

7. Appendix

7.1. Proofs

Proof of Proposition 1

Proposition. Let $C = \{e_1, \dots, e_k\}$ be a cycle with $\theta_{e_1} \leq -\epsilon$ and $\theta_{e_l} \geq \epsilon$ for $l > 1$. Then, the dual lower bound $L(\theta)$ can be increased by ϵ by including a triangulation of C .

Proof. Let cycle C have vertices $\{v_1, \dots, v_k\}$ and assume that $e_i = v_i v_{i+1}$ with $v_{k+1} = v_1$ for notational purposes. After triangulation, triangle factors on vertices $v_1 v_2 v_3, \dots, v_1 v_{k-1} v_k$ will be present in the model. Let the current reparametrization be θ .

The triangle factors corresponding to cycle C will enforce the cycle inequality (3)

$$x_{e_1} \leq \sum_{i=1, \dots, k} x_{e_i}. \quad (19)$$

It holds that

$$\begin{aligned} -\theta_{e_1} &= \min_{x_{e_1}, \dots, x_{e_k} \in [0, 1]} \sum_{i=1}^k \theta_{e_i} x_{e_i} \\ &\leq -\epsilon + \min_{x_{e_1}, \dots, x_{e_k} \in [0, 1]} \sum_{i=1}^k \theta_{e_i} x_{e_i} \text{ s.t. (19)} \\ &\leq \epsilon + \left\{ \begin{array}{c} \max_{\theta_{e_1}, \dots, \theta_{e_k}} \\ \theta_{v_1 v_2 v_3, \dots, v_1 v_{k-1} v_k} \\ \text{a reparametrization} \end{array} \right\} L_C(\theta) \quad (20) \end{aligned}$$

where $L_C(\theta) = \sum_{i=1}^k \min(0, \theta_{e_i}) + \sum_{i=2}^{k-1} \min\{\theta_{v_1 v_i v_{i+1}}\}$ the dual lower bound on cycle C . The first inequality above is due to either $x_{e_1} = 0$ in the optimal solution or one x_{e_2}, \dots, x_{e_k} being one due to (19). The second inequality is due to the fact that (i) $\max_{\theta \text{ a reparametrization}} L(\theta) = \min_{\mu \in \Lambda} \langle \theta, \mu \rangle$ by linear programming duality and (ii) the triangle factors enforce more inequalities than only (19). \square

Proof of Proposition 2

Proposition. Let O an odd wheel with center node u and cycle nodes v_1, \dots, v_k . Adding the lollipop subproblems for O increases $L(\theta)$ by at least ϵ if the costs $\theta_{uv_i v_{i+1}}$ of each triangle $uv_i v_{i+1}$ are such that the minimal cost of any edge labeling of the triangle cutting precisely one edge incident to u is smaller by ϵ than the minimal cost of any edge labeling of the triangle cutting 0 or 2 edges incident to u . That is:

$$\begin{aligned} &\min_{\{x: x_{uv_i} + x_{uv_{i+1}} = 1\}} \theta_{uv_i v_{i+1}}(x) + \epsilon \\ &\leq \min_{\{x: x_{uv_i} + x_{uv_{i+1}} \neq 1\}} \theta_{uv_i v_{i+1}}(x). \quad (21) \end{aligned}$$

Proof. Condition (16) means that in all triangles in the odd wheel O , the minimal assignment with regard to the current reparametrization, has exactly one edge incident to u . All other assignments have cost greater by at least ϵ . As k is odd, there is no possibility to combine those local assignments to a global assignment on O .

On the other hand, our construction of lollipop factors ensures exactness on odd wheels. As at least one triangle must then be assigned costs that are not locally optimal and which is larger by ϵ than its minimal reparametrized cost, the result follows. \square

Proof of Proposition 3

Proposition. Assume θ maximizes the dual lower bound $L(\theta)$ and the relaxation is tight, i.e.

$$L(\theta) = \min_{\{x \in \{0, 1\}^E \mid x^{-1}(1) \in \mathcal{M}_G\}} \langle \theta, x \rangle. \quad (22)$$

Moreover, let $\hat{x} \in \{0, 1\}^E$ such that $\hat{x}^{-1}(1)$ is an optimal multicut of G . Then,

$$\theta_e \begin{cases} \leq 0 & \text{if } \hat{x}_e = 1 \\ \geq 0, & \text{if } \hat{x}_e = 0 \end{cases} \quad (23)$$

Proof. Follows from the complementary slackness conditions in linear programming duality. \square

7.2. Detailed experimental evaluation

In Table 2 a detailed per instance evaluation of all algorithms considered in the experimental section can be found.

Table 2: Per instance evaluation of the considered eight datasets. UB means primal solution energy, LB dual lower bound and runtime(s) the runtime in seconds. **Bold** numbers indicate lowest primal energy, highest lower bound and smallest runtime. † means method not applicable.

Instance		MP-C	MC-COW	CGC	MC-ILP	CC-Fusion-RWS	CC-Fusion-RHC
knott-3d-450							
gm_knott_3d_096	UB	-89946.94	-89959.41	-80280.68	-89959.41	-89779.76	-89786.75
	LB	-89965.19	-89960.53	-94633.08	-89959.41	0.00	0.00
	runtime(s)	333.20	307.70	48.66	1438.60	16.89	153.09
gm_knott_3d_097	UB	-73474.82	-73477.25	-67333.36	-73477.55	-73386.50	-73364.35
	LB	-73482.05	-73478.09	-78026.62	-73477.55	0.00	0.00
	runtime(s)	146.60	255.79	26.83	1075.22	15.59	177.63
gm_knott_3d_098	UB	-86571.34	-86592.24	-78396.23	-86593.97	-86470.44	-86498.10
	LB	-86607.77	-86602.77	-91051.86	-86593.97	0.00	0.00
	runtime(s)	543.17	796.30	11.39	1507.50	15.15	69.82
gm_knott_3d_099	UB	-86248.96	-86248.11	-78712.34	-85956.93	-86184.26	-86180.01
	LB	-86262.13	-86260.36	-91020.66	-86449.40	0.00	0.00
	runtime(s)	333.42	859.70	34.96	3732.50	11.12	90.11
gm_knott_3d_100	UB	-76691.38	-76697.61	-68324.61	-76699.37	-76561.72	-76523.90
	LB	-76706.49	-76700.74	-81763.95	-76699.37	0.00	0.00
	runtime(s)	265.22	550.50	40.53	1076.28	22.65	141.63
gm_knott_3d_101	UB	-74529.16	-74529.51	-66796.31	-74529.51	-74500.99	-74495.74
	LB	-74531.14	-74529.51	-79463.73	-74529.51	0.00	0.00
	runtime(s)	202.22	154.58	33.62	1149.05	19.52	110.02
gm_knott_3d_102	UB	-66474.00	-66480.24	-60651.60	-66482.68	-66454.83	-66455.86
	LB	-66495.70	-66488.12	-71160.72	-66482.68	0.00	0.00
	runtime(s)	455.64	939.95	19.75	907.64	14.44	131.82
gm_knott_3d_103	UB	-73791.74	-73793.51	-66427.02	-73431.17	-73750.61	-73743.79
	LB	-73829.85	-73827.95	-79062.14	-73988.21	0.00	0.00
	runtime(s)	801.62	924.27	36.71	3836.96	16.82	79.70
knott-3d-550							
gm_knott_3d_112	UB	-152992.56	-153009.58	-136448.39	-153021.45	-152908.44	-152675.73
	LB	-153035.56	-153025.94	-160981.94	-153024.89	0.00	0.00
	runtime(s)	2469.78	3164.36	151.77	3716.74	88.55	462.34
gm_knott_3d_113	UB	-135578.01	-135574.00	-122181.33	-134820.65	-135466.46	-135386.34
	LB	-135623.31	-135624.43	-144181.42	-135924.12	0.00	0.00
	runtime(s)	1286.22	1354.84	105.96	3614.44	57.52	540.01
gm_knott_3d_114	UB	-149712.08	-149715.50	-134889.26	-149716.68	-149683.60	-149526.15
	LB	-149741.56	-149732.46	-157228.92	-149722.18	0.00	0.00
	runtime(s)	1150.88	1804.33	108.75	3614.56	99.81	384.54
gm_knott_3d_115	UB	-149829.79	-149850.05	-135760.55	-148726.82	-149736.65	-149777.01
	LB	-149906.03	-149902.08	-158348.78	-150325.94	0.00	0.00
	runtime(s)	1056.74	2030.40	111.32	3747.40	47.97	384.89
gm_knott_3d_116	UB	-130736.14	-130753.03	-118822.35	-130757.57	-130720.10	-130580.01
	LB	-130767.89	-130760.65	-138934.59	-130761.25	0.00	0.00
	runtime(s)	1120.40	1532.90	88.56	3688.88	92.77	976.88
gm_knott_3d_117	UB	-123458.56	-123442.93	-112948.58	-122646.08	-123368.23	-123448.71
	LB	-123510.55	-123508.20	-131937.12	-123810.61	0.00	0.00
	runtime(s)	1961.72	2303.68	90.49	3617.71	54.49	582.92
gm_knott_3d_118	UB	-123516.35	-123523.75	-112812.30	-122526.33	-123520.61	-123483.58
	LB	-123545.31	-123534.34	-131313.86	-123538.09	0.00	0.00
	runtime(s)	1763.03	1932.48	77.63	3777.86	83.73	869.68
gm_knott_3d_119	UB	-126318.25	-126318.25	-116868.98	-123919.61	-126308.30	-126289.59
	LB	-126437.44	-126424.69	-134702.52	-126935.77	0.00	0.00
	runtime(s)	3625.46	3626.10	84.92	3688.17	58.68	555.56
knott-3d-150							
gm_knott_3d_032	UB	-5811.47	-5811.47	-5365.34	-5811.47	-5745.79	-5767.34
	LB	-5812.24	-5811.47	-6052.81	-5811.47	0.00	0.00
	runtime(s)	0.81	1.13	0.03	2.70	0.17	0.48
gm_knott_3d_033	UB	-2545.84	-2545.84	-2536.26	-2545.84	-2517.65	-2545.84
	LB	-2545.84	-2545.84	-3029.52	-2545.84	0.00	0.00
	runtime(s)	0.44	0.43	0.02	1.28	0.22	0.35
gm_knott_3d_034	UB	-4064.87	-4064.87	-3921.65	-4064.87	-3971.60	-3972.66
	LB	-4065.78	-4064.87	-4337.06	-4064.87	0.00	0.00
	runtime(s)	0.48	0.51	0.02	3.69	0.47	0.58

Table 2: Per instance evaluation of the considered eight datasets. UB means primal solution energy, LB dual lower bound and runtime(s) the runtime in seconds. **Bold** numbers indicate lowest primal energy, highest lower bound and smallest runtime. † means method not applicable.

Instance		MP-C	MC-COW	CGC	MC-ILP	CC-Fusion-RWS	CC-Fusion-RHC
gm_knott_3d_035	UB	-4595.84	-4595.84	-4238.86	-4595.84	-4568.88	-4595.84
	LB	-4595.84	-4595.84	-4916.47	-4595.84	0.00	0.00
	runtime(s)	0.52	0.53	0.08	1.18	0.27	0.46
gm_knott_3d_036	UB	-5197.99	-5198.37	-4678.23	-5198.37	-5159.18	-5198.37
	LB	-5198.39	-5198.37	-5440.26	-5198.37	0.00	0.00
	runtime(s)	0.69	0.78	0.09	1.32	0.32	0.62
gm_knott_3d_037	UB	-4635.53	-4638.72	-4312.24	-4638.99	-4616.39	-4638.03
	LB	-4640.99	-4639.21	-4928.85	-4638.99	0.00	0.00
	runtime(s)	1.71	1.94	0.03	6.27	0.23	0.76
gm_knott_3d_038	UB	-4625.80	-4625.80	-4235.69	-4625.80	-4616.99	-4619.01
	LB	-4625.80	-4625.80	-4818.46	-4625.80	0.00	0.00
	runtime(s)	0.46	0.46	0.05	0.65	0.19	0.46
gm_knott_3d_039	UB	-5092.32	-5092.32	-4476.96	-5092.32	-5081.58	-5082.97
	LB	-5092.78	-5092.32	-5318.04	-5092.32	0.00	0.00
	runtime(s)	0.87	0.74	0.03	1.88	0.23	0.58
knott-3d-300							
gm_knott_3d_072	UB	-32994.78	-32999.85	-29632.23	-32999.85	-32883.17	-32875.45
	LB	-33000.58	-32999.85	-34512.56	-32999.85	0.00	0.00
	runtime(s)	25.40	35.19	4.60	49.78	4.26	3.40
gm_knott_3d_073	UB	-25863.15	-25863.38	-23433.69	-25863.38	-25738.25	-25740.05
	LB	-25864.27	-25863.38	-27464.92	-25863.38	0.00	0.00
	runtime(s)	9.34	13.64	3.24	55.72	2.72	8.88
gm_knott_3d_074	UB	-25717.45	-25721.90	-23513.94	-25721.90	-25625.74	-25627.65
	LB	-25724.00	-25721.90	-27196.88	-25721.90	0.00	0.00
	runtime(s)	12.35	13.83	1.19	39.63	2.18	10.73
gm_knott_3d_075	UB	-30478.37	-30478.37	-27294.35	-30478.37	-30429.06	-30471.30
	LB	-30478.41	-30478.41	-31854.88	-30478.37	0.00	0.00
	runtime(s)	5.48	7.22	2.89	14.29	2.18	3.57
gm_knott_3d_076	UB	-27055.03	-27056.62	-24789.93	-27056.99	-27004.21	-27031.94
	LB	-27064.67	-27058.47	-28550.56	-27056.99	0.00	0.00
	runtime(s)	32.44	66.39	2.62	50.33	3.32	10.79
gm_knott_3d_077	UB	-29480.77	-29480.77	-27122.85	-29482.24	-29476.76	-29481.33
	LB	-29483.33	-29482.48	-31159.83	-29482.24	0.00	0.00
	runtime(s)	11.15	16.57	2.40	47.23	4.12	10.59
gm_knott_3d_078	UB	-20207.36	-20210.01	-19374.02	-20211.55	-20189.82	-20157.27
	LB	-20213.81	-20212.24	-22015.47	-20211.55	0.00	0.00
	runtime(s)	27.23	73.08	1.56	1451.04	2.44	10.97
gm_knott_3d_079	UB	-26601.32	-26604.84	-23755.71	-26607.98	-26589.21	-26593.33
	LB	-26610.60	-26608.57	-28457.54	-26607.98	0.00	0.00
	runtime(s)	31.21	48.23	3.37	110.62	2.46	6.24
modularity clustering							
adjnoun	UB	-0.31	-0.31	-0.17	0.00	0.00	-0.29
	LB	-0.50	-0.45	-0.79	-0.45	0.00	0.00
	runtime(s)	161.03	1440.76	0.29	7288.07	0.01	92.48
dolphins	UB	-0.53	-0.53	-0.34	-0.53	0.00	-0.52
	LB	-0.56	-0.54	-0.83	-0.53	0.00	0.00
	runtime(s)	11.51	136.12	0.03	44.61	0.00	0.79
football	UB	-0.60	-0.60	-0.34	-0.60	0.00	-0.49
	LB	-0.61	-0.61	-0.90	-0.60	0.00	0.00
	runtime(s)	107.87	2742.70	0.37	71.91	0.01	7.63
karate	UB	-0.42	-0.42	-0.28	-0.42	0.00	-0.32
	LB	-0.42	-0.42	-0.66	-0.42	0.00	0.00
	runtime(s)	0.99	5.49	0.00	0.40	0.00	0.08
lesmis	UB	-0.56	-0.56	-0.37	-0.56	0.00	-0.50
	LB	-0.56	-0.56	-0.72	-0.56	0.00	0.00
	runtime(s)	19.44	165.51	0.03	3.77	0.00	0.72
polbooks	UB	-0.53	-0.53	-0.33	-0.53	0.00	-0.51
	LB	-0.57	-0.56	-0.83	-0.54	0.00	0.00
	runtime(s)	221.30	1936.45	0.17	10057.85	0.01	4.97

Table 2: Per instance evaluation of the considered eight datasets. UB means primal solution energy, LB dual lower bound and runtime(s) the runtime in seconds. **Bold** numbers indicate lowest primal energy, highest lower bound and smallest runtime. † means method not applicable.

Instance		MP-C	MC-COW	CGC	MC-ILP	CC-Fusion-RWS	CC-Fusion-RHC
		image-seg					
101087.bmp	UB	2789.90	2789.90	2853.56	2789.90	2800.22	2789.90
	LB	2789.21	2789.90	2622.38	2789.90	0.00	0.00
	runtime(s)	0.24	0.23	0.02	5.11	0.63	0.81
102061.bmp	UB	2944.18	2944.18	3090.33	2943.77	2963.42	2944.46
	LB	2943.48	2943.68	2750.99	2943.77	0.00	0.00
	runtime(s)	2.54	11.82	0.05	8.75	0.61	0.96
103070.bmp	UB	4200.41	4200.01	4457.13	4199.38	4205.03	4200.64
	LB	4198.39	4198.66	3842.84	4199.38	0.00	0.00
	runtime(s)	5.38	18.88	0.15	6.97	1.32	0.98
105025.bmp	UB	6056.43	6057.40	6290.71	6055.33	6070.84	6061.05
	LB	6053.20	6054.68	5506.01	6055.33	0.00	0.00
	runtime(s)	14.44	45.09	0.35	32.15	1.91	1.59
106024.bmp	UB	1599.25	1599.25	1654.27	1599.25	1618.18	1599.83
	LB	1598.60	1598.88	1466.60	1599.25	0.00	0.00
	runtime(s)	0.27	2.55	0.02	4.76	0.18	0.25
108005.bmp	UB	6578.26	6578.06	6855.33	6578.03	6584.62	6578.18
	LB	6577.53	6577.80	6151.29	6578.03	0.00	0.00
	runtime(s)	5.92	15.77	0.26	10.86	2.49	1.54
108070.bmp	UB	8423.52	8422.88	8612.32	8422.24	8445.73	8424.36
	LB	8421.95	8422.14	7818.70	8422.24	0.00	0.00
	runtime(s)	16.35	41.49	0.41	26.67	3.35	1.75
108082.bmp	UB	4802.59	4800.15	5090.88	4800.15	4815.78	4806.04
	LB	4798.87	4799.35	4380.66	4800.15	0.00	0.00
	runtime(s)	5.96	21.36	0.16	13.51	1.20	1.89
109053.bmp	UB	4422.17	4421.56	4616.61	4421.13	4424.05	4421.13
	LB	4419.85	4419.96	4021.22	4421.13	0.00	0.00
	runtime(s)	4.99	20.00	0.17	9.64	1.50	0.84
119082.bmp	UB	4530.71	4530.71	4642.96	4530.71	4535.85	4532.29
	LB	4530.71	4530.71	4346.24	4530.71	0.00	0.00
	runtime(s)	0.23	0.34	0.02	0.48	1.53	1.57
12084.bmp	UB	7284.48	7284.57	7443.28	7284.45	7301.17	7287.68
	LB	7284.13	7284.45	6941.02	7284.45	0.00	0.00
	runtime(s)	0.92	1.27	0.15	2.47	2.20	3.99
123074.bmp	UB	3844.10	3842.74	4031.03	3842.74	3856.82	3847.83
	LB	3841.67	3841.93	3439.47	3842.74	0.00	0.00
	runtime(s)	11.49	35.05	0.14	23.01	0.47	0.86
126007.bmp	UB	2685.44	2685.44	2747.72	2684.83	2706.76	2685.26
	LB	2684.75	2684.78	2512.08	2684.83	0.00	0.00
	runtime(s)	0.12	0.22	0.01	0.79	0.38	0.82
130026.bmp	UB	5363.89	5362.82	5580.58	5350.83	5369.95	5354.31
	LB	5347.89	5348.67	4828.82	5350.83	0.00	0.00
	runtime(s)	56.04	124.59	0.26	19.66	0.99	1.40
134035.bmp	UB	6579.48	6579.70	6679.89	6578.98	6595.87	6579.62
	LB	6577.89	6578.48	6166.95	6578.98	0.00	0.00
	runtime(s)	5.48	7.92	0.19	28.82	1.81	1.33
14037.bmp	UB	1383.14	1383.14	1431.56	1383.14	1393.66	1383.14
	LB	1383.14	1383.14	1274.27	1383.14	0.00	0.00
	runtime(s)	0.04	0.06	0.01	0.25	0.13	0.22
143090.bmp	UB	1714.38	1714.38	1807.41	1714.38	1725.88	1715.76
	LB	1714.38	1714.38	1595.54	1714.38	0.00	0.00
	runtime(s)	0.07	0.07	0.01	0.56	0.44	0.34
145086.bmp	UB	3322.50	3322.50	3407.83	3322.21	3329.14	3322.59
	LB	3322.16	3322.16	3197.53	3322.21	0.00	0.00
	runtime(s)	0.13	0.22	0.01	0.83	0.41	1.59
147091.bmp	UB	3974.62	3973.71	4129.72	3973.71	3982.30	3975.15
	LB	3973.48	3973.71	3734.67	3973.71	0.00	0.00
	runtime(s)	0.34	0.58	0.10	13.24	0.90	0.83

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Instance		MP-C	MC-COW	CGC	MC-ILP	CC-Fusion-RWS	CC-Fusion-RHC
148026.bmp	UB	8206.18	8205.98	8436.70	8205.98	8226.20	8207.72
	LB	8205.78	8205.98	7780.68	8205.98	0.00	0.00
	runtime(s)	0.47	0.64	0.15	4.49	3.10	2.73
148089.bmp	UB	6440.69	6440.55	6666.83	6439.58	6455.48	6440.33
	LB	6439.17	6439.21	6030.94	6439.58	0.00	0.00
	runtime(s)	7.35	17.99	0.17	18.64	2.00	1.53
156065.bmp	UB	5236.26	5235.41	5429.45	5234.15	5248.18	5234.76
	LB	5233.37	5233.30	4857.14	5234.15	0.00	0.00
	runtime(s)	6.63	47.04	0.15	18.51	1.49	1.12
157055.bmp	UB	4685.26	4685.26	4768.87	4685.17	4696.42	4685.17
	LB	4684.78	4684.78	4472.39	4685.17	0.00	0.00
	runtime(s)	0.19	0.29	0.02	2.60	1.37	1.48
159008.bmp	UB	4541.78	4540.87	4814.85	4540.87	4569.12	4541.22
	LB	4540.64	4540.87	4217.95	4540.87	0.00	0.00
	runtime(s)	0.36	0.48	0.10	16.64	0.83	1.29
160068.bmp	UB	3089.66	3089.33	3264.31	3089.32	3103.23	3089.32
	LB	3089.08	3089.20	2866.45	3089.32	0.00	0.00
	runtime(s)	0.21	2.31	0.05	1.80	0.61	1.07
16077.bmp	UB	4230.34	4227.88	4408.62	4227.88	4236.46	4228.78
	LB	4227.85	4227.88	3921.75	4227.88	0.00	0.00
	runtime(s)	0.74	1.40	0.07	3.22	1.18	1.63
163085.bmp	UB	4386.86	4384.15	4577.62	4381.13	4406.98	4384.59
	LB	4379.36	4379.97	3983.52	4381.13	0.00	0.00
	runtime(s)	7.48	17.43	0.15	9.82	0.91	1.30
167062.bmp	UB	1273.72	1273.72	1281.48	1273.72	1275.87	1275.67
	LB	1273.44	1273.72	1233.39	1273.72	0.00	0.00
	runtime(s)	0.04	0.09	0.00	1.00	0.11	0.17
167083.bmp	UB	8331.64	8331.63	8545.37	8331.63	8344.35	8331.90
	LB	8331.41	8331.63	7921.06	8331.63	0.00	0.00
	runtime(s)	1.50	1.71	0.23	11.74	2.27	1.80
170057.bmp	UB	3269.37	3269.37	3355.95	3266.17	3273.20	3266.73
	LB	3263.12	3263.67	2989.38	3266.17	0.00	0.00
	runtime(s)	18.87	22.13	0.07	18.20	0.53	0.67
175032.bmp	UB	11550.97	11550.67	11926.00	11542.63	11566.74	11547.67
	LB	11537.63	11538.95	10543.16	11542.63	0.00	0.00
	runtime(s)	162.07	380.08	1.27	165.83	3.25	3.98
175043.bmp	UB	7818.80	7818.80	8224.34	7816.92	7844.49	7822.01
	LB	7816.92	7816.92	7136.44	7816.92	0.00	0.00
	runtime(s)	0.96	1.58	0.39	13.17	4.30	3.46
182053.bmp	UB	3582.11	3582.11	3714.59	3579.24	3602.74	3582.99
	LB	3577.81	3578.32	3321.74	3579.24	0.00	0.00
	runtime(s)	11.00	20.32	0.07	14.11	0.65	0.92
189080.bmp	UB	1078.24	1078.24	1095.38	1077.47	1086.93	1078.41
	LB	1077.42	1077.42	972.41	1077.47	0.00	0.00
	runtime(s)	0.04	0.05	0.00	0.25	0.11	0.49
19021.bmp	UB	4516.53	4515.90	4693.10	4515.08	4521.63	4515.27
	LB	4514.97	4515.06	4178.50	4515.08	0.00	0.00
	runtime(s)	0.35	0.50	0.08	12.93	1.16	1.41
196073.bmp	UB	545.47	545.47	572.73	545.47	548.39	545.47
	LB	545.47	545.47	508.04	545.47	0.00	0.00
	runtime(s)	0.06	0.07	0.00	0.76	0.07	0.24
197017.bmp	UB	2798.77	2798.77	2857.86	2798.77	2801.34	2798.77
	LB	2798.77	2798.77	2663.98	2798.77	0.00	0.00
	runtime(s)	0.16	0.22	0.01	0.77	0.38	0.77
208001.bmp	UB	6274.40	6274.29	6605.51	6272.68	6277.73	6275.37
	LB	6269.16	6269.50	5773.37	6272.68	0.00	0.00
	runtime(s)	24.45	81.54	0.25	78.10	2.12	1.69
210088.bmp	UB	1896.19	1895.44	2034.65	1895.44	1901.54	1896.58
	LB	1895.44	1895.44	1726.75	1895.44	0.00	0.00
	runtime(s)	0.15	0.13	0.02	1.26	0.49	0.51

Table 2: Per instance evaluation of the considered eight datasets. UB means primal solution energy, LB dual lower bound and runtime(s) the runtime in seconds. **Bold** numbers indicate lowest primal energy, highest lower bound and smallest runtime. † means method not applicable.

Instance		MP-C	MC-COW	CGC	MC-ILP	CC-Fusion-RWS	CC-Fusion-RHC
21077.bmp	UB	2946.71	2946.71	3001.74	2946.71	2966.82	2946.71
	LB	2946.69	2946.71	2793.10	2946.71	0.00	0.00
	runtime(s)	2.77	1.26	0.01	4.60	0.44	1.13
216081.bmp	UB	4158.73	4158.73	4324.17	4158.73	4176.92	4158.73
	LB	4158.67	4158.67	3952.38	4158.73	0.00	0.00
	runtime(s)	0.21	0.26	0.03	1.37	1.13	1.08
219090.bmp	UB	2501.27	2501.27	2624.72	2501.27	2507.80	2501.27
	LB	2501.27	2501.27	2379.46	2501.27	0.00	0.00
	runtime(s)	0.10	0.15	0.02	0.14	0.48	0.97
220075.bmp	UB	3115.95	3115.95	3155.80	3115.95	3124.04	3115.95
	LB	3115.95	3115.95	2939.67	3115.95	0.00	0.00
	runtime(s)	0.11	0.13	0.01	0.38	0.75	1.65
223061.bmp	UB	6580.49	6579.52	6751.72	6576.83	6591.07	6578.02
	LB	6574.33	6574.59	5983.87	6576.83	0.00	0.00
	runtime(s)	30.80	109.42	0.34	37.60	1.67	1.53
227092.bmp	UB	1999.49	2001.53	2051.77	1998.46	2001.58	2001.19
	LB	1997.45	1997.66	1816.59	1998.46	0.00	0.00
	runtime(s)	2.79	19.63	0.03	4.25	0.37	0.34
229036.bmp	UB	6126.29	6126.29	6250.80	6125.73	6153.41	6126.61
	LB	6125.39	6125.50	5729.80	6125.73	0.00	0.00
	runtime(s)	8.34	24.22	0.09	4.83	1.53	2.00
236037.bmp	UB	9062.68	9062.68	9507.86	9060.84	9071.82	9060.84
	LB	9059.90	9060.10	8261.54	9060.84	0.00	0.00
	runtime(s)	21.34	40.49	0.75	20.14	5.02	4.38
24077.bmp	UB	4762.86	4761.98	4892.44	4761.98	4779.26	4761.98
	LB	4761.87	4761.98	4531.31	4761.98	0.00	0.00
	runtime(s)	1.29	3.02	0.03	5.10	1.14	2.40
241004.bmp	UB	1057.14	1057.14	1077.34	1057.14	1060.90	1057.42
	LB	1057.14	1057.14	984.58	1057.14	0.00	0.00
	runtime(s)	0.10	0.13	0.00	0.15	0.13	0.26
241048.bmp	UB	4734.71	4732.33	4945.52	4730.95	4750.17	4731.19
	LB	4729.01	4730.92	4343.63	4730.95	0.00	0.00
	runtime(s)	1.06	2.27	0.10	17.26	0.79	2.00
253027.bmp	UB	6607.26	6607.04	6950.82	6606.62	6614.15	6606.62
	LB	6606.17	6606.34	6371.53	6606.62	0.00	0.00
	runtime(s)	10.97	31.11	0.32	7.39	2.90	2.36
253055.bmp	UB	1502.16	1502.16	1549.84	1502.16	1518.91	1502.16
	LB	1502.04	1502.16	1408.65	1502.16	0.00	0.00
	runtime(s)	0.05	0.05	0.01	1.31	0.09	0.19
260058.bmp	UB	1084.26	1084.26	1091.96	1084.26	1085.01	1084.26
	LB	1084.26	1084.26	1017.59	1084.26	0.00	0.00
	runtime(s)	0.02	0.03	0.00	0.18	0.10	0.15
271035.bmp	UB	3622.64	3621.34	3875.99	3621.00	3657.57	3621.48
	LB	3620.21	3620.46	3326.25	3621.00	0.00	0.00
	runtime(s)	0.28	0.40	0.10	12.45	0.78	1.75
285079.bmp	UB	5611.41	5610.46	5773.41	5610.12	5640.22	5612.68
	LB	5609.32	5609.37	5246.17	5610.12	0.00	0.00
	runtime(s)	3.01	8.82	0.10	26.96	1.30	2.02
291000.bmp	UB	10209.05	10209.05	10401.10	10208.87	10222.91	10210.47
	LB	10208.76	10208.76	9626.61	10208.87	0.00	0.00
	runtime(s)	5.80	10.85	0.56	39.32	2.46	2.24
295087.bmp	UB	4292.12	4292.12	4509.08	4290.54	4299.90	4291.66
	LB	4289.70	4289.55	3985.98	4290.54	0.00	0.00
	runtime(s)	5.20	10.03	0.11	6.34	1.43	1.83
296007.bmp	UB	2293.13	2293.13	2384.77	2293.13	2306.24	2293.83
	LB	2293.13	2293.13	2115.21	2293.13	0.00	0.00
	runtime(s)	0.12	0.16	0.03	0.34	0.28	0.47
296059.bmp	UB	2044.72	2044.71	2160.60	2044.71	2045.98	2044.71
	LB	2044.66	2044.71	1891.55	2044.71	0.00	0.00
	runtime(s)	0.19	0.26	0.02	1.14	0.27	0.24

Table 2: Per instance evaluation of the considered eight datasets. UB means primal solution energy, LB dual lower bound and runtime(s) the runtime in seconds. **Bold** numbers indicate lowest primal energy, highest lower bound and smallest runtime. † means method not applicable.

Instance		MP-C	MC-COW	CGC	MC-ILP	CC-Fusion-RWS	CC-Fusion-RHC
299086.bmp	UB	1557.24	1557.24	1650.07	1557.24	1561.49	1557.24
	LB	1557.24	1557.24	1450.72	1557.24	0.00	0.00
	runtime(s)	0.07	0.09	0.02	0.11	0.29	0.41
300091.bmp	UB	1495.10	1495.10	1508.08	1495.10	1498.51	1495.10
	LB	1494.83	1495.10	1426.88	1495.10	0.00	0.00
	runtime(s)	0.12	0.08	0.01	1.91	0.21	0.16
302008.bmp	UB	2543.23	2543.23	2583.88	2543.23	2557.22	2543.23
	LB	2543.23	2543.23	2481.98	2543.23	0.00	0.00
	runtime(s)	0.07	0.09	0.01	0.23	0.17	0.46
304034.bmp	UB	7836.00	7836.04	8159.63	7835.47	7850.78	7836.49
	LB	7833.63	7834.74	7212.50	7835.47	0.00	0.00
	runtime(s)	13.03	29.85	0.40	24.88	3.14	2.05
304074.bmp	UB	3891.88	3891.88	4138.04	3891.88	3897.33	3891.88
	LB	3891.88	3891.88	3543.87	3891.88	0.00	0.00
	runtime(s)	0.18	0.23	0.11	0.51	0.83	0.79
306005.bmp	UB	4291.16	4290.25	4497.63	4290.25	4305.78	4290.25
	LB	4290.04	4290.12	4004.02	4290.25	0.00	0.00
	runtime(s)	0.98	1.66	0.11	17.38	0.91	0.98
3096.bmp	UB	396.90	396.90	396.90	396.90	411.23	396.90
	LB	396.90	396.90	389.83	396.90	0.00	0.00
	runtime(s)	0.01	0.01	0.00	0.02	0.00	0.06
33039.bmp	UB	8077.24	8074.46	8555.09	8069.67	8082.04	8070.48
	LB	8067.31	8067.79	7384.06	8069.67	0.00	0.00
	runtime(s)	24.21	119.33	0.44	20.06	3.61	3.29
351093.bmp	UB	6105.67	6105.67	6342.60	6105.28	6111.45	6107.08
	LB	6104.61	6104.70	5679.10	6105.28	0.00	0.00
	runtime(s)	10.94	27.96	0.20	19.54	2.10	2.30
361010.bmp	UB	3361.02	3361.02	3459.05	3361.02	3368.12	3364.98
	LB	3361.02	3361.02	3189.41	3361.02	0.00	0.00
	runtime(s)	0.39	0.65	0.02	0.39	0.91	0.79
37073.bmp	UB	1975.00	1975.00	2044.68	1975.00	1982.24	1975.00
	LB	1975.00	1975.00	1904.57	1975.00	0.00	0.00
	runtime(s)	0.08	0.12	0.01	0.15	0.31	0.49
376043.bmp	UB	5863.83	5865.41	6054.26	5863.83	5872.57	5864.00
	LB	5863.78	5863.59	5433.92	5863.83	0.00	0.00
	runtime(s)	10.70	33.77	0.17	9.80	1.32	0.94
38082.bmp	UB	8063.61	8063.34	8492.54	8060.34	8065.57	8066.62
	LB	8058.23	8059.42	7359.93	8060.34	0.00	0.00
	runtime(s)	14.17	62.08	0.62	26.23	2.41	2.49
38092.bmp	UB	4071.86	4071.86	4213.16	4071.86	4085.14	4071.86
	LB	4071.86	4071.86	3814.35	4071.86	0.00	0.00
	runtime(s)	0.16	0.21	0.09	0.46	0.91	0.99
385039.bmp	UB	3745.97	3745.97	3876.12	3745.53	3752.81	3745.53
	LB	3744.80	3744.80	3565.10	3745.53	0.00	0.00
	runtime(s)	0.14	0.20	0.06	2.51	1.20	0.81
41033.bmp	UB	1994.24	1994.24	2050.50	1994.24	2001.06	1997.55
	LB	1994.24	1994.24	1841.58	1994.24	0.00	0.00
	runtime(s)	0.12	0.29	0.01	0.94	0.28	0.40
41069.bmp	UB	5125.93	5125.93	5182.46	5110.96	5115.12	5122.63
	LB	5105.34	5106.56	4896.23	5110.96	0.00	0.00
	runtime(s)	190.78	246.34	0.07	35.77	0.36	0.54
42012.bmp	UB	3250.45	3250.45	3524.18	3248.70	3252.28	3251.04
	LB	3245.91	3246.18	3005.31	3248.70	0.00	0.00
	runtime(s)	3.47	28.98	0.08	8.95	0.59	0.73
42049.bmp	UB	1069.22	1069.22	1098.96	1069.22	1076.54	1069.22
	LB	1069.22	1069.22	997.53	1069.22	0.00	0.00
	runtime(s)	0.04	0.05	0.00	0.22	0.10	0.31
43074.bmp	UB	2332.83	2332.83	2374.97	2332.83	2340.46	2333.36
	LB	2331.80	2332.19	2166.88	2332.83	0.00	0.00
	runtime(s)	0.43	0.79	0.03	5.82	0.32	0.28

Table 2: Per instance evaluation of the considered eight datasets. UB means primal solution energy, LB dual lower bound and runtime(s) the runtime in seconds. **Bold** numbers indicate lowest primal energy, highest lower bound and smallest runtime. † means method not applicable.

Instance		MP-C	MC-COW	CGC	MC-ILP	CC-Fusion-RWS	CC-Fusion-RHC
45096.bmp	UB	977.78	977.78	1024.96	977.78	1031.06	977.78
	LB	977.78	977.78	911.19	977.78	0.00	0.00
	runtime(s)	0.04	0.05	0.01	0.10	0.05	0.25
54082.bmp	UB	3796.87	3796.36	3914.51	3796.36	3806.52	3797.18
	LB	3795.69	3796.36	3486.46	3796.36	0.00	0.00
	runtime(s)	1.22	1.04	0.10	5.80	0.43	1.03
55073.bmp	UB	7839.57	7839.57	8179.32	7835.96	7844.37	7838.99
	LB	7834.02	7835.13	7189.03	7835.96	0.00	0.00
	runtime(s)	18.69	65.96	0.39	13.66	2.54	1.67
58060.bmp	UB	9884.25	9884.05	10133.69	9881.86	9891.88	9882.77
	LB	9880.92	9881.27	9389.50	9881.86	0.00	0.00
	runtime(s)	13.50	42.07	0.27	20.02	4.15	6.54
62096.bmp	UB	3419.40	3419.40	3485.31	3419.40	3421.38	3420.03
	LB	3419.32	3419.35	3233.59	3419.40	0.00	0.00
	runtime(s)	1.08	2.61	0.05	5.70	0.54	0.56
65033.bmp	UB	7364.67	7364.67	7647.20	7364.57	7372.11	7365.30
	LB	7364.38	7364.38	6865.71	7364.57	0.00	0.00
	runtime(s)	0.44	0.56	0.18	11.87	2.04	2.05
66053.bmp	UB	4427.97	4427.97	4522.25	4427.25	4434.10	4427.25
	LB	4427.16	4427.25	4172.50	4427.25	0.00	0.00
	runtime(s)	1.22	1.58	0.07	4.29	0.65	0.80
69015.bmp	UB	4024.80	4024.45	4183.41	4024.45	4032.54	4025.34
	LB	4024.09	4024.45	3778.35	4024.45	0.00	0.00
	runtime(s)	0.55	0.90	0.06	6.14	1.00	1.27
69020.bmp	UB	5180.94	5180.14	5527.82	5179.29	5183.07	5179.29
	LB	5178.53	5179.05	4796.09	5179.29	0.00	0.00
	runtime(s)	3.24	26.70	0.20	12.34	1.16	0.97
69040.bmp	UB	7981.91	7980.11	8255.62	7974.58	7994.72	7983.83
	LB	7971.85	7972.94	7213.49	7974.58	0.00	0.00
	runtime(s)	64.40	156.34	0.55	31.49	2.16	3.76
76053.bmp	UB	4516.71	4515.41	4823.04	4514.99	4527.26	4516.03
	LB	4512.77	4514.22	4073.68	4514.99	0.00	0.00
	runtime(s)	3.91	13.74	0.14	18.85	1.59	1.21
78004.bmp	UB	3255.01	3254.61	3380.61	3254.61	3271.58	3254.85
	LB	3254.30	3254.54	3109.85	3254.61	0.00	0.00
	runtime(s)	0.12	0.95	0.01	2.70	0.34	0.64
8023.bmp	UB	4024.11	4026.12	4108.08	4023.38	4032.92	4026.67
	LB	4021.90	4022.01	3679.77	4023.38	0.00	0.00
	runtime(s)	32.73	57.23	0.14	32.67	0.84	0.60
85048.bmp	UB	5852.48	5852.48	6186.51	5851.38	5863.69	5852.33
	LB	5850.73	5851.13	5452.57	5851.38	0.00	0.00
	runtime(s)	1.48	2.75	0.16	8.07	2.03	1.91
86000.bmp	UB	4634.35	4634.35	4769.21	4633.86	4643.13	4633.96
	LB	4633.29	4633.29	4414.10	4633.86	0.00	0.00
	runtime(s)	1.75	3.62	0.03	4.90	1.49	1.76
86016.bmp	UB	6619.25	6618.85	6654.84	6618.85	6619.75	6620.36
	LB	6618.83	6618.85	6502.79	6618.85	0.00	0.00
	runtime(s)	1.23	3.52	0.02	2.62	0.36	0.53
86068.bmp	UB	5204.32	5200.40	5289.20	5198.87	5207.65	5205.68
	LB	5198.49	5198.64	4731.74	5198.87	0.00	0.00
	runtime(s)	8.51	22.42	0.23	9.72	1.10	0.60
87046.bmp	UB	4315.53	4315.53	4470.22	4315.53	4321.55	4315.53
	LB	4315.32	4315.53	3985.82	4315.53	0.00	0.00
	runtime(s)	3.00	1.07	0.09	11.52	1.14	0.60
89072.bmp	UB	3933.75	3933.75	4159.28	3933.75	3948.57	3934.47
	LB	3933.32	3933.75	3707.83	3933.75	0.00	0.00
	runtime(s)	0.18	0.23	0.06	3.69	1.07	1.00
97033.bmp	UB	4320.69	4320.69	4583.48	4320.69	4336.56	4322.76
	LB	4320.69	4320.69	3996.62	4320.69	0.00	0.00
	runtime(s)	0.34	0.43	0.06	1.62	0.83	1.31

Table 2: Per instance evaluation of the considered eight datasets. UB means primal solution energy, LB dual lower bound and runtime(s) the runtime in seconds. **Bold** numbers indicate lowest primal energy, highest lower bound and smallest runtime. † means method not applicable.

Instance		MP-C	MC-COW	CGC	MC-ILP	CC-Fusion-RWS	CC-Fusion-RHC
CREMI-small							
gm_small_1	UB	-301673.23	-301672.88	-278423.74	-301674.02	0.00	-301674.02
	LB	-301674.60	-301674.42	-302379.69	-301673.92	0.00	0.00
	runtime(s)	475.67	938.66	56.18	998.95	0.00	191.08
gm_small_2	UB	-127477.96	-127484.22	-114182.56	-116678.57	-127414.03	-127292.54
	LB	-127525.18	-127519.65	-131075.52	-127520.30	0.00	0.00
	runtime(s)	1694.22	1801.45	369.68	3722.33	3607.92	3752.86
gm_small_3	UB	-210416.41	-210422.18	-191243.51	-210430.88	-210396.31	-210386.96
	LB	-210437.50	-210434.95	-212966.73	-210432.60	0.00	0.00
	runtime(s)	577.22	975.66	531.16	3606.16	3479.31	3722.51
CREMI-large							
gm_large_1	UB	-5647849.59	-5647849.59	0.00	0.00	-5628646.55	-5524856.64
	LB	-5648219.39	-5648215.44	0.00	0.00	0.00	0.00
	runtime(s)	4239.42	4446.06	0.00	0.00	3863.35	10197.66
gm_large_2	UB	-2368925.73	-2368876.90	0.00	0.00	-1916548.20	-1713523.77
	LB	-2371443.65	-2371482.98	0.00	0.00	0.00	0.00
	runtime(s)	4208.11	3978.70	0.00	0.00	8092.81	36081.14
gm_large_3	UB	-3644563.67	-3644510.14	0.00	0.00	0.00	0.00
	LB	-3645722.55	-3645747.77	0.00	0.00	0.00	0.00
	runtime(s)	3818.01	3856.96	0.00	0.00	0.00	0.00