Learning random-walk label propagation for weakly-supervised semantic segmentation: Supplemental material

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1. Additional results

1.1. Additional qualitative results

An expanded version of the VOC 2012 result figure is shown in Fig. 1.

2. Derivations

2.1. Probabilistic justification of loss

In Sec. 2.1, it is claimed that Eq. 2 follows from marginalizing Eq. 1 over the unobserved labels, given certain assumptions. This is proved here.

Marginalizing the objective in Eq. 1 over y is expressed as:

$$\sum_{\tilde{y},\tilde{B}} P_{y,B|\hat{y},I}(\tilde{y},\tilde{B}) \sum_{x\in X} H(\delta_{\tilde{y}(x)}, Q_{\theta,I}(x)). \tag{1}$$

Using the conditional independence assumptions and the assumption that B is a deterministic function of I yields

$$\sum_{\tilde{y}} P_{y|\hat{y},B_I}(\tilde{y}) \sum_{x\in X} H(\delta_{\tilde{y}(x)}, Q_{\theta,I}(x)), \tag{2}$$

where B_I denotes the boundaries as a function of I. Assuming that $y(x)$ and $y(x')$ are conditionally independent given $\hat{y}, B, \forall x \neq x' \in X$ allows us to represent $P_{y|\hat{y},B_I}$ as a product of factors:

$$\sum_{\tilde{y}} \left(\prod_{x'\in X} P_{y(x')|\hat{y},B_I}(\tilde{y}(x')) \right) \sum_{x\in X} H(\delta_{\tilde{y}(x)}, Q_{\theta,I}(x)). \tag{3}$$

Factorizing out sums equal to one results in

$$\sum_{x\in X} \sum_{\tilde{y}(x)} P_{y(x)|\hat{y},B_I}(\tilde{y}(x)) H(\delta_{\tilde{y}(x)}, Q_{\theta,I}(x)). \tag{4}$$

Expanding the definition of cross-entropy yields the desired expression:

$$\sum_{x\in X} H(P_{y(x)|\hat{y},B_I}, Q_{\theta,I}(x)) \tag{5}$$

2.2. Fréchet derivative of matrix inverse

This result is used in the derivation of the derivative of the random-walk partition function. An intuitive derivation is provided here.

Suppose $Ax = b$. We wish to find an expansion linear in ϵV for $\tilde{x} := (A + \epsilon V)^{-1} b$, assuming the inverse exists.

$$\begin{aligned}
(A+\epsilon V)\tilde{x} &= b && (6)\\
A\tilde{x} &= b - \epsilon V \tilde{x} && (7)\\
\tilde{x} &= A^{-1}b - \epsilon A^{-1} V \tilde{x} && (8)\\
&= A^{-1}b - \epsilon A^{-1} V (A^{-1} b - O(\epsilon)) && (9)\\
&= A^{-1}b - \epsilon A^{-1} V A^{-1} b + O(\epsilon^2) && (10)
\end{aligned}$$

where the second-to-last line follows from recursive expansion of the same expression.

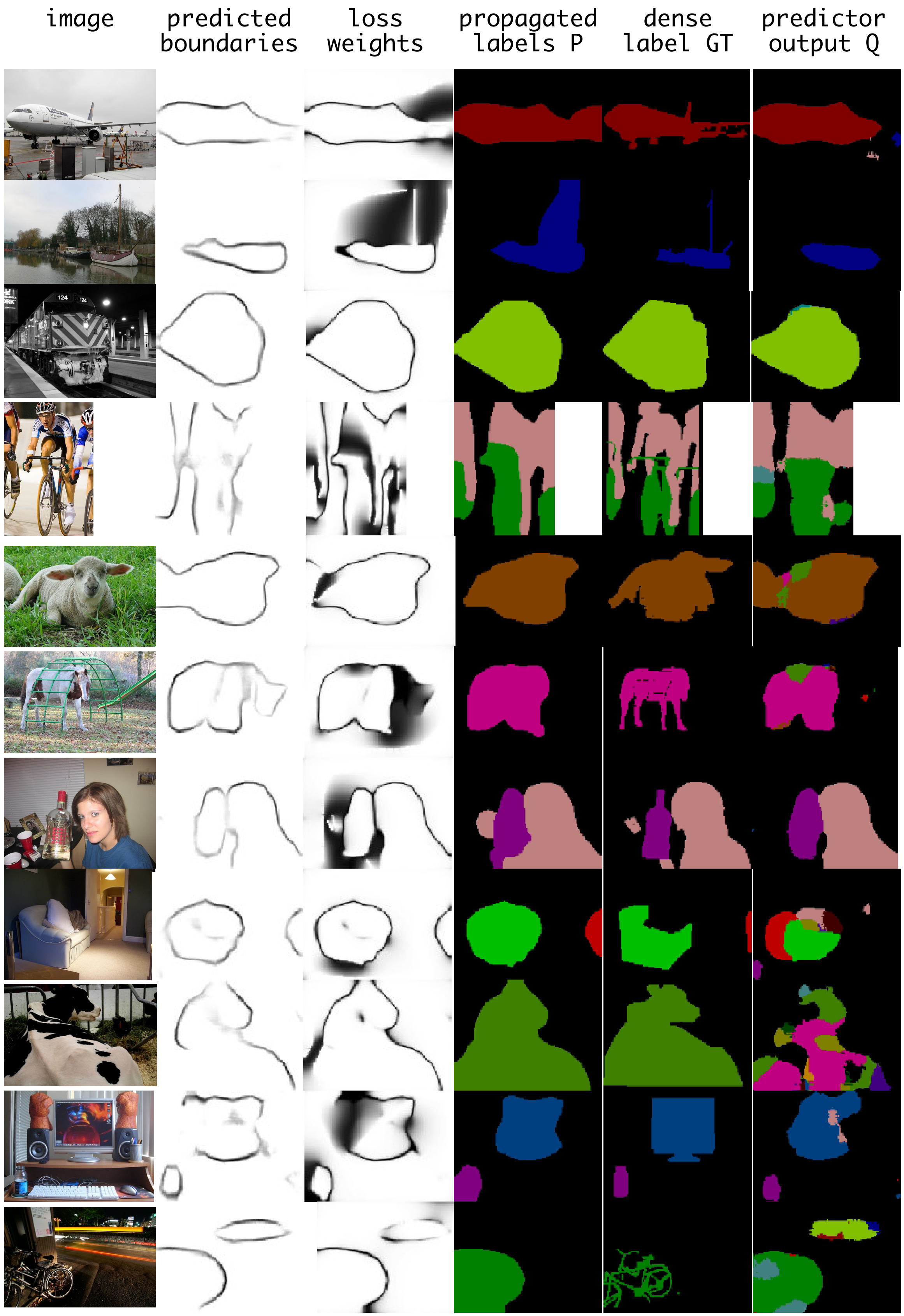

Figure 1: Validation set results on VOC 2012 dataset