A Geodesic-Preserving Method for Image Warping

Dongping Li[1], Kaiming He[2], Jian Sun[2], and Kun Zhou[1]

[1]Zhejiang University , [2]Microsoft Research

Supplementary Materials

0.1. Definitions of the terms

In Eqn.(1) of the main body, the rotation matrix $\mathrm{R}_{\theta,\phi}$ is:

$$\mathrm{Q}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}\mathrm{Q}^{\mathrm{T}}, \quad (1)$$

Here Q is a 3×3 orthogonal matrix in the form $[\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{n}}]$, where $\hat{\mathbf{n}}$ is the normal vector of the plane spanned by $\hat{\mathbf{p}}_1$, $\hat{\mathbf{p}}_2$, and $\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2$ are arbitrary two orthogonal vectors that are also orthogonal to $\hat{\mathbf{n}}$.

In Eqn.(7) of the main body, the shape-preserving term is [1]:

$$E_S(\mathbf{V}) = \frac{1}{N}\sum_q \|(A_q(A_q^{\mathrm{T}}A_q)^{-1}A_q^{\mathrm{T}} - I)\mathbf{v}_q\|^2, \quad (2)$$

where N is the quad number, q is a quad index, and

$$A_q = \begin{bmatrix} \hat{x}_{q,0} & -\hat{y}_{q,0} & 1 & 0 \\ \hat{y}_{q,0} & \hat{x}_{q,0} & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \hat{x}_{q,3} & -\hat{y}_{q,3} & 1 & 0 \\ \hat{y}_{q,3} & \hat{x}_{q,3} & 0 & 1 \end{bmatrix}, \quad \mathbf{v}_q = \begin{bmatrix} x_{q,0} \\ y_{q,0} \\ \vdots \\ x_{q,3} \\ y_{q,3} \end{bmatrix}. \quad (3)$$

Here $(x_{q,0}, y_{q,0})$, ..., $(x_{q,3}, y_{q,3})$ denote a warped quad, and $(\hat{x}_{q,0}, \hat{y}_{q,0})$, ..., $(\hat{x}_{q,3}, \hat{y}_{q,3})$ the input quad.

0.2. Preserving local smoothness of curves

To compare with local-smoothness-preserving, we implement a simple method to minimize the differences of directions of adjacent segments. We replace the geodesic-preserving term with:

$$E_C(\mathbf{V}) = \frac{1}{N_c}\sum_j (\mathbf{e}_j \cdot \mathbf{e}_{j+1} - |\mathbf{e}_j| \cdot |\mathbf{e}_{j+1}|)^2. \quad (4)$$

where N_c is the number of segments; $\mathbf{e}_j, \mathbf{e}_{j+1}$ are two adjacent segments on a curve (each represented by the difference of its two endpoints). The adjacent segments are encouraged to have similar directions. We combine this term with the shape and boundary terms

$$E(\mathbf{V}) = \lambda_B E_B(\mathbf{V}) + \lambda_S E_S(\mathbf{V}) + \lambda_C E_C(\mathbf{V}). \quad (5)$$

Here $\lambda_C = 100$ is the weight of the curvature term. We solve this energy function via the Gauss-Newton method.

References

[1] G. Zhang, M. Cheng, S. Hu, and R. Martin. A shape-preserving approach to image resizing. In *Computer Graphics Forum*, pages 1897–1906. Wiley Online Library, 2009.

978-1-4673-6964-0/15/$31.00 ©2015 IEEE