

The new formulation of the problem is to find $\widehat{X}$ such that the equation (15) is respected.

$$\widehat{X} = \underset{X \in Ker(D)}{\text{argmin}} \left(X^T A X + B X + C\right) \tag{15}$$

We solve Eq. (15) by Lagrange multipliers.