Line-Based Multi-Label Energy Optimization for Fisheye Image Rectification and Calibration - Supplementary Material

Mi Zhang Jian Yao † Menghan Xia Yi Zhang Kai Li Yaping Liu

School of Remote Sensing and Information Engineering, Wuhan University, Hubei, China

[†]EMail: jian.yao@whu.edu.cn Web: http://cvrs.whu.edu.cn

This document shows the details on Multi-Label Energy Optimization (MLEO) method used for circular arcs selection and additional results for Line-Based Multi-Label Energy Optimization for Fisheye Image Rectification and Calibration, described in the submitted manuscript (CVPR submission 1687). The Structure of this document is as follows:

- The next section describes details about the relationship between our circular arcs selection function and the Multi-Label Energy Optimization (MLEO) method. Our proposed function can be seen as a specific case for MLEO function presented in the manuscript.
- Figure 1 provides the graph cut like relationship of our circular arcs selection method and MLEO method.
- Figure 2 shows more experimental results for circular arcs selection algorithm.
- Figure 3, 4 and 5depict additional successfully rectified results for images download from Internet.
- Figure 6 and Figure 7 illustrate the results for synthetic approach where the images is captured by the same fisheye camera
- Table 1 demonstrates our synthetic results of intrinsic parameters for Figure 7 and the result of existed approach using chessboard¹.
- Figure 8 and Figure 9 display the failed examples of proposed algorithm.

For the rectified examples of Internet images illustrated in the following, the first row is the source image download from Internet, the second row is the clustered circular arcs using our proposed algorithm and and the last row always depicts corresponding rectified results.

In our experiments, the initial value of LM algorithm for line-based fisheye image rectification is as follow:

$$R = k\max\left(w,h\right),\tag{1}$$

$$z_0 = \frac{R}{3},\tag{2}$$

where the k is the regulate coefficients:

$$k = \begin{cases} 0.45 & \text{for circular fisheye} \\ 0.55 & \text{for full-frame fisheye} \end{cases},$$
(3)

and w, h represent the width and height of source fisheye image respectively. R and z_0 have the same meaning as mentioned in the manuscript.

¹http://www-sop.inria.fr/icare/personnel/Christopher.Mei/index.html

1. Relationship Between Circular Arcs Selection Algorithm and MLEO Method

As described in the manuscript, the abstract form of the Multi-Label Energy Optimization function is as follow:

$$E(f) = \sum_{p \in P} D_p(f_p) + \sum_{pq \in N} V_{pq}(f_p, f_q) + \sum_{l \in L} h_l \cdot \delta_l(f).$$

$$\tag{4}$$

And the specific formulation for circular arcs selection in the manuscript is

$$E_{s}\left(f;\hat{\theta}_{s}\right) = \sum_{i=1}^{N} \sum_{j=1}^{4} \ln\gamma\left(r_{i} - r_{j}^{p}\right)^{2} + \sum_{i=1}^{N} \sum_{j=1}^{4} \ln\beta\left(k_{i} - k_{j}^{p}\right)^{2} + \sum_{m=1}^{4} \sum_{n=1}^{4} \lambda \|r_{m}^{p} - r_{n}^{p}\|,\tag{5}$$

which indicating four clusters belonging to four directions in perspective image plane Π_P are grouped. The data term $\epsilon_j = \sum_{i=1}^N \ln \gamma (r_i - r_j^p)^2 + \sum_{i=1}^N \ln \beta (k_i - k_j^p)^2 (j = 1 \cdots 4)$ represents the total deviations within *j*th clusters. The smooth term $s_m^m = \lambda ||r_m^p - r_n^p||$ denotes the difference between predefined radius r_m^p and radius r_n^p , which also indicates the difference between these four clusters. Unlike the circular arc extraction process, we do not give any penalty to this function.

This model can be explained as a special case for graph cut like MLEO method as is shown in Figure 1(a) and Figure 1(b). The minimization process corresponds to the minimization of Eq. (4). Given a set of detected circular arcs $\omega_j \in \Pi_F$, each circular arc is connected to its terminal and each detected circular arc has n - link to its neighbour circular arc. The t - link corresponds to the data term ϵ_j while the n - link represents the hidden difference between the four groups which is formulated by smooth term s_m^n . The minimum cut of four groups reached until these two terms obtaining its local minimum value. This demonstrates that all the circular arcs belonging to the same groups are clustered. Four groups circulars arcs are classified in this manner. The reason why we choose four groups rather than three clustered groups is that four directions including west, east, south and north are taking into consideration. This will provide the freedom for us to choose the three final circular arcs for intrinsic parameters estimation.



(a) (b) Figure 1. Relationship of our circular arcs selection method and MLEO method: (a) Initial graph corresponding to the terms in Eq. (5); (b) Four circular arcs groups Ω_m^1 , Ω_k^2 , Ω_j^3 and Ω_p^4 are clustered Minimum using MLEO method.

	focal length	u_0	v_0	aspect ratio
Ground Truth	3200.000	3000.000	2000.000	0.000
Our Method	3133.100	3034.300	2100.400	7.10548870e - 11
Chessboard Method	3576.629	3025.500	2015.060	0.000

Table 1. Comparison of our synthetic method corresponding to Figure 7 and chessboard method with the same camera.



Figure 2. Circular arcs selection results: The first row is the clustered circular arcs for source image. The second row is the corresponding line information in undistorted perspective image plane. And the last row depicts the selected circular arcs for intrinsic parameters estimation.



Figure 3. Examples # 1



Figure 4. Examples # 2



Figure 5. Examples # 3



Figure 6. Circular arcs extraction results for fisheye images captured by the same camera.



Figure 7. Synthetic method results: the first image illustrates the synthetic result for extracted circular arcs in Figure 6. The second image depicts the selected circular arcs for intrinsic parameters estimation.



Figure 8. Failed Example # 1: Poorly detected circular arcs due to low texture regions in source image. The Canny edge detector could not perform well as expected, thus leading to fewer clustered circular arcs in source image.



Figure 9. Failed Example # 2: Wrongly rectified result due to the circular structure of target object. The yellow line itself is the circular arc in real scene and this makes the incorrect rectified result.