# Supplementary Material for "GIFT: A Real-time and Scalable 3D Shape Search Engine" 

In this supplementary material, we first illustrate the procedure of projection rendering in our 3D shape search engine. Then we list several formal definitions in fuzzy set theory, which are used to formulate our re-ranking component. As a by-product, the classification performances on ModelNet dataset of our search engine are presented later, followed by the GUI of our search engine which gives some example retrieval results of some selected queries at last.

## 1. The Details of Projection

As illustrated in Fig. 1, $\theta_{a z}$ is the polar angle in $x y$ plane and $\theta_{e l}$ is the angle between the camera and $x y$ plane. We evenly divide $[0,2 \pi]$ into 8 parts to get the values of $\theta_{a z}$, and divide $[0, \pi]$ into 8 parts to get the values of $\theta_{e l}$. For each tuple $\left(\theta_{a z}, \theta_{e l}\right)$, a virtual camera is set on the unit sphere. The depth information is recorded on a $224 \times 224$ image in the format of uint8.

As a result, for each 3D shape, we use 64 depth projections to represent it.

## 2. Fuzzy Set Theory

In fuzzy set theory, a fuzzy set is a pair $(\mathcal{N}, m)$, where $\mathcal{N}$ denotes the set and a membership function

$$
m: \mathcal{N} \rightarrow[0,1]
$$



Figure 1. The illustration of the projection rendering.
$\forall x \in \mathcal{N}, m(x)$ measures the membership grade of $x$ in ( $\mathcal{N}, m$ ).

The conventional (crisp) set theory can be assumed as a special case of fuzzy set theory, where $m(x)$ is either 0 or 1 , indicating $x$ belongs to or does not belong to $\mathcal{N}$.

### 2.1. Fuzzy Intersection

Definition. In fuzzy set theory, the intersection of two fuzzy sets $\left(\mathcal{N}^{(1)}, m^{(1)}\right)$ and $\left(\mathcal{N}^{(2)}, m^{(2)}\right)$ requires for a function $h$

$$
h:[0,1] \times[0,1] \rightarrow[0,1] .
$$

Let $(\mathcal{N}, m)$ denote the intersection set, $\forall x \in \mathcal{N}$,

$$
m(x)=h\left(m^{(1)}(x), m^{(2)}(x)\right) .
$$

## Axiomatic requirements.

1. $h(m(x), 1)=m(x), \forall x$.
2. If $m^{(2)}(x) \leq m^{(3)}(x)$, then $h\left(m^{(1)}(x), m^{(2)}(x)\right) \leq$ $h\left(m^{(1)}(x), m^{(3)}(x)\right), \forall x$.
3. $h\left(m^{(1)}(x), m^{(2)}(x)\right)=h\left(m^{(2)}(x), m^{(1)}(x)\right), \forall x$.
4. $\forall x \in \mathcal{N}, h\left(m^{(1)}(x), h\left(m^{(2)}(x), m^{(3)}(x)\right)\right)=$ $h\left(h\left(m^{(1)}(x), m^{(2)}(x)\right), m^{(3)}(x)\right)$.

Examples. Indeed, any functions that meet the above axiomatic requirements can be used to define the fuzzy intersection operation. In this paper, we consider a widely-used one as

$$
m(x)=\min \left(m^{(1)}(x), m^{(2)}(x)\right), \forall x \in \mathcal{N}
$$

### 2.2. Fuzzy Union

Definition. The union of two fuzzy sets $\left(\mathcal{N}^{(1)}, m^{(1)}\right)$ and $\left(\mathcal{N}^{(2)}, m^{(2)}\right)$ requires for a function $h$

$$
h:[0,1] \times[0,1] \rightarrow[0,1] .
$$

Let $(\mathcal{N}, m)$ denote the union set, $\forall x \in \mathcal{N}$,

$$
m(x)=h\left(m^{(1)}(x), m^{(2)}(x)\right)
$$

## Axiomatic requirements.

1. $h(m(x), 0)=m(x), \forall x$.
2. If $m^{(2)}(x) \leq m^{(3)}(x)$, then $h\left(m^{(1)}(x), m^{(2)}(x)\right) \leq$ $h\left(m^{(1)}(x), m^{(3)}(x)\right), \forall x$.
3. $h\left(m^{(1)}(x), m^{(2)}(x)\right)=h\left(m^{(2)}(x), m^{(1)}(x)\right), \forall x$.
4. $\forall x \in \mathcal{N}, h\left(m^{(1)}(x), h\left(m^{(2)}(x), m^{(3)}(x)\right)\right)=$ $h\left(h\left(m^{(1)}(x), m^{(2)}(x)\right), m^{(3)}(x)\right)$.

Examples. Similar to fuzzy intersection, the fuzzy union considered in this paper is

$$
m(x)=\max \left(m^{(1)}(x), m^{(2)}(x)\right), \forall x \in \mathcal{N}
$$

### 2.3. Fuzzy Aggregation

Definition. Aggregation is a specific operation in fuzzy set theory, and it does not exist in the conventional set theory (aggregation operation degenerates into union operation in crisp logic).

In fuzzy logic, aggregation on $k(k \geq 2)$ fuzzy sets requires for a function

$$
h:[0,1]^{k} \rightarrow[0,1] .
$$

Applied to fuzzy sets $\mathcal{N}^{(1)}, \mathcal{N}^{(2)} \ldots, \mathcal{N}^{(k)}$, function $h$ generates an aggregated fuzzy set $\mathcal{N} . \forall x \in \mathcal{N}, m(x)$ is computed as

$$
m(x)=h\left(m^{(1)}(x), m^{(2)}(x), \ldots, m^{(k)}(x)\right)
$$

## Axiomatic requirements.

1. $h(0,0, \ldots, 0)=0$ and $h(1,1, \ldots, 1)=1$.
2. $h$ is monotonic increasing in all its arguments: for any $x$ and $y$, such that $0 \leq$ $m^{(i)}(x) \leq m^{(i)}(y) \leq 1$ for $i=1,2, \ldots, k$, then $\quad h\left(m^{(1)}(x), m^{(2)}(x), \ldots, m^{(k)}(x)\right) \quad \leq$ $h\left(m^{(1)}(y), m^{(2)}(y), \ldots, m^{(k)}(y)\right)$.
3. $h$ is a symmetric function in all its arguments: for any permutation $p$ on $\{1,2, \ldots, k\}$, $h\left(m^{(1)}(x), m^{(2)}(x), \ldots, m^{(k)}(x)\right)$ $h\left(m^{\left(p_{1}\right)}(x), m^{\left(p_{2}\right)}(x), \ldots, m^{\left(p_{k}\right)}(x)\right)$.
4. $h$ is idempotent: for any $x$ such that $0 \leq m(x) \leq 1$, then $h(m(x), m(x), \ldots, m(x))=m(x)$.

Examples. A widely-used aggregation function is generalized means

$$
\begin{aligned}
& h\left(m^{(1)}(x), m^{(2)}(x), \ldots, m^{(k)}(x)\right) \\
& =\left(\frac{m^{(1)}(x)^{\alpha}+m^{(2)}(x)^{\alpha}+\cdots+m^{(k)}(x)^{\alpha}}{k}\right)^{\frac{1}{\alpha}}
\end{aligned}
$$

In the main paper, we set $\alpha$ to 0.5 to suppress the dominant influence of some larger values on the similarity calculation.

### 2.4. The Cardinality of Fuzzy Set

In fuzzy set theory, the cardinality of a fuzzy set is the sum of all the membership values, as

$$
|(\mathcal{N}, m)|=\sum_{x \in \mathcal{N}} m(x) .
$$

## 3. Additional Results on ModelNet

Our proposed system focuses on real-time 3D shape search in large scale, and 3D shape classification task is beyond our scope. Nevertheless, ModelNet defines the two classification tracks, so we report the classification results only here. The parameter setup is as follows: the inverted file in approximate Hausdorff matching use 256 entries, and MA is set to 2 ; re-ranking is not used; linear SVM is used on training shapes represented by feature $L_{i}(5 \leq i \leq 7)$.

The confusion matrices generated by our system in ModelNet10 dataset and ModelNet40 dataset are given in Fig. 2 and Fig. 3 respectively. In ModelNet10 dataset, the average classification performance of $L_{5}, L_{6}$ and $L_{7}$ is the same, i.e. accuracy $\mathbf{9 1 . 5 0 \%}$. In ModelNet40 dataset, the average classification accuracy of $L_{5}, L_{6}$ and $L_{7}$ are $\mathbf{8 9 . 5 0 \%}$, $88.13 \%$ and $87.38 \%$ respectively.

## 4. Example Retrieval

We build a GUI for the proposed search engine. Some representative retrieval results are given in Fig. 4 ,


Figure 2. The confusion matrix of our system in ModelNet10 dataset, using feature $L_{5}$ (a), $L_{6}$ (b) and $L_{7}$ (c) respectively.


Figure 3. The confusion matrix of our system in ModelNet40 dataset, using feature $L_{5}$ (a), $L_{6}$ (b) and $L_{7}$ (c) respectively.


Figure 4. Some representative retrieval results in GUI.

