A Novel Tensor-based Video Rain Streaks Removal Approach via Utilizing Discriminatively Intrinsic Priors
Supplementary material

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Abstract

This supplementary appendix provides additional details of the convergency analysis of our algorithm, and details of the conduction of our experiments. Sections 1 illustrates that our algorithm fits the typical ADMM framework and its convergency is theoretically ensured. Section 2 gives some details of our experiments and an extra experimental result.

1. Convergency

The minimization problem in our paper is

$$\begin{aligned}\min_{\mathcal{R},\mathcal{Y},\mathcal{S},\mathcal{X},\mathcal{T},\mathcal{L},} \quad & \alpha_1\|\mathcal{Y}\|_1+\alpha_2\|\mathcal{S}\|_1 \\ & +\alpha_3\|\mathcal{X}\|_1+\alpha_4\|\mathcal{T}\|_1+\|\mathcal{L}\|_* \\ \text{s.t.} \quad \mathcal{Y} & = \nabla_y\mathcal{R}, \quad \mathcal{S}=\mathcal{R}, \\ \mathcal{X} & = \nabla_x(\mathcal{O}-\mathcal{R}), \\ \mathcal{T} & = \nabla_t(\mathcal{O}-\mathcal{R}), \\ \mathcal{L} & = \mathcal{O}-\mathcal{R}, \quad \mathcal{O}\geqslant\mathcal{R}\geqslant 0,\end{aligned} \tag{1}$$

where $\mathcal{S},\mathcal{Y},\mathcal{X},\mathcal{T}$ and $\mathcal{L}\in\mathbb{R}^{m\times n\times t}$.

Although there are five components in the objective function, they can be categorized as the l_1 part and the nuclear norm part. Actually, let

$$\mathcal{A}=\begin{pmatrix}\alpha_1\mathcal{Y}\\ \alpha_2\mathcal{S}\\ \alpha_3\mathcal{X}\\ \alpha_4\mathcal{T}\end{pmatrix}, \tag{2}$$

where $\mathcal{A}\in\mathbb{R}^{m\times n\times t\times 4}$ and we can get that

$$\begin{aligned}\|\mathcal{A}\|_1 &= \left\|\begin{matrix}\alpha_1\mathcal{Y}\\ \alpha_2\mathcal{S}\\ \alpha_3\mathcal{X}\\ \alpha_4\mathcal{T}\end{matrix}\right\|_1 \\ &= \|\alpha_1\mathcal{Y}\|_1+\|\alpha_2\mathcal{S}\|_1+\|\alpha_3\mathcal{X}\|_1+\|\alpha_4\mathcal{T}\|_1 \\ &= \alpha_1\|\mathcal{Y}\|_1+\alpha_2\|\mathcal{S}\|_1+\alpha_3\|\mathcal{X}\|_1+\alpha_4\|\mathcal{T}\|_1.\end{aligned} \tag{3}$$

Besides, because of the constraint $\mathcal{L}=\mathcal{O}-\mathcal{R}$,

$$\begin{aligned}\mathcal{A} &= \begin{pmatrix}\alpha_1\mathcal{Y}\\ \alpha_2\mathcal{S}\\ \alpha_3\mathcal{X}\\ \alpha_4\mathcal{T}\end{pmatrix} = \begin{pmatrix}\alpha_1\nabla_y\mathcal{R}\\ \alpha_2\mathcal{R}\\ \alpha_3\nabla_x(\mathcal{O}-\mathcal{R})\\ \alpha_4\nabla_t(\mathcal{O}-\mathcal{R})\end{pmatrix} \\ &= \begin{pmatrix}\alpha_1\nabla_y(\mathcal{O}-\mathcal{L})\\ \alpha_2(\mathcal{O}-\mathcal{L})\\ \alpha_3\nabla_x\mathcal{L}\\ \alpha_4\nabla_t\mathcal{L}\end{pmatrix} \\ &= \begin{pmatrix}-\alpha_1\nabla_y\\ -\alpha_2\\ \alpha_3\nabla_x\\ \alpha_4\nabla_t\end{pmatrix}\cdot\mathcal{L}+\begin{pmatrix}\alpha_1\nabla_y\\ \alpha_2\\ 0\\ 0\end{pmatrix}\cdot\mathcal{O}\end{aligned} \tag{4}$$

Thus, the minimization problem (1) can be rewrote as:

$$\begin{aligned}\min_{\mathcal{A},\mathcal{L},} \quad & \|\mathcal{A}\|_1+\alpha_2\|\mathcal{L}\|_* \\ \text{s.t.} \quad & \mathcal{A}+\begin{pmatrix}-\alpha_1\nabla_y\\ -\alpha_2\\ \alpha_3\nabla_x\\ \alpha_4\nabla_t\end{pmatrix}\cdot\mathcal{L}=\begin{pmatrix}\alpha_1\nabla_y\\ \alpha_2\\ 0\\ 0\end{pmatrix}\cdot\mathcal{O}, \\ & \mathcal{O}\geqslant\mathcal{L}\geqslant 0.\end{aligned} \tag{5}$$

The problem fits the framework of ADM. The optimization problem is well-structured since both sets of variables $\mathcal{A}$ and $\mathcal{L}$ are separated. The convergence of the algorithm is theoretically guaranteed [1].

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2. Details

Preprocessing Since the fast Fourier transform is involved in our algorithm, the boundary conditions need to be circular. To lower impact from circular boundary, we pad the input tensor $\mathcal{O} \in \mathbb{R}^{m\times n\times t}$ with mirror reflections of itself.

Parameters In Fig. 1, a parameter analysis is given. The peak signal-to-noise ratio (PSNR) is selected. It can be seen from Fig. 2 that α_2 and α_4 are a little bit more sensitive than α_1, α_3 and β. With the guidance from parameter analysis, our tuning strategy is as following: (1) set all parameters as 50, (2) tune α_2 and α_4, until the results are barely satisfactory, (3) then fix α_2 and α_4, tune α_1 and α_3 for better results. The tuning principle is that, when the texture or details of the clean video are extracted into the rain streaks part, we enlarge α_2 and α_1 or decrease α_4 and α_3, and do the opposite when the rain streaks remain in rain-free part. Our recommended set of candidate values for α_1 to α_4 are {10, 10^2, 10^3} and $\beta = 50$. The parameters' empirical tuning does not cost so much time.

Rotation When the angle between the y-direction and the real falling direction of raindrops is large, our approach also works. But the results (the fifth column in Fig. 1) is not desirable, for the remaining rain streaks. As previously stated in our paper, we rotate the frames. We can determine the rotation angle empirically from our observation. As long as the direction of the rotated rain streaks is not far deviated from the y-axis (less than 10 degree), our approach works well. Although the rotating image in MATLAB causes distortion (mainly due to interpolation), it can be found in Fig. 1 that the rotation indeed handle the large angle case.

The real video shown in Fig. 2 is named as "postbox"[1]. Qualitatively, although the distortion affect our performance, our method removes almost all the rain streaks in "postbox". Finally, all the results in an uncompressed audio video interleave (AVI) format will be uploaded the authors' personal homepage. These AVI format videos provide a better instantiated presentation of the outperformance of our method.

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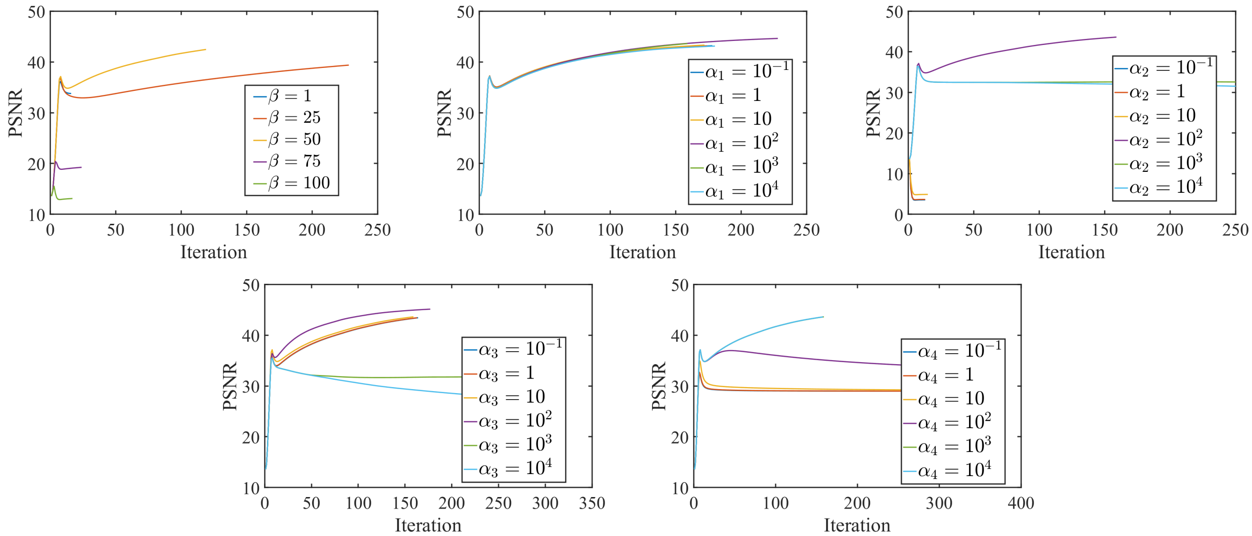

Figure 1. The PSNR values with respect to iteration for α_1, α_2, α_3, α_4 and β with different values.

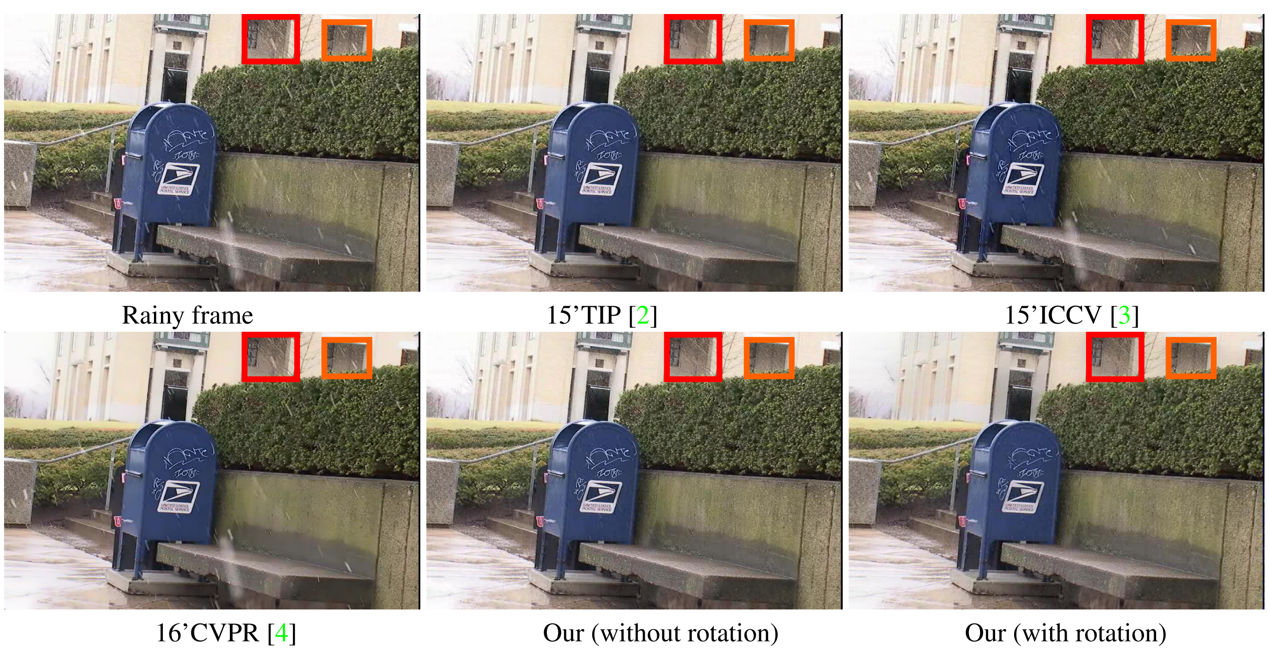

Figure 2. The rotation case. Results on the video "postbox".