

# Robust Image Filtering Using Joint Static and Dynamic Guidance

## Supplementary Material

Bumsub Ham<sup>1,\*</sup>

Minsu Cho<sup>1,\*</sup>

Jean Ponce<sup>2,\*</sup>

<sup>1</sup>Inria

<sup>2</sup>École Normale Supérieure / PSL Research University

### 1. Proof

**Proposition 1.**  $\Psi_\nu^y(x)$  is a surrogate function of  $\psi_\nu(x)$ .

*Proof.* A function  $g^y(x)$  is said to a surrogate function of  $f(x)$  provided

$$\begin{cases} f(x) \leq g^y(x), \forall x, y \in [0, \infty) \\ f(y) = g^y(y), \forall y \in [0, \infty) \end{cases}. \quad (1)$$

Let us consider the following function:

$$f_\nu(x) = (1 - \exp(-\nu x))/\nu. \quad (2)$$

Since  $f_\nu''(x) < 0$ , this function is strictly concave,  $\forall x, y$ ,

$$f_\nu(x) \leq f_\nu(y) + (x - y)f'_\nu(y), \quad (3)$$

with equality holding at  $x = y$ . Thus, the surrogate function  $g_\nu^y(x)$  is

$$\begin{aligned} g_\nu^y(x) &= f_\nu(y) + (x - y)f'_\nu(y) \\ &= f_\nu(y) + (x - y)\exp(-\nu y) \\ &= f_\nu(y) + (x - y)(1 - \nu f_\nu(y)). \end{aligned} \quad (4)$$

Let us substitute  $x$  and  $y$  with  $x^2$  and  $y^2$ , respectively, then

$$\begin{aligned} \psi_\nu(x) &= f_\nu(x^2) \\ &\leq g_\nu^{y^2}(x^2), \end{aligned} \quad (5)$$

and

$$\begin{aligned} g_\nu^{y^2}(x^2) &= f_\nu(y^2) + (x^2 - y^2)(1 - \nu f_\nu(y^2)) \\ &= \psi_\nu(y) + (x^2 - y^2)(1 - \nu \psi_\nu(y)) \\ &= \Psi_\nu^y(x). \end{aligned} \quad (6)$$

□

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\*WILLOW project-team, Département d’Informatique de l’Ecole Normale Supérieure, ENS/Inria/CNRS UMR 8548.

## 2. More Results

Table 1. Quantitative Evaluation of Upsampled Depth Maps on the Middlebury Test Bed [8]

$\mathbf{u}^0 = \mathbb{I}$	Tsukuba			Venus			Teddy			Cones			Art	Books	Dolls	Lau.	Moe.	Rei.
Method	$\mathcal{O}_{all}$	$\mathcal{O}_{disc}$	$\mathcal{O}_{all}$	$\mathcal{O}_{disc}$	$\mathcal{O}_{all}$	$\mathcal{O}_{disc}$	$\mathcal{O}_{all}$	$\mathcal{O}_{disc}$	$\mathcal{O}_{all}$									
Bilinear Int.	10.40	46.30	3.29	37.10	11.80	35.30	14.60	35.80	34.59	13.07	14.59	22.85	17.15	17.48				
GF [3]	9.87	43.20	2.74	26.50	15.50	37.50	15.50	34.40	45.31	19.14	19.64	26.89	22.22	21.70				
Park et al. [6]	—	—	—	—	10.10	25.40	9.71	19.90	27.81	11.18	15.23	18.27	13.82	12.35				
TGV [2]	5.40	23.90	1.31	14.40	9.82	29.00	10.20	23.60	23.59	13.14	16.14	15.12	18.26	10.42				
WMF [5]	6.14	28.00	1.03	10.10	7.88	22.20	8.10	19.20	22.13	<b>8.67</b>	<b>9.58</b>	14.26	10.52	10.04				
Ours	<b>2.39</b>	<b>10.40</b>	<b>0.55</b>	<b>5.10</b>	<b>7.39</b>	<b>20.30</b>	<b>5.24</b>	<b>12.40</b>	<b>10.04</b>	9.79	10.84	<b>8.11</b>	<b>9.49</b>	<b>6.93</b>				

## References

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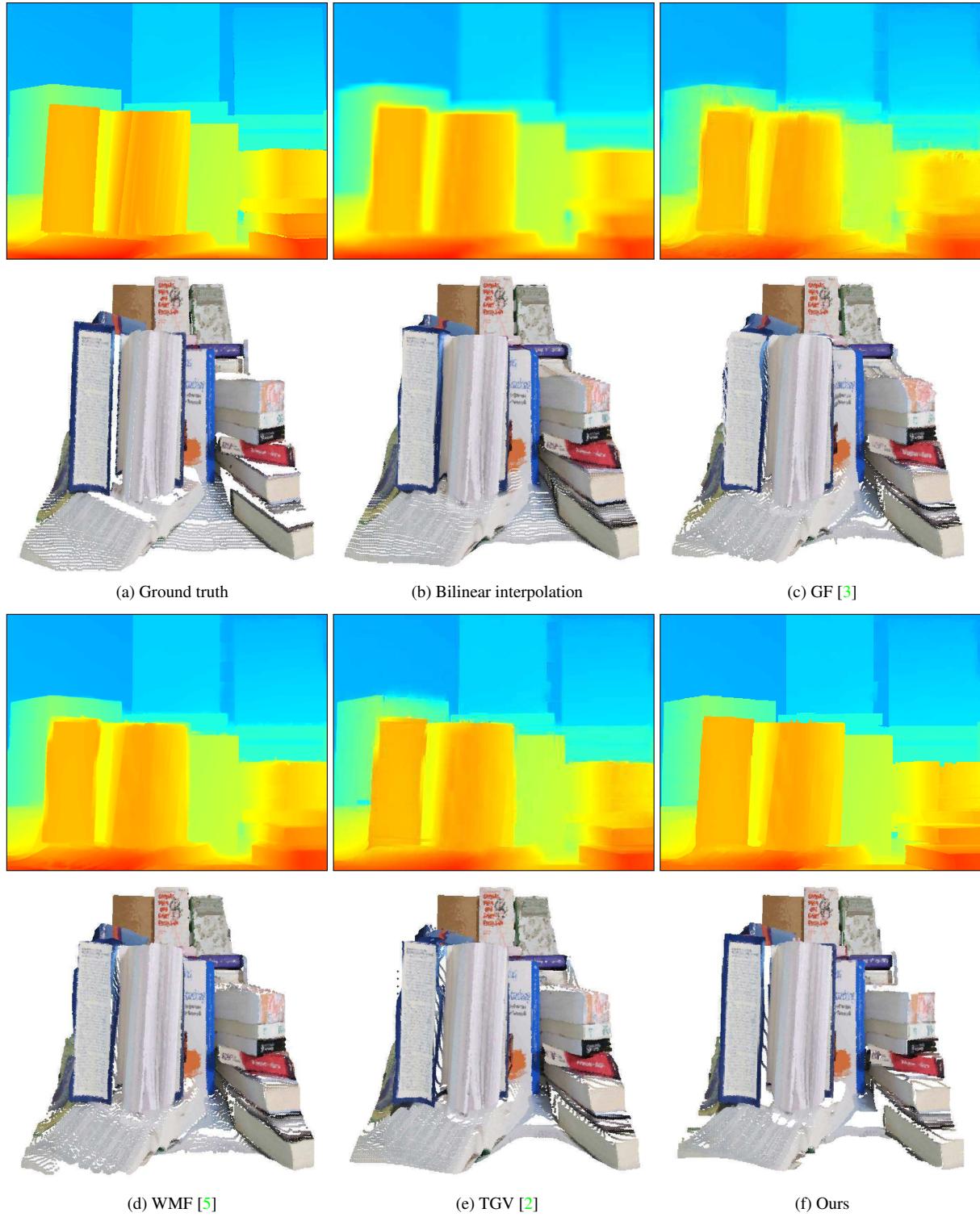


Figure 1. Visual comparison of ( $\times 8$ ) upsampled depth maps and point cloud reconstructions on *books* sequence in the Middlebury test bed [8].

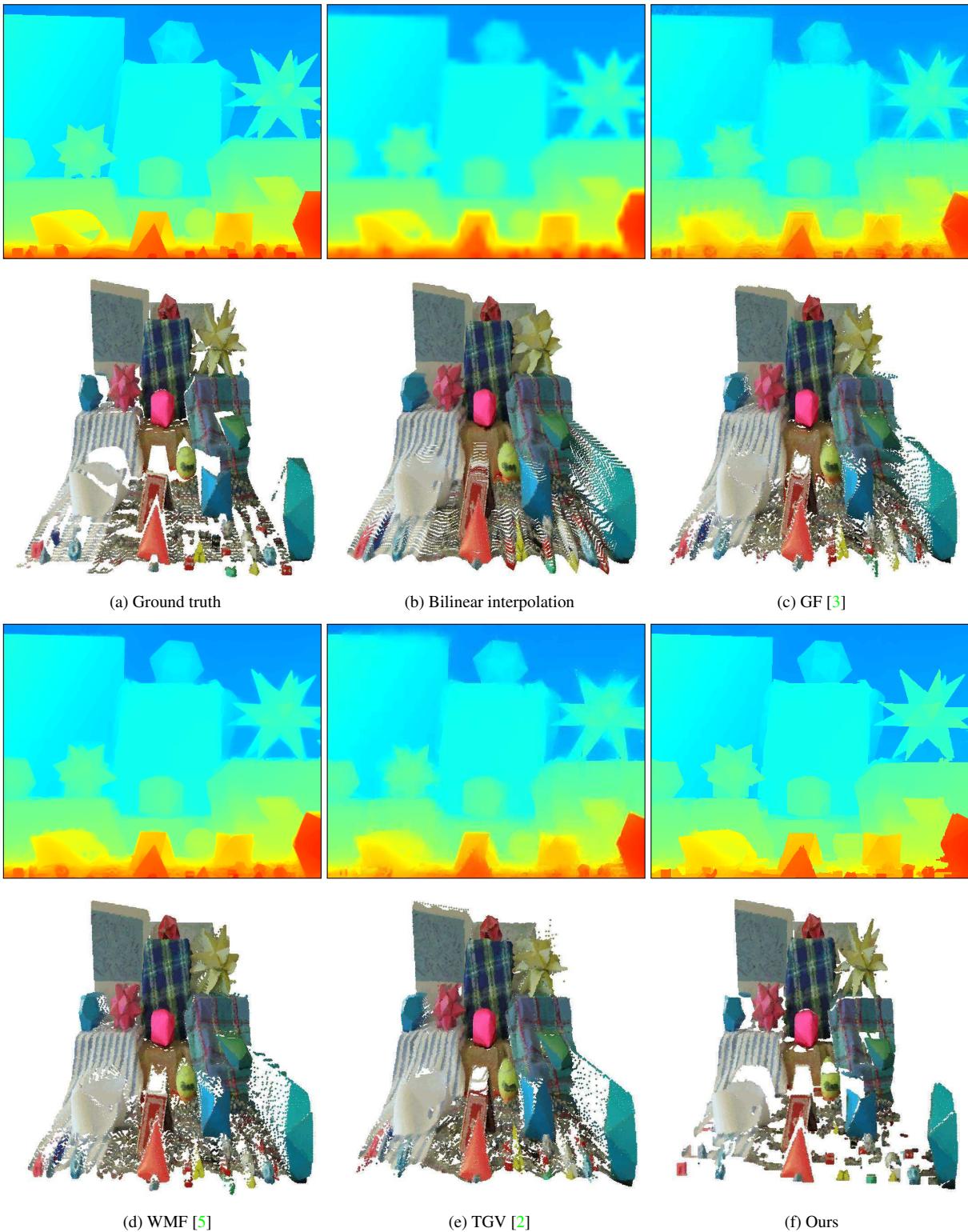


Figure 2. Visual comparison of ( $\times 8$ ) upsampled depth maps and point cloud reconstructions on *moebius* sequence in the Middlebury test bed [8].

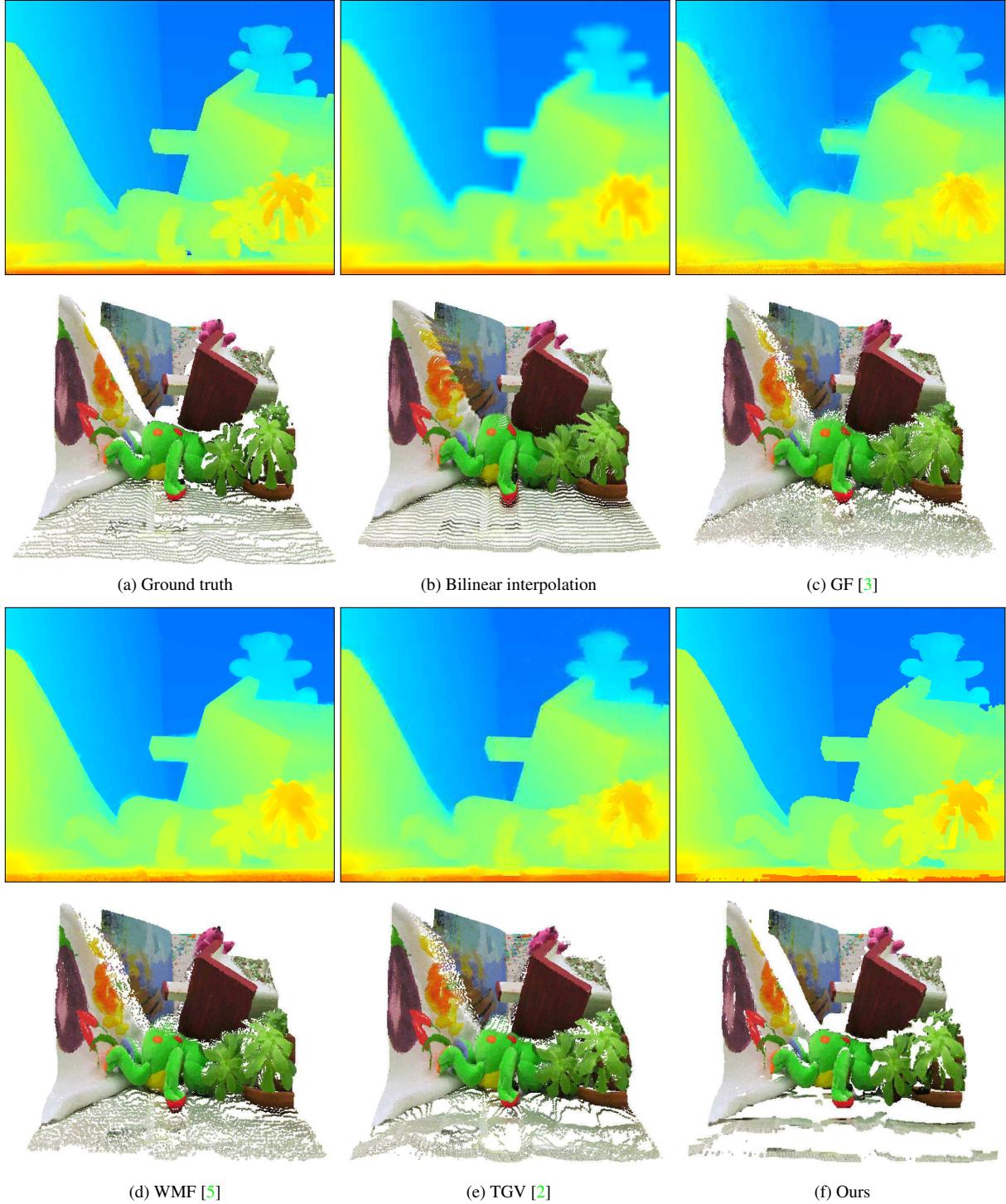


Figure 3. Visual comparison of ( $\times 8$ ) upsampled depth maps and point cloud reconstructions on *teddy* sequence in the Middlebury test bed [8].

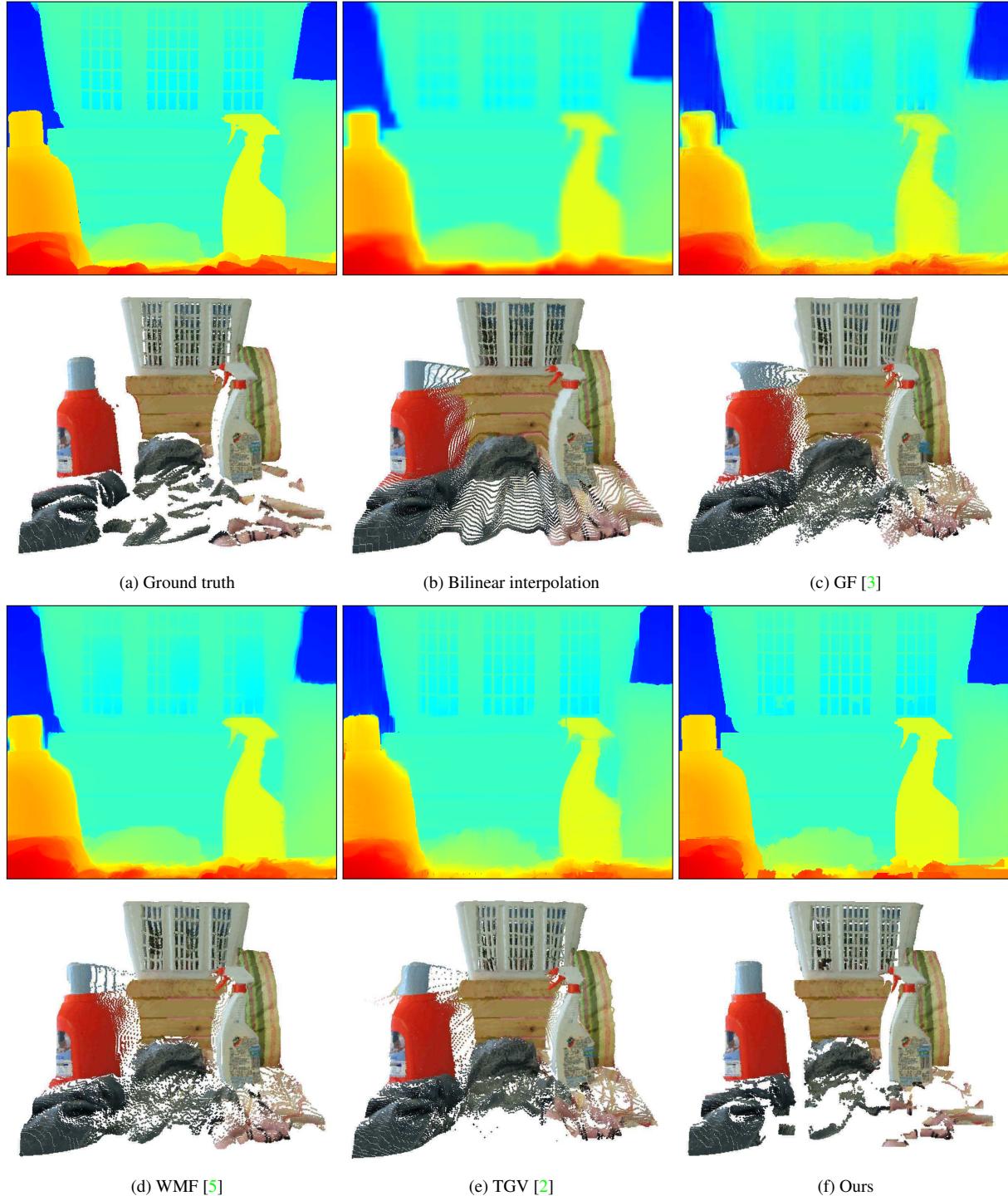


Figure 4. Visual comparison of ( $\times 8$ ) upsampled depth maps and point cloud reconstructions on *laundry* sequence in the Middlebury test bed [8].

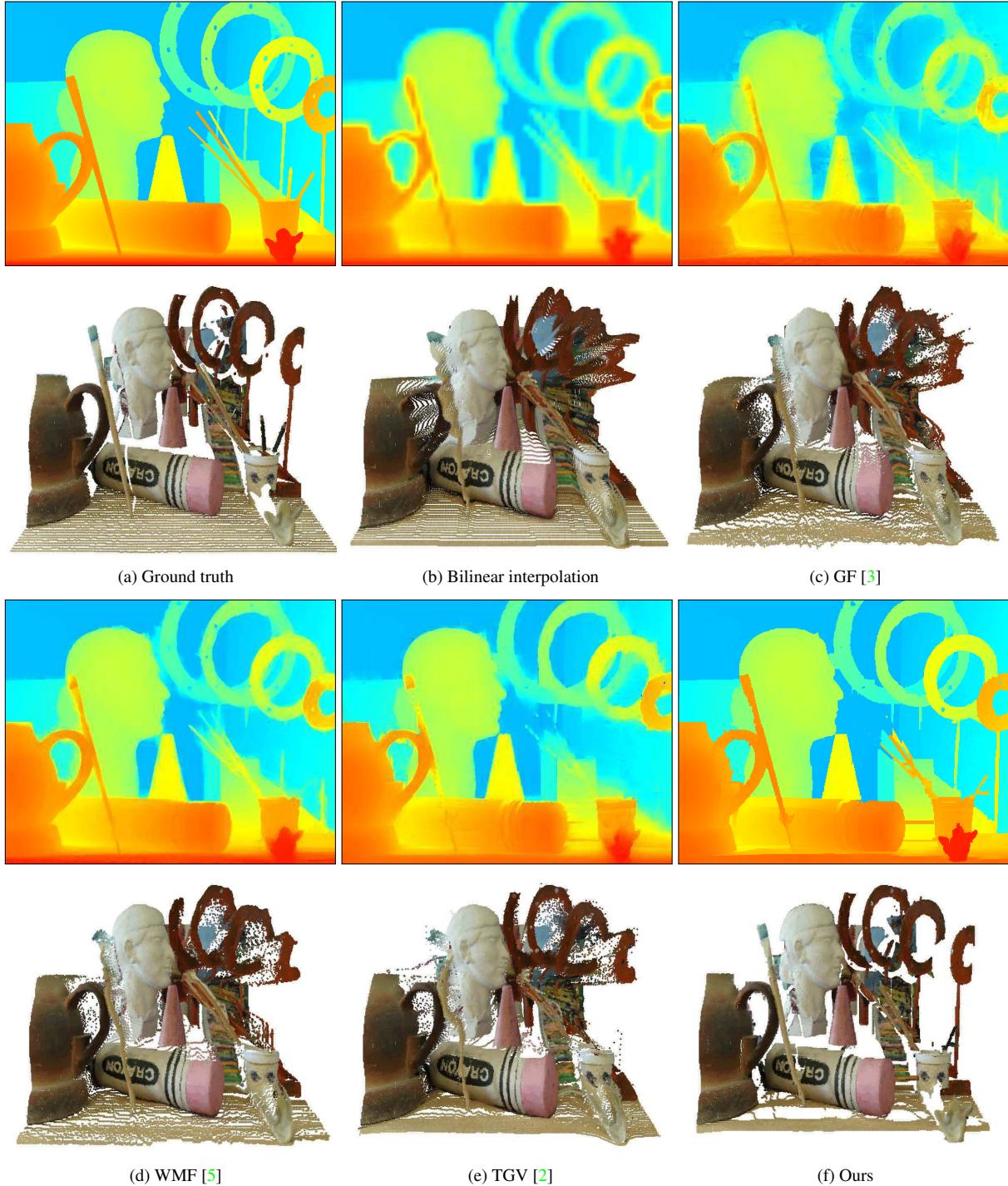


Figure 5. Visual comparison of ( $\times 8$ ) upsampled depth maps and point cloud reconstructions on *art* sequence in the Middlebury test bed [8].

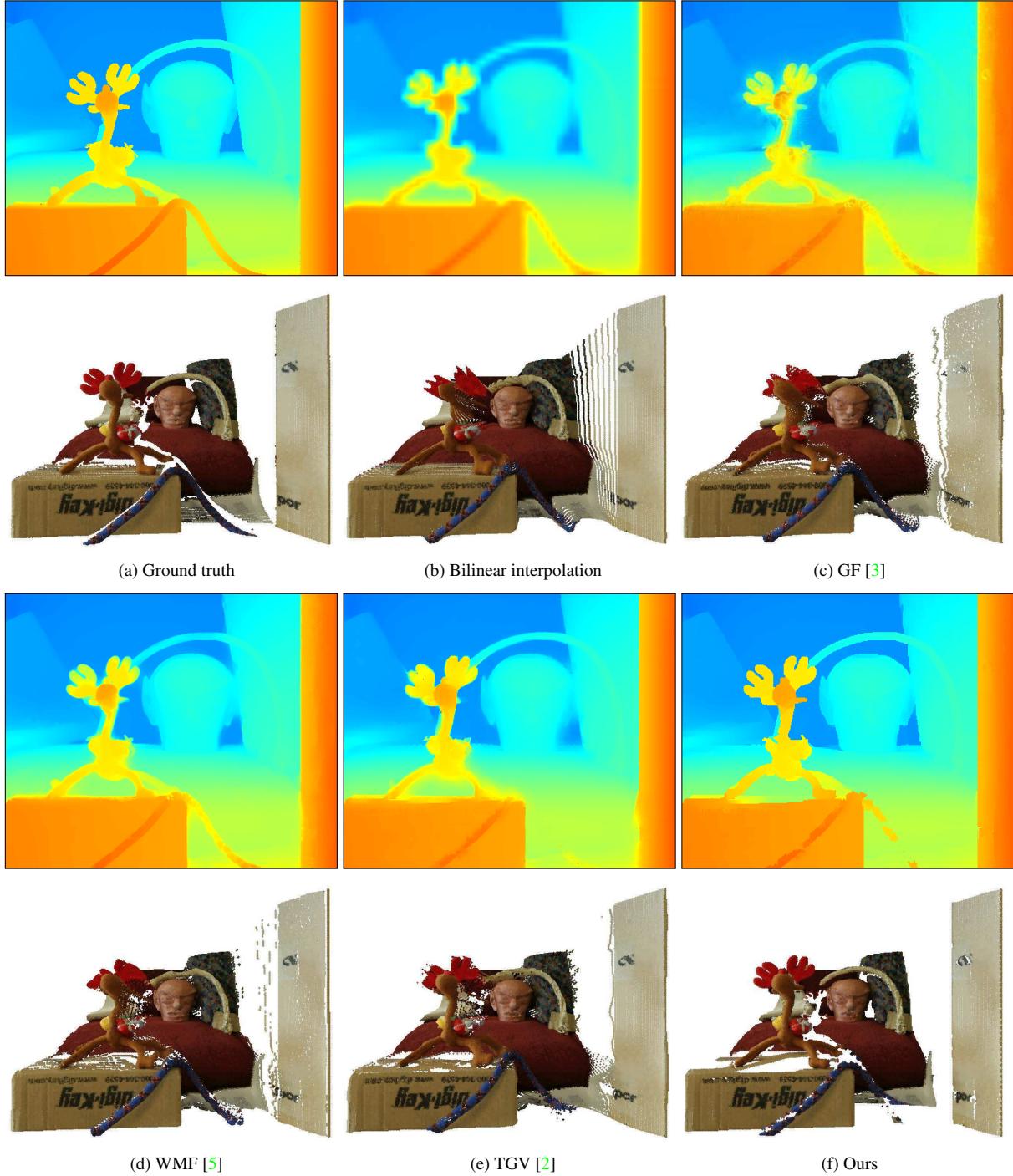


Figure 6. Visual comparison of ( $\times 8$ ) upsampled depth maps and point cloud reconstructions on *reindeer* sequence in the Middlebury test bed [8].

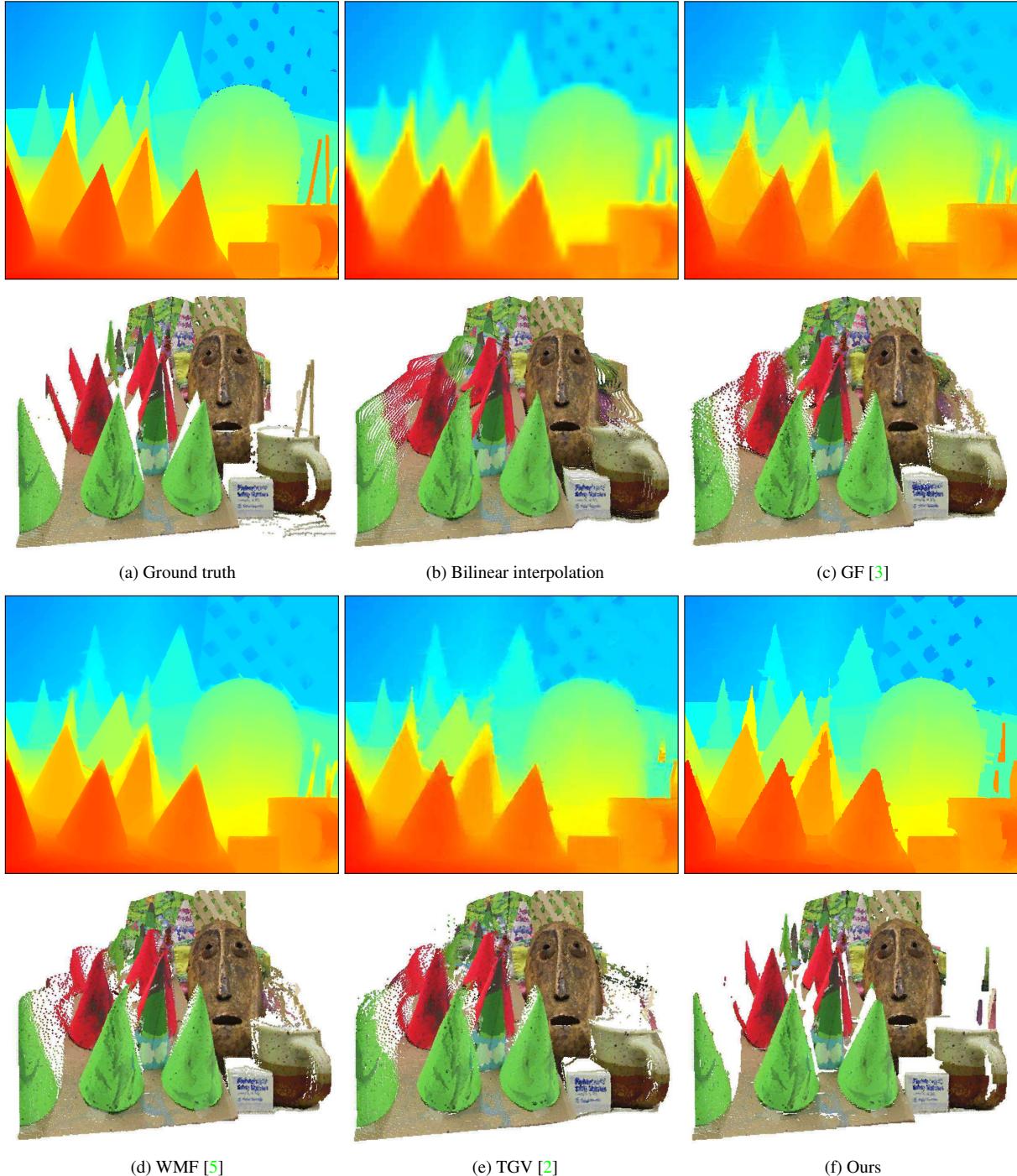


Figure 7. Visual comparison of ( $\times 8$ ) upsampled depth maps and point cloud reconstructions on *cones* sequence in the Middlebury test bed [8].

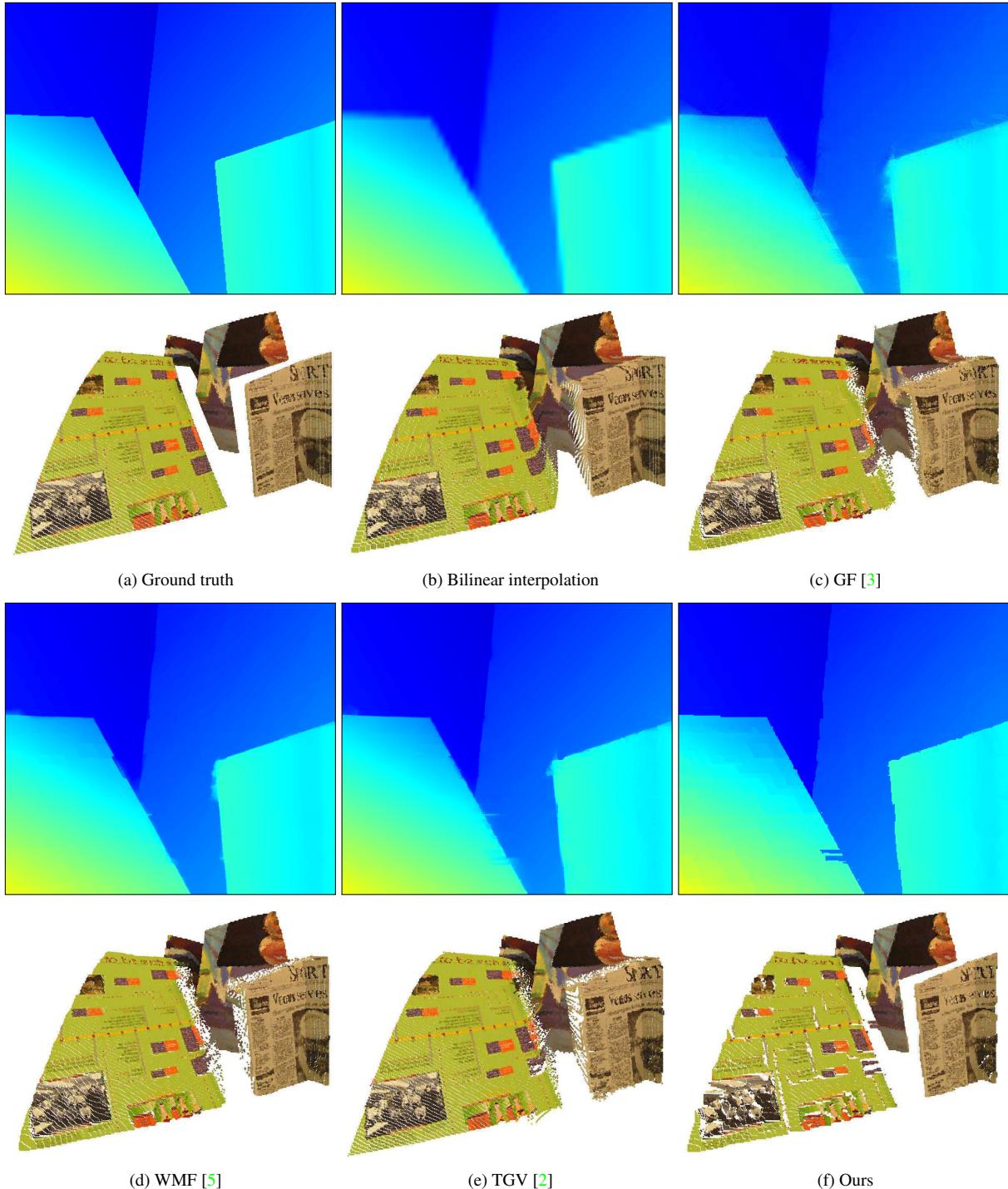


Figure 8. Visual comparison of ( $\times 8$ ) upsampled depth maps and point cloud reconstructions on *venus* sequence in the Middlebury test bed [8].

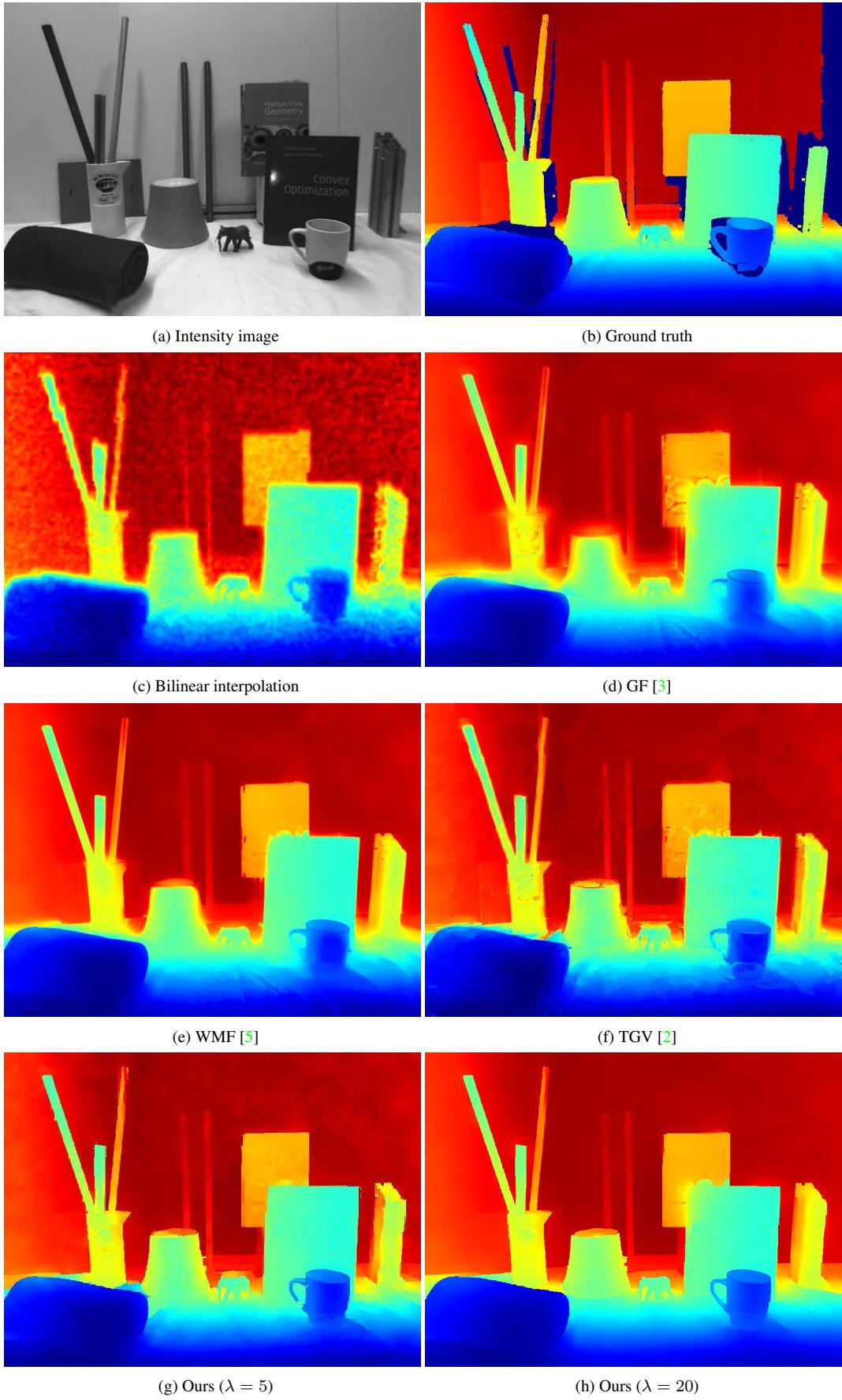


Figure 9. Visual comparison of upsampled depth maps on *books* sequence in the Graz data sets [2].

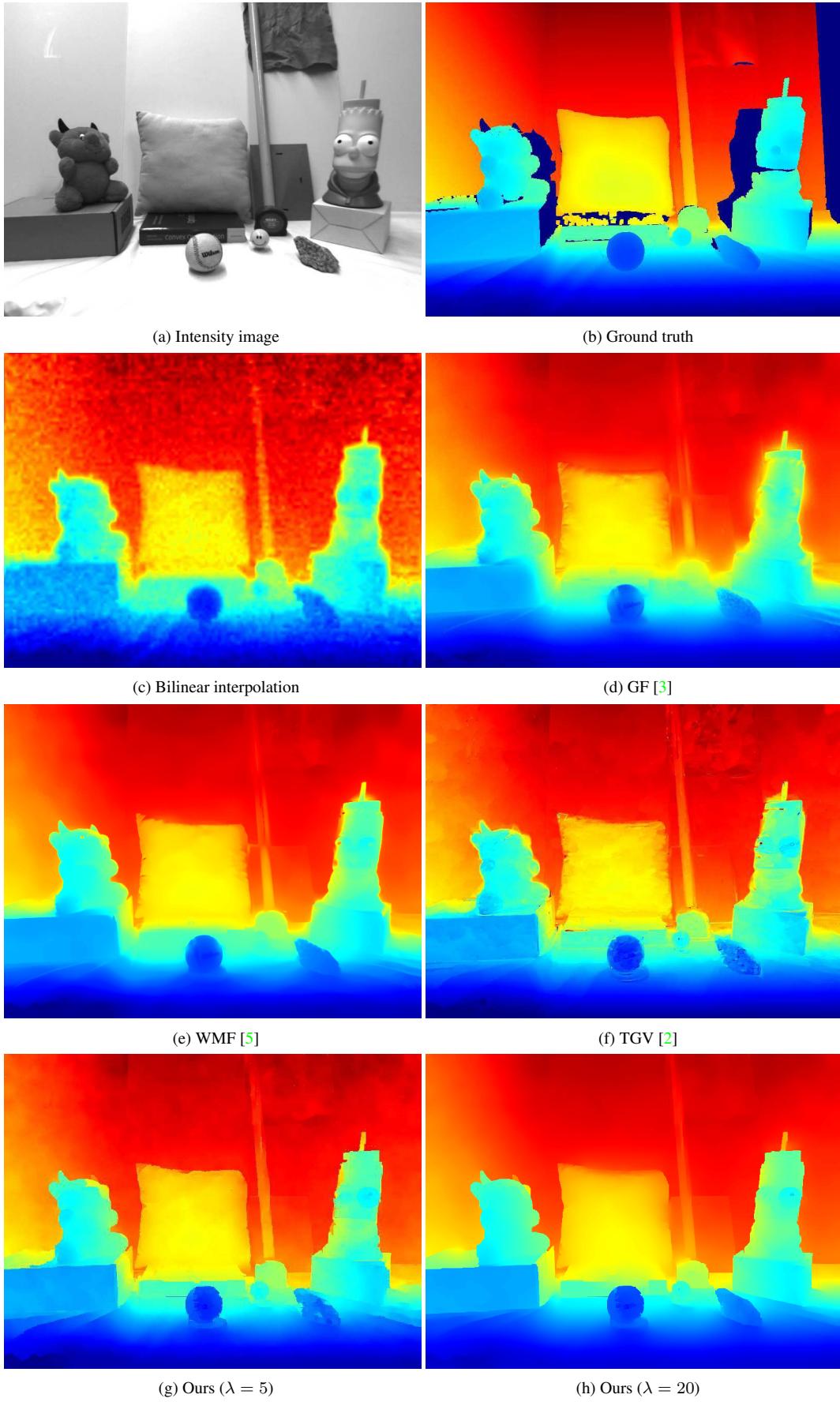


Figure 10. Visual comparison of upsampled depth maps on *devil* sequence in the Graz data sets [2].

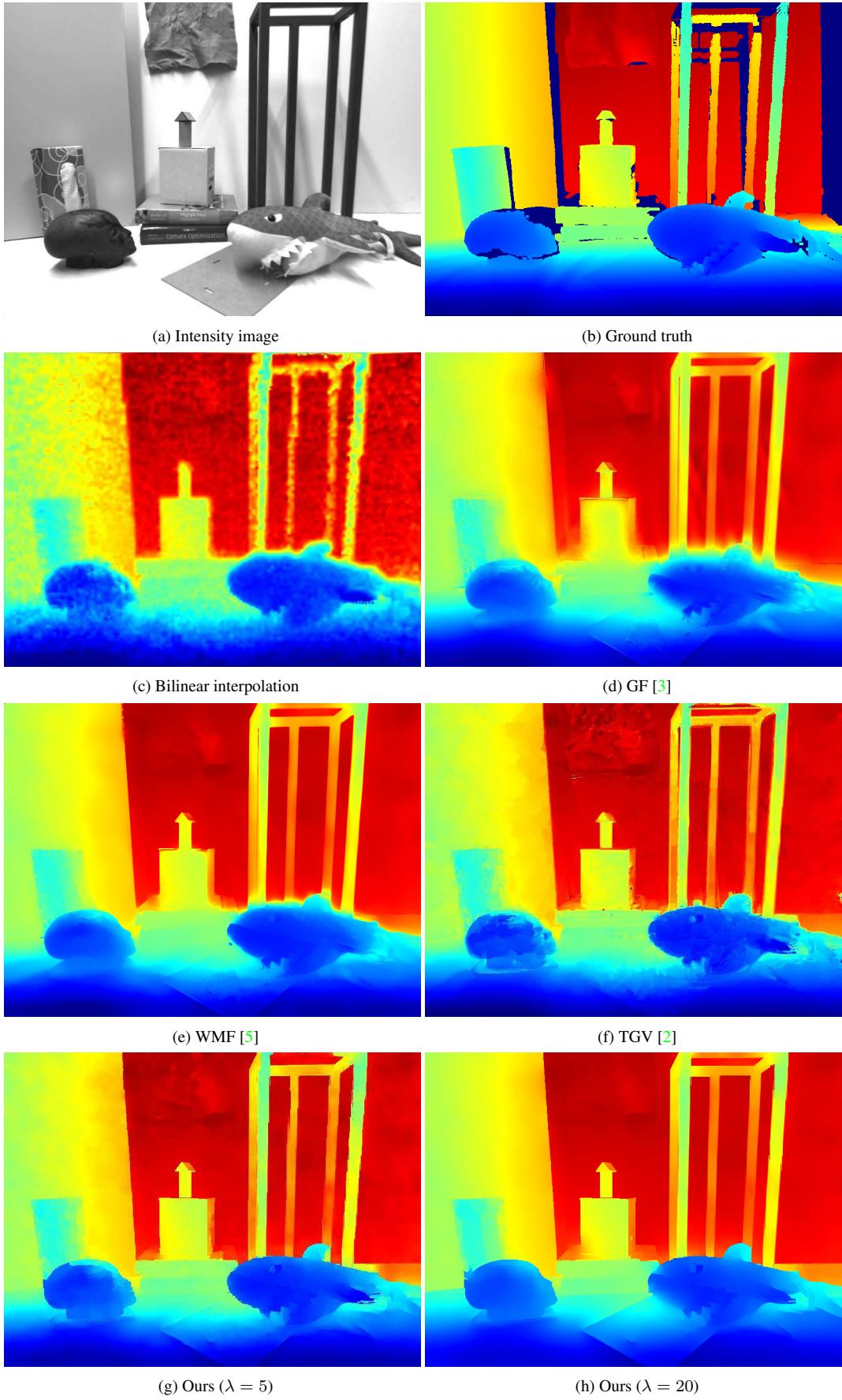


Figure 11. Visual comparison of upsampled depth maps on *shark* sequence in the Graz data sets [2].

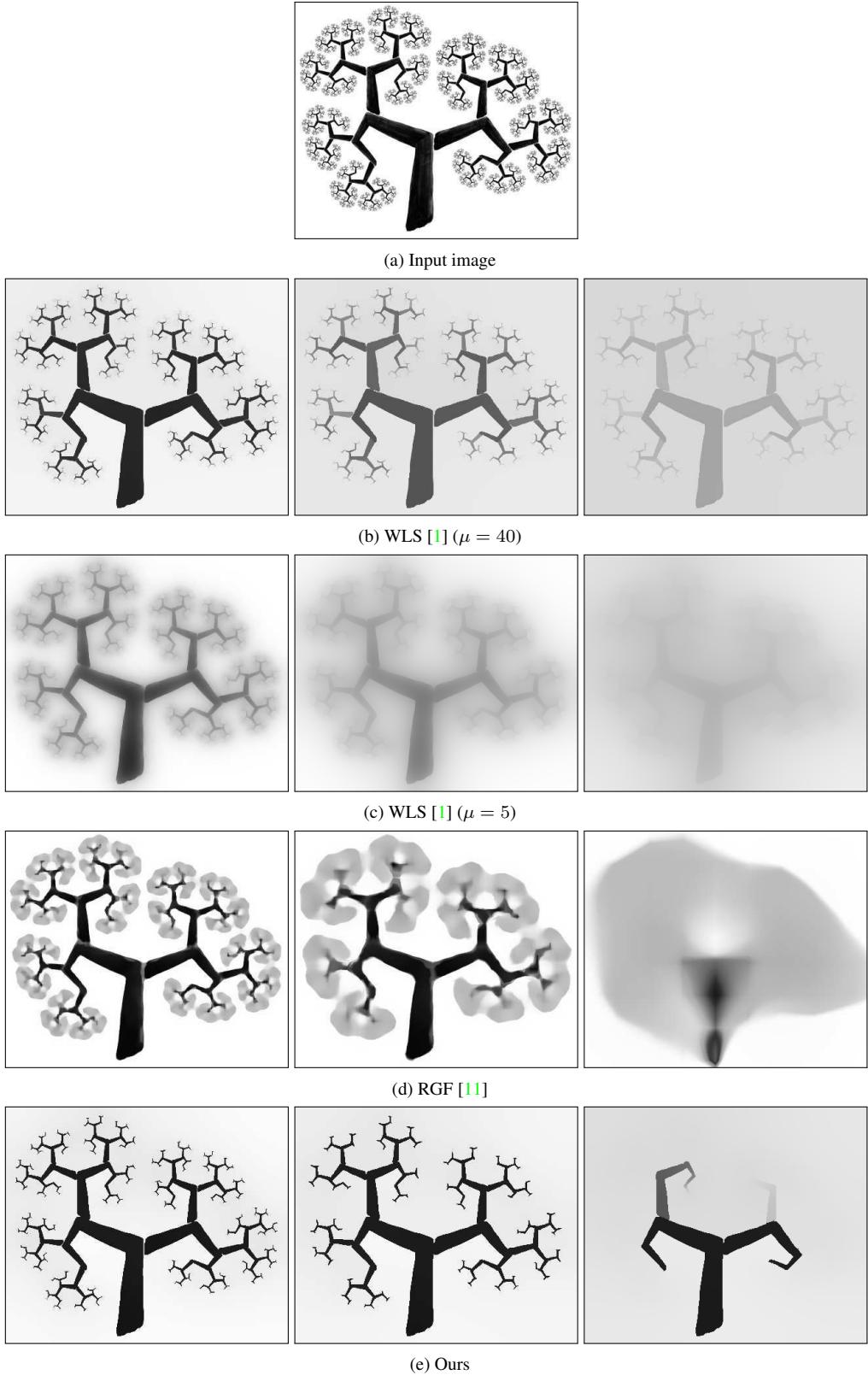


Figure 12. Examples of the scale space for (a) the input image, constructed by (b) WLS [1] [(from left to right)  $\lambda = 5 \times 10^3, 3 \times 10^4, 2 \times 10^5$ ,  $\mu = 40$ ], (c) WLS [1] [(from left to right)  $\lambda = 5 \times 10, 3 \times 10^2, 2 \times 10^3$ ,  $\mu = 5$ ], (d) RGF [11] [(from left to right)  $\sigma_s = 5, 10, 50$ ,  $\sigma_r = 0.05, k = 5$ ], (e) our model [ $\mathbf{u}^0 = \mathbf{u}_{l_1}$ , (from left to right)  $\lambda = 1 \times 10^3, 3 \times 10^3, 1 \times 10^4$ ,  $\mu = 5, \nu = 40, k = 5$ ].



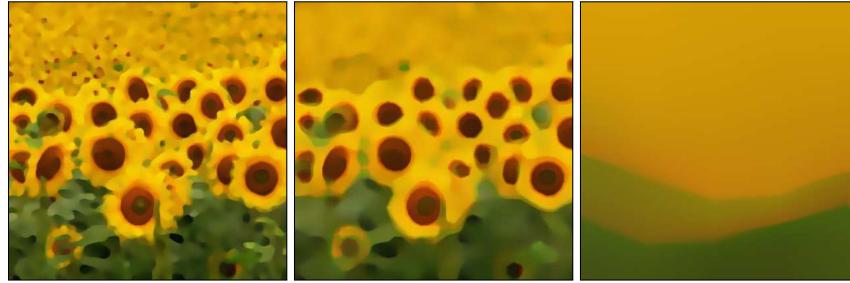
(a) Input image



(b) WLS [1] ( $\mu = 40$ )



(c) WLS [1] ( $\mu = 5$ )



(d) RGF [11]



(e) Ours

Figure 13. Examples of the scale space for (a) the input image, constructed by (b) WLS [1] [(from left to right)  $\lambda = 5 \times 10^3, 3 \times 10^4, 4 \times 10^5, \mu = 40$ ], (c) WLS [1] [(from left to right)  $\lambda = 5 \times 10, 3 \times 10^2, 4 \times 10^3, \mu = 5$ ], (d) RGF [11] [(from left to right)  $\sigma_s = 5, 10, 100, \sigma_r = 0.05, k = 5$ ], (e) our model [ $\mathbf{u}^0 = \mathbf{u}_{l_1}$ , (from left to right)  $\lambda = 3 \times 10^2, 1 \times 10^3, 4 \times 10^3, \mu = 5, \nu = 40, k = 5$ ].

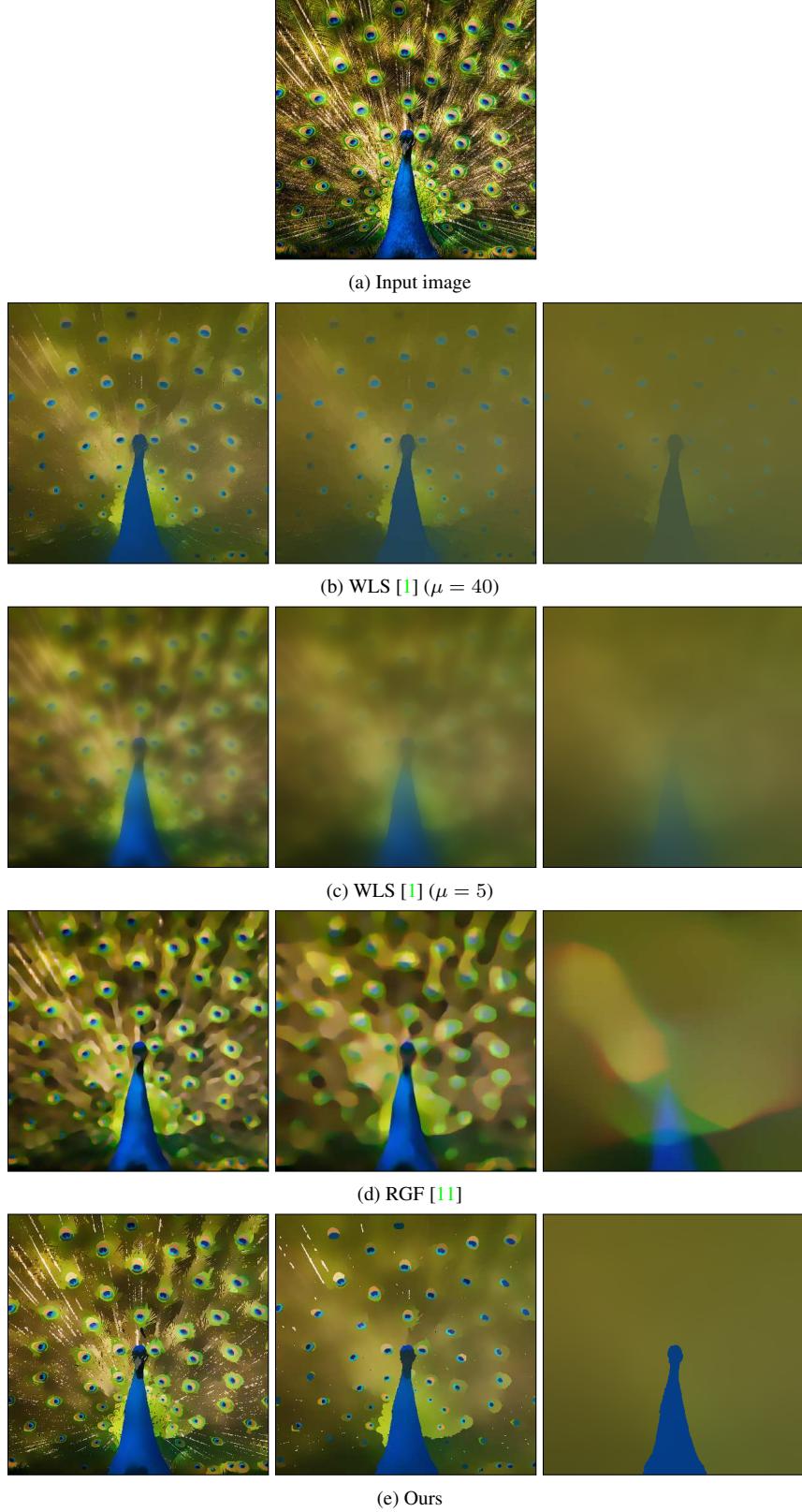


Figure 14. Examples of the scale space for (a) the input image, constructed by (b) WLS [1] [(from left to right)  $\lambda = 5 \times 10^3, 3 \times 10^4, 2 \times 10^5, \mu = 40$ ], (c) WLS [1] [(from left to right)  $\lambda = 5 \times 10, 3 \times 10^2, 2 \times 10^3, \mu = 5$ ], (d) RGF [11] [(from left to right)  $\sigma_s = 5, 10, 50, \sigma_r = 0.05, k = 5$ ], (e) our model [ $\mathbf{u}^0 = \mathbf{u}_{l_1}$ , (from left to right)  $\lambda = 5 \times 10, 3 \times 10^2, 1 \times 10^4, \mu = 5, \nu = 40, k = 5$ ].

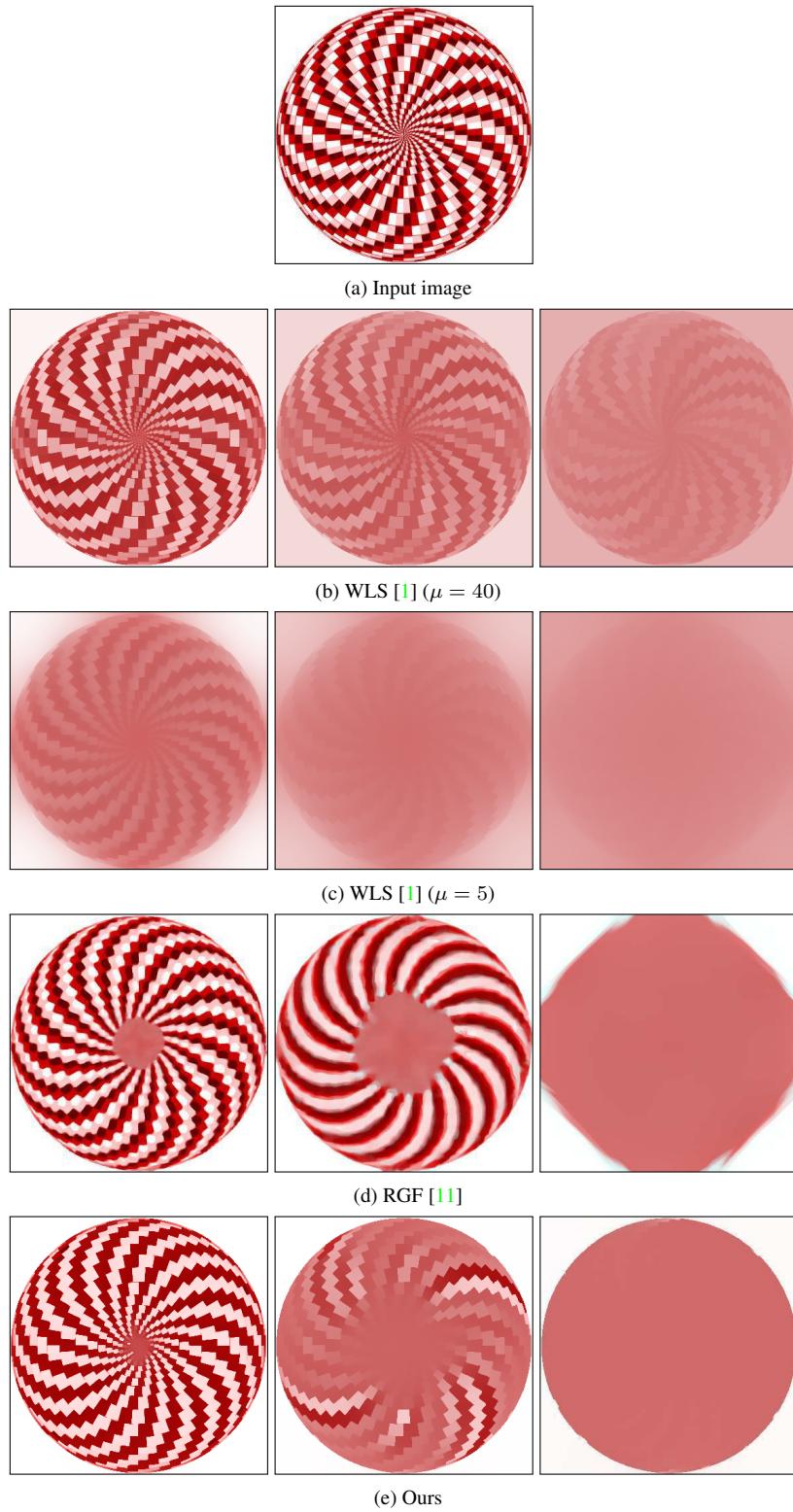


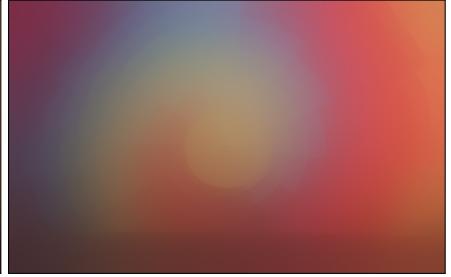
Figure 15. Examples of the scale space for (a) the input image, constructed by (b) WLS [1] [(from left to right)  $\lambda = 3 \times 10^4, 2 \times 10^5, 1 \times 10^6$ ,  $\mu = 40$ ], (c) WLS [1] [(from left to right)  $\lambda = 3 \times 10^2, 2 \times 10^3, 1 \times 10^4$ ,  $\mu = 5$ ], (d) RGF [11] [(from left to right)  $\sigma_s = 5, 10, 50$ ,  $\sigma_r = 0.05$ ,  $k = 5$ ], (e) our model [ $\mathbf{u}^0 = \mathbb{1}$ , (from left to right)  $\lambda = 3 \times 10^2, 2 \times 10^3, 1 \times 10^4$ ,  $\mu = 5, \nu = 40, k = 5$ ].



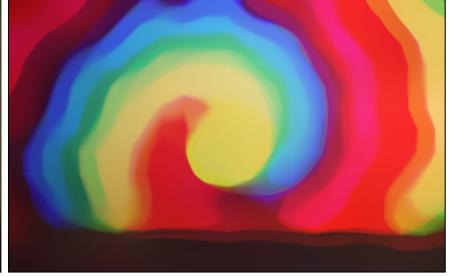
(a) Input image



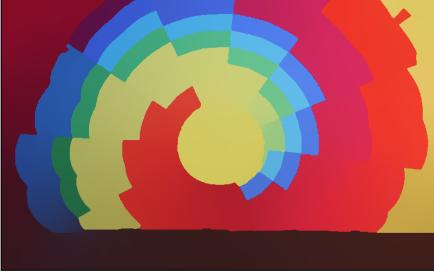
(b) WLS [1] ( $\mu = 40$ )



(c) WLS [1] ( $\mu = 5$ )



(d) RGF [11]

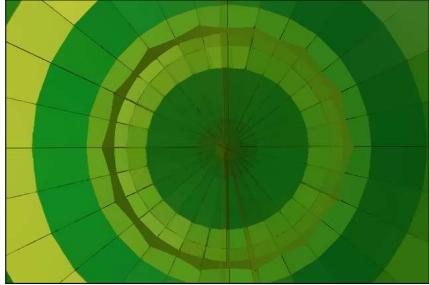


(e) Ours

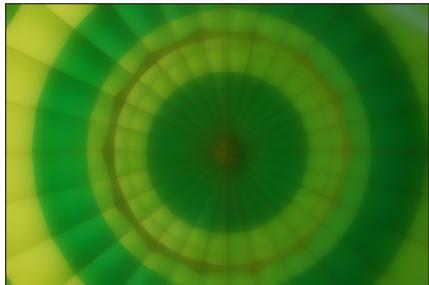
Figure 16. Examples of the scale space for (a) the input image, constructed by (b) WLS [1] [(from left to right)  $\lambda = 3 \times 10^4, 1 \times 10^5, 5 \times 10^5, \mu = 40$ ], (c) WLS [1] [(from left to right)  $\lambda = 3 \times 10^2, 2 \times 10^3, 5 \times 10^3, \mu = 5$ ], (d) RGF [11] [(from left to right)  $\sigma_s = 10, 20, 40, \sigma_r = 0.05, k = 5$ ], (e) our model [ $\mathbf{u}^0 = \mathbf{u}_{t_1}$ , (from left to right)  $\lambda = 1 \times 10^3, 4 \times 10^3, 1 \times 10^4, \mu = 5, \nu = 40, k = 5$ ].



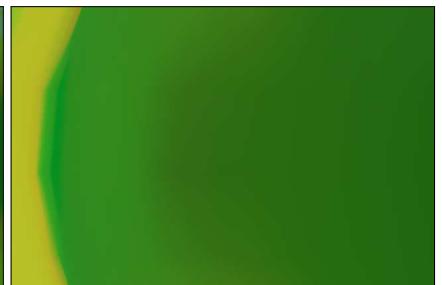
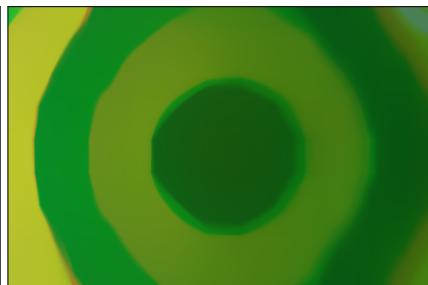
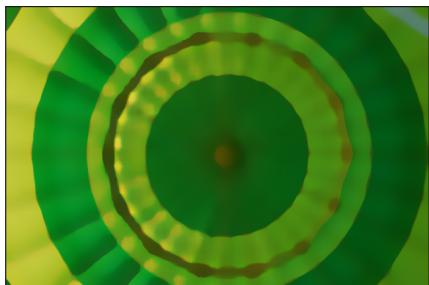
(a) Input image



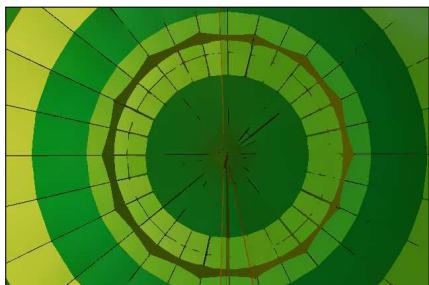
(b) WLS [1] ( $\mu = 40$ )



(c) WLS [1] ( $\mu = 5$ )



(d) RGF [11]

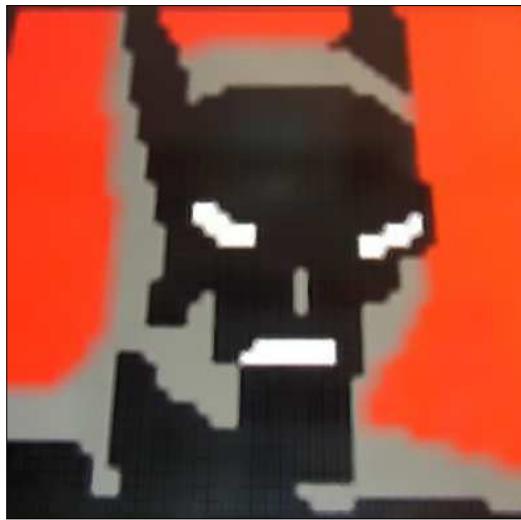


(e) Ours

Figure 17. Examples of the scale space for (a) the input image, constructed by (b) WLS [1] [(from left to right)  $\lambda = 5 \times 10^3, 2 \times 10^5, 5 \times 10^6, \mu = 40$ ], (c) WLS [1] [(from left to right)  $\lambda = 5 \times 10, 2 \times 10^3, 5 \times 10^4, \mu = 5$ ], (d) RGF [11] [(from left to right)  $\sigma_s = 10, 20, 40, \sigma_r = 0.05, k = 5$ ], (e) our model [ $\mathbf{u}^0 = \mathbf{u}_{l_1}$ , (from left to right)  $\lambda = 3 \times 10^2, 4 \times 10^3, 1 \times 10^4, \mu = 5, \nu = 40, k = 5$ ].



(a) Input



(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]



(e) Ours

Figure 18. Examples of the texture removal for regular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ]), (c) RTV [9] [ $\lambda = 0.01, \sigma = 6$ ], (d) RGF [11] [ $\sigma_s = 5, \sigma_r = 0.05, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbf{u}_{l_1}, \lambda = 1000, \sigma = 2, \mu = 5, \nu = 40, k = 10$ ].



(a) Input



(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]



(e) Ours

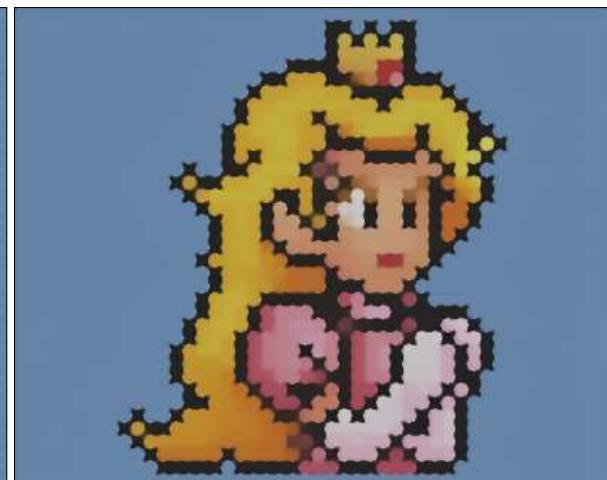
Figure 19. Examples of the texture removal for regular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ], (c) RTV [9] [ $\lambda = 0.01, \sigma = 6$ ], (d) RGF [11] [ $\sigma_s = 5, \sigma_r = 0.1, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbf{u}_{l_1}, \lambda = 2000, \sigma = 2, \mu = 5, \nu = 40, k = 10$ ].



(a) Input



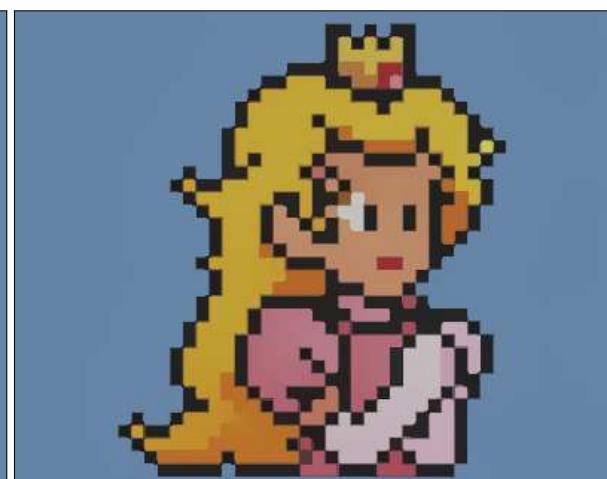
(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]



(e) Ours

Figure 20. Examples of the texture removal for regular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ], (c) RTV [9] [ $\lambda = 0.01, \sigma = 6$ ], (d) RGF [11] [ $\sigma_s = 5, \sigma_r = 0.1, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbf{u}_{l_1}, \lambda = 2000, \sigma = 2, \mu = 5, \nu = 40, k = 10$ ].



(a) Input



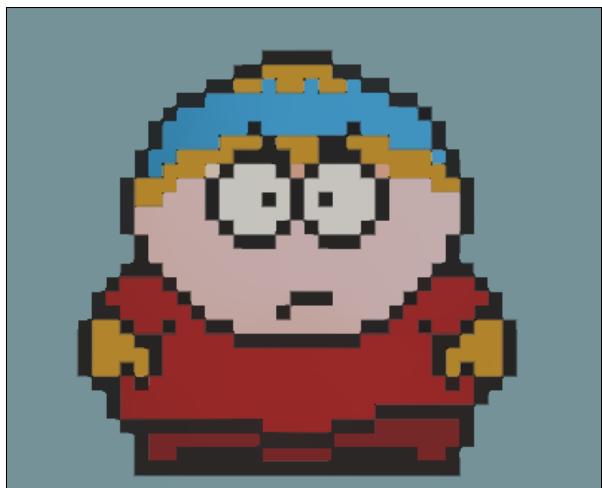
(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]



(e) Ours

Figure 21. Examples of the texture removal for regular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ], (c) RTV [9] [ $\lambda = 0.01, \sigma = 6$ ], (d) RGF [11] [ $\sigma_s = 5, \sigma_r = 0.1, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbf{u}_{l_1}, \lambda = 2000, \sigma = 2, \mu = 5, \nu = 40, k = 10$ ].



(a) Input



(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]



(e) Ours

Figure 22. Examples of the texture removal for regular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ]), (c) RTV [9] [ $\lambda = 0.01, \sigma = 3$ ], (d) RGF [11] [ $\sigma_s = 3, \sigma_r = 0.05, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbf{u}_1, \lambda = 2000, \sigma = 1, \mu = 5, \nu = 40, k = 10$ ].



(a) Input



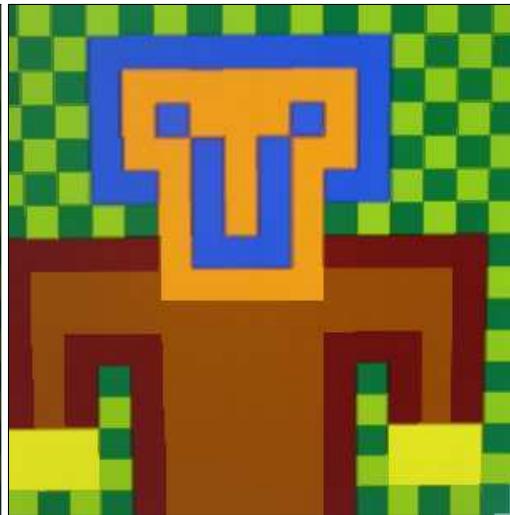
(b) Cov. M1 [4]



(c) RTV [9]

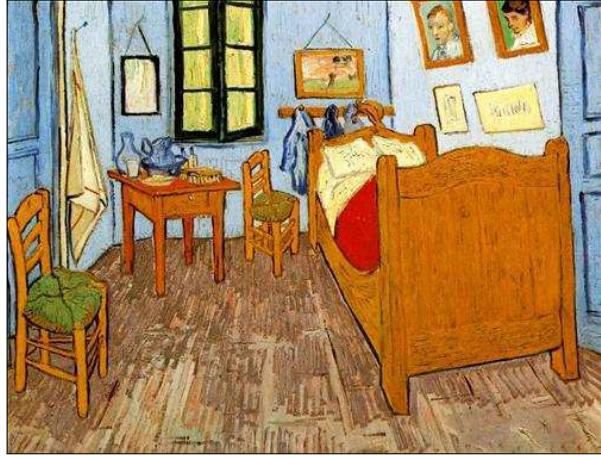


(d) RGF [11]



(e) Ours

Figure 23. Examples of the texture removal for regular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ], (c) RTV [9] [ $\lambda = 0.01, \sigma = 3$ ], (d) RGF [11] [ $\sigma_s = 3, \sigma_r = 0.05, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbf{u}_1, \lambda = 500, \sigma = 1, \mu = 5, \nu = 40, k = 10$ ].



(a) Input



(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]



(e) Ours

Figure 24. Examples of the texture removal for irregular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ], (c) RTV [9] [ $\lambda = 0.01, \sigma = 6$ ], (d) RGF [11] [ $\sigma_s = 3, \sigma_r = 0.05, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbf{u}_1, \lambda = 50, \sigma = 1, \mu = 5, \nu = 40, k = 10$ ].



(a) Input



(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]



(e) Ours

Figure 25. Examples of the texture removal for irregular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ], (c) RTV [9] [ $\lambda = 0.01, \sigma = 6$ ], (d) RGF [11] [ $\sigma_s = 3, \sigma_r = 0.1, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbf{u}_l, \lambda = 2000, \sigma = 2, \mu = 5, \nu = 40, k = 10$ ].



(a) Input



(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]



(e) Ours

Figure 26. Examples of the texture removal for irregular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ], (c) RTV [9] [ $\lambda = 0.01, \sigma = 6$ ], (d) RGF [11] [ $\sigma_s = 5, \sigma_r = 0.05, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbb{1}, \lambda = 50, \sigma = 2, \mu = 5, \nu = 40, k = 5$ ].



(a) Input



(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]



(e) Ours

Figure 27. Examples of the texture removal for irregular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ]], (c) RTV [9] [ $\lambda = 0.01, \sigma = 9$ ], (d) RGF [11] [ $\sigma_s = 5, \sigma_r = 0.05, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbb{1}, \lambda = 100, \sigma = 2, \mu = 5, \nu = 40, k = 5$ ].



(a) Input



(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]

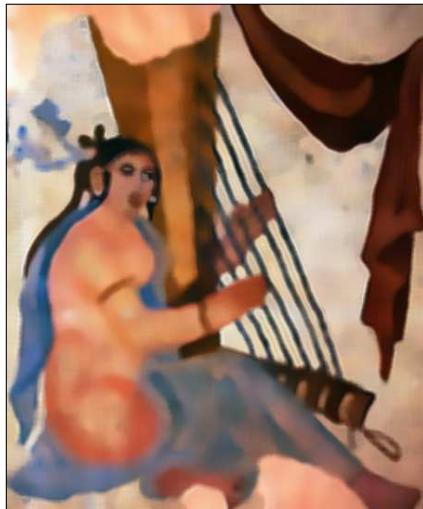


(e) Ours

Figure 28. Examples of the texture removal for irregular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ]), (c) RTV [9] [ $\lambda = 0.01, \sigma = 3$ ], (d) RGF [11] [ $\sigma_s = 3, \sigma_r = 0.05, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbb{1}, \lambda = 50, \sigma = 1, \mu = 5, \nu = 40, k = 5$ ].



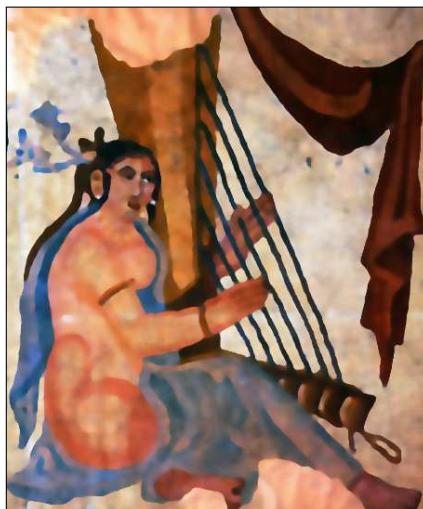
(a) Input



(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]



(e) Ours

Figure 29. Examples of the texture removal for irregular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ], (c) RTV [9] [ $\lambda = 0.01, \sigma = 6$ ], (d) RGF [11] [ $\sigma_s = 3, \sigma_r = 0.1, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbb{1}, \lambda = 50, \sigma = 2, \mu = 5, \nu = 40, k = 5$ ].



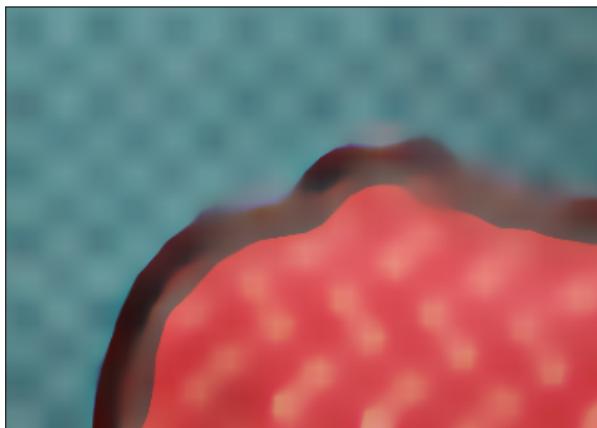
(a) Input



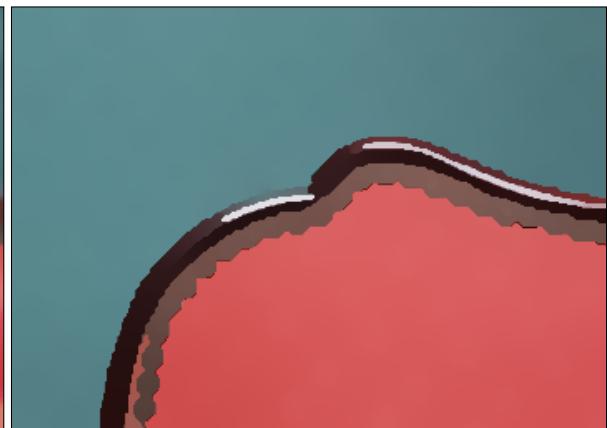
(b) Cov. M1 [4]



(c) RTV [9]



(d) RGF [11]



(e) Ours

Figure 30. Examples of the texture removal for irregular textures. (a) Input image, (b) Cov. M1 [4] [ $\sigma = 0.3, r = 10$ ], (c) RTV [9] [ $\lambda = 0.05, \sigma = 3$ ], (d) RGF [11] [ $\sigma_s = 9, \sigma_r = 0.1, k = 5$ ], (e) ours [ $\mathbf{u}^0 = \mathbb{1}, \lambda = 1000, \sigma = 1, \mu = 5, \nu = 40, k = 5$ ].



(a) RGB image



(b) Flash NIR image



(c) GF [3]



(d) Ours

Figure 31. RGB and flash NIR image restoration on *teapot* sequence [10]: (a) RGB image, (b) NIR image, (c) GF [3] [ $r = 3, \varepsilon = 0.0004$ ], (d) ours [ $\mathbf{u}^0 = \mathbb{1}, \lambda = 15, \mu = 60, \nu = 30, k = 5$ ].

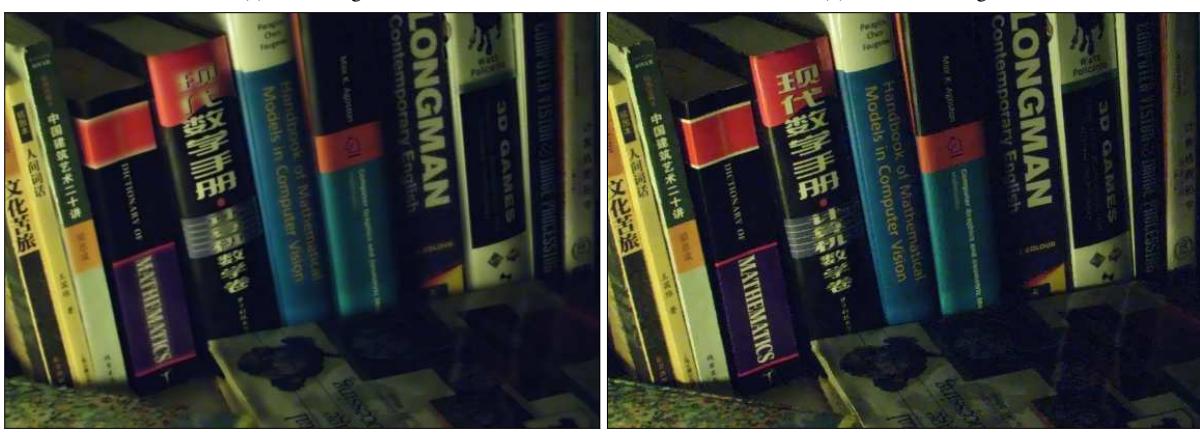
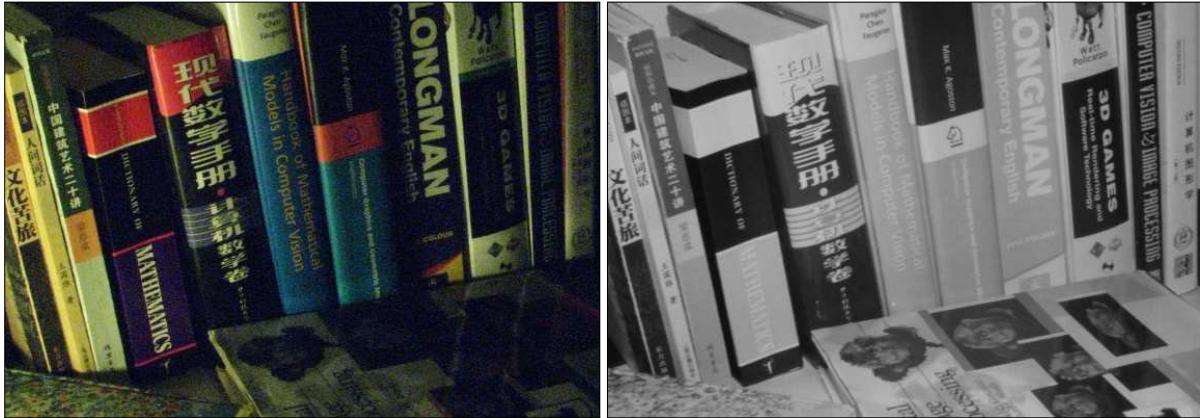


Figure 32. RGB and flash NIR image restoration on *books* sequence [10]: (a) RGB image, (b) NIR image, (c) GF [3] [ $r = 3, \varepsilon = 0.0004$ ], (d) ours [ $\mathbf{u}^0 = \mathbf{1}, \lambda = 15, \mu = 60, \nu = 30, k = 5$ ].

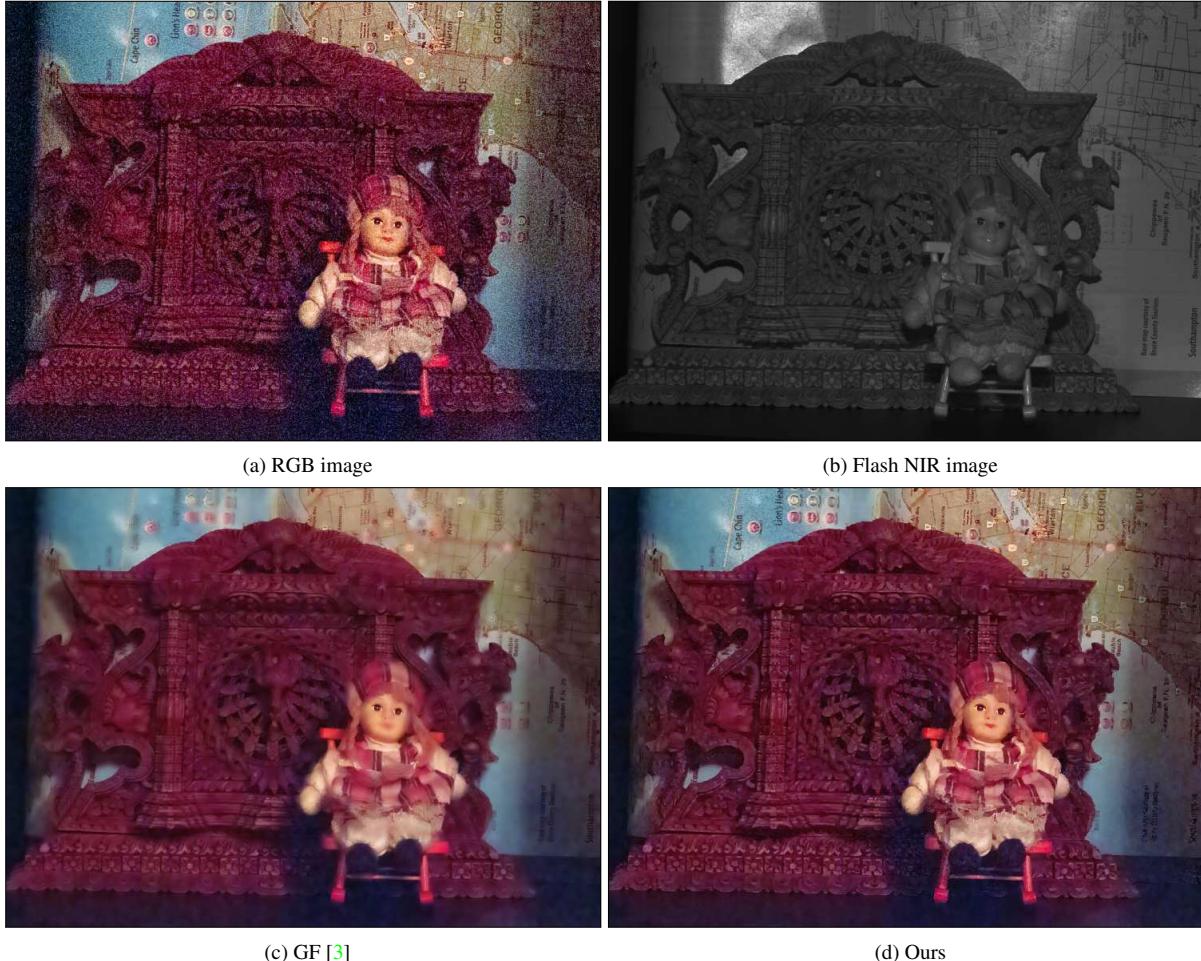


Figure 33. RGB and flash NIR image restoration on *window* sequence [10]: (a) RGB image, (b) NIR image, (c) GF [3] [ $r = 3, \varepsilon = 0.0004$ ], (d) ours [ $\mathbf{u}^0 = \mathbb{1}, \lambda = 15, \mu = 60, \nu = 30, k = 5$ ].



(a) RGB image

(b) Flash NIR image



(c) GF [3]

(d) Ours

Figure 34. RGB and flash NIR image restoration on *bowl*s sequence [10]: (a) RGB image, (b) NIR image, (c) GF [3] [ $r = 3, \varepsilon = 0.0004$ ], (d) ours [ $\mathbf{u}^0 = \mathbf{1}, \lambda = 7, \mu = 30, \nu = 30, k = 5$ ].



(a) Flash image



(b) Non-flash image



(c) GF [3]



(d) Result of [10]



Ours

Figure 35. Flash and non-flash image restoration on *cave* sequence [7]. (a) Flash image, (b) non-flash image, (c) GF [3] [ $r = 3$ ,  $\varepsilon = 0.0004$ ], (d) result of [10], and (f) ours [ $\mathbf{u}^0 = \mathbb{1}$ ,  $\lambda = 15$ ,  $\mu = 60$ ,  $\nu = 30$ ,  $k = 5$ ]. The results of (d) is obtained from the project webpage.



(a) Flash image



(b) Non-flash image



(c) GF [3]



(d) Result of [10]



Ours

Figure 36. Flash and non-flash image restoration on *pot* sequence [7]. (a) Flash image, (b) non-flash image, (c) GF [3] [ $r = 1, \varepsilon = 0.0004$ ], (d) result of [10], and (f) ours [ $\mathbf{u}^0 = \mathbb{1}, \lambda = 3, \mu = 60, \nu = 30, k = 5$ ]. The results of (d) is obtained from the project webpage.