1 Confidence measures

We report the exhaustive list of state-of-the-art confidence measures, grouped into stand-alone and machine-learning based measures, considered in our paper.

1.1 Stand-alone measures

We recall the definition of each stand-alone confidence measure according to literature. As shown in Figure 1, given a pixel \( p = (p_x, p_y) \), we will refer to its minimum cost as \( c_1(p) \), the second minimum as \( c_2(p) \) and the second local minimum as \( c_{2m}(p) \). We refer to a matching cost for any disparity hypothesis \( d \) as \( c_d(p) \). The disparity hypothesis corresponding to \( c_1(p) \) will be referred to as \( d_1(p) \), the one to \( c_2(p) \) as \( d_2(p) \) and so on. If not specified, costs and disparities refers to left image pixels (L). When talking about right image (R), we introduce the \( R \) notations on both costs (e.g., \( c_1^R(p) \)) and disparities. We denote as \( p' = (p'_x, p'_y) \) the matching pixel for \( p \) according to \( d_1 \) (i.e., \( p'_x = p_x + d_1(p) \), \( p'_y = p_y \)). Finally, we denote with \( LL \) matching costs and disparities related to self-matching stereo on the left image (i.e., stereo algorithm processing the left image as both reference and target).

- **PKRN** (Peak Ratio Naive), reviewed in [1]

\[
C_{PKRN}(p) = \frac{c_2(p)}{c_1(p)}
\]
– **PKR** (Peak Ratio), reviewed in [1]

\[ C_{PKR}(p) = \frac{c_{2m}(p)}{c_1(p)} \]  (2)

– **MSM** (Matching Score Measure), reviewed in [1]

\[ C_{MSM}(p) = -c_1(p) \]  (3)

– **MMN** (Maximum Margin), reviewed in [1]

\[ C_{MMN}(p) = c_2(p) - c_1(p) \]  (4)

– **WMN** (Winner Margin), reviewed in [1]

\[ C_{WMN}(p) = \frac{c_{2m}(p) - c_1(p)}{\sum_d c_d(p)} \]  (5)

– **MLM** (Maximum Likelihood Measure), reviewed in [1]

\[ C_{MLM}(p) = \frac{e^{\frac{-c_1(p)}{2\sigma^2_{MLM}}} - e^{\frac{-c_d(p)}{2\sigma^2_{MLM}}}}{e^{\frac{-c_1(p)}{2\sigma^2_{MLM}}}} \]  (6)

– **PER** (Perturbation), proposed in [2] and adopted in [3]

\[ C_{PER}(p) = \sum_{d \neq d_1} e^{-\frac{(c_1(p) - c_d(p))^2}{s^2}} \]  (7)

– **NEM** (Negative Entropy Measure), reviewed in [1]
\[ C_{NEM} = -\sum d \; p(d) \log p(d) \]

\[ p(d) = \frac{e^{-c_1}}{\sum_d e^{-c_d(p)}} \]  

- **LRD** (Left-Right Difference), reviewed in [1]

\[ C_{LRD}(p) = \frac{c_2(p) - c_1(p)}{|c_1(p) - \min_d c_d^R(p')|} \]  

- **CUR** (Curvature), reviewed in [1]

\[ C_{CUR} = -2c_1(p) + c_{d-1}(p) + c_{d+1}(p) \]  

- **DSM** (Distinctiveness Similarity Measure), reviewed in [1]

\[ C_{DSM} = \frac{C_{L.DTS}(p) \times C_{R.DTS}(p')}{c_1^2(p)} \]

\[ C_{L.DTS} = \min_{d \in d_s, d \neq 0} c_d^{LL}(p) \]

\[ C_{R.DTS} = \min_{d \in d_s, d \neq 0} c_d^{RR}(p) \]  

- **AML** (Attainable Maximum Likelihood), reviewed in [1]

\[ C_{AML} = \frac{1}{e^{\frac{c_d(p)}{2\sigma_{AML}^2}}} \]  

- **NOI** (Number Of Inflections), reviewed in [1]

\[ C_{NOI} = |M| \]

\[ M = \{ d_i : c_{d_i-1}(p) > c_{d_i}(p) \wedge c_{d_i}(p) < c_{d_i+1}(p) \} \]  

- **SAMM** (Self-Aware Matching Measure), proposed in [4] and reviewed in [1]

\[ C_{SAMM} = \sum_d (c_{d-d_1}(p) - \mu) - \frac{(c_d^{LL}(p) - \mu_{LL})}{\sigma_{LL}} \]  

\[ \sigma_{LL} \]
- **WMNN** (Winner Margin Naive), reviewed in [1]
  \[
  C_{WMNN} = \frac{c_2(p) - c_1(p)}{\sum_d c_d(p)}
  \] (15)

- **LRC** (Left-Right Consistency), reviewed in [1]
  \[
  C_{LRC}(x, y) = -|d_1 - d_R(p')|
  \] (16)

- **LC** (Local Curve), proposed in [5]
  \[
  C_{LC} = \frac{\max (c_{d_1-1}(p), c_{d_1+1}(p)) - c_1(p)}{\gamma}
  \] (17)

- **UC** (Uniqueness Constraint), proposed in [6]
  \[
  UC(p) = \begin{cases} 
  0, & \text{if } d_1(p) \neq d_R(p - d_1(p)) \text{ and } c_1 \neq \min_{q \in Q} c_1(q) \\
  1, & \text{otherwise}
  \end{cases}
  \] (18)

  being Q the set of pixels matching the same pixel on the right image

### 1.2 Machine-learning based measures

We briefly review the five machine-learning measures considered in the paper.

- **Ensemble**, proposed in [3]. It combines 23 confidence measures (some computed at multiple resolutions) fed to a random forest trained in classification mode.

- **GCP**, proposed in [7]. It combines 5 confidence measures (MSM, MMN, AML, LRC, LRD) and three additional features (i.e., *distance to border*, *distance to discontinuities* and *median deviation of disparity*), fed to a random forest trained in regression mode.

- **Park**, proposed in [8]. It combines PKR, PKRN, MSM, MMN, WMN, MLM, NEM, LRD, CUR, LRC and PER confidence measures plus *distance to border*, *distance to edges*, *horizontal gradient magnitude*, *median deviation* and *variance of disparity* (the latter two computed on 5 × 5, 7 × 7, 9 × 9 and 11 × 11 patches) in a vector of 22 features, fed to a random forest trained in regression mode.

- **O1**, proposed in [9]. It relies on a vector of 20 features extracted from the disparity map fed to a random forest trained in regression mode. The features are: *disparity agreement*, *disparity scattering*, *median*, *variance* and *median deviation* of disparity (the latter three computed on 5 × 5, 7 × 7, 9 × 9 and 11 × 11 patches).

- **CCNN**, proposed in [10]. The confidence map is inferred from scratch processing the raw disparity map by means of a convolutional neural network with perceptive field of size 9 × 9.
2 Confidence maps

In this section, we report a qualitative comparison between confidence maps computed by stand-alone or machine-learning based confidence measures and their proposed plus counterpart training our networks, as for any experimental results reported in this document and in the submitted paper, on 25 images of the KITTI 2012 datasets. In most cases, the benefit yielded by our framework can be clearly perceived comparing the original confidence map and its plus version. We report detailed experimental results for image #93 from KITTI 2015 dataset and for Motorcycle image from Middlebury 2014 dataset.
Left image | Disparity map
--- | ---
PKRN | PKRN$^+$
PKR | PKR$^+$
MSM | MSM$^+$
MMN | MMN$^+$
WMN | WMN$^+$
MLM | MLM$^+$
PER | PER$^+$
References