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109	163	
110	There are four parts in this supplementary material:	164
111		165
112	• The first part (part 1) presents the details of the optimization procedure.	166
113		167
114	• The second part (part 2) presents additional comparison results against state-of-	168
115	the-art methods.	169
116		170
117	• The third part (part 3) presents additional despeckled results produced from our	171
118	method.	172
119		173
120	• The last part (part 4) presents additional segmentation results with comparison.	174
121		175
122	Note that all the clinical images presented in this work are obtained from the public	176
123	ultrasound data set downloaded from the following webpage:	177
124	http://www.ultrasoundcases.info .	178
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Part 1. Details of Our Optimization Method

In this part, we present how we iteratively solve (minimize) the low-rank recovery model presented in Eq. 10 of our submitted paper.

Rewriting the Low-rank Recovery Model. First, according to the definition of w_i (see Eq. 7 in our paper), we set w_i to be zero for those smallest singular values, and there are λ of them. Hence, we can re-write the truncated and weighted nuclear norm (TWNN) by skipping the first λ terms in Eq. 7 in the paper as

$$\|\Psi_D\|_{tw} = \sum_{i=\lambda+1}^M w_i \sigma_i(\Psi_D), \quad (1)$$

where Ψ_D is the low-rank component of input Ψ_I (see paper); M is the total number of the singular values of Ψ_D ; and w_i is the weight on the i -th singular value σ_i of Ψ_D .

Next, we can devise an alternating direction method of multipliers (ADMM) method to solve our low-rank recovery model (after substituting Eq. 1 above into Eq. 10 in our submitted paper):

$$\min_{\Psi_D, \Psi_\eta} \|\Psi_D\|_{tw} + \alpha \sum_{g \in \Psi_\eta} \|g\|_\infty + \langle Y, \Psi_I - \Psi_D - \Psi_\eta \rangle + \beta \|\Psi_I - \Psi_D - \Psi_\eta\|_F^2, \quad (2)$$

where $\|\cdot\|_F$ is the Frobenius norm; α is a parameter set to be 1.0 in the current implementation; g is each 3×3 submatrix in Ψ_η (see paper); Y is the Lagrange multiplier; β is a parameter set to 2; and $\langle \cdot, \cdot \rangle$ denotes the inner product. Note that we set the initial values of Ψ_D and Ψ_η as Ψ_I and a zero matrix, respectively, and Y as:

$$Y_0 = \text{sgn}(\Psi_I) / \max(\|\text{sgn}(\Psi_I)\|_2, \lambda^{-1} \|\text{sgn}(\Psi_I)\|_\infty), \quad (3)$$

where sgn is the sign function; and $\|\cdot\|_\infty$ denotes the maximum absolute value of all the matrix elements. For details about this initialization, readers may refer to [6, 5].

The core idea of the ADMM is to separate the optimization in Eq. 2 above into two subproblems, and then to solve them iteratively by updating Ψ_D and Ψ_η alternatively.

324 **Subproblem 1: Update $(\Psi_D)_{t+1}$:** Given $(\Psi_\eta)_t$ and Y_t , we compute $(\Psi_D)_{t+1}$ by solv-
 325 ing (minimizing) the following objective function:
 326

$$\min_{\Psi_D} \sum_{i=\lambda+1}^M \|\Psi_D\|_{tw} + \frac{\beta}{2} \|\Psi_I - \Psi_D - (\Psi_\eta)_t - \frac{1}{\beta} Y_t\|_F^2. \quad (4)$$

327 To solve Eq. 4, we need to expand the nonlinear term $\|\Psi_D\|_{tw}$. Regarding this, we
 328 first prove the following inequality, which is a generalization of Theorem 3.1 in [12]:
 329

330 *To prove:* For any given matrix $X \in \mathbb{R}^{m \times n}$, any matrices $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{m \times n}$, such
 331 that $AA^T = \mathbb{I}$ and $BB^T = \mathbb{I}$, and any diagonal matrix $Q \in \mathbb{R}^{n \times n}$, for non-negative
 332 integer $r = \text{rank}(A) = \text{rank}(B)$ (where $0 \leq r \leq \min(m, n)$), we have
 333

$$\text{Tr}(AXQB^T) \leq \sum_{i=1}^r \rho_i \sigma_i(X), \quad (5)$$

334 where $\sigma_i(X)$ is the i -th singular value of X ; and ρ_1, \dots, ρ_n are diagonal elements in Q .
 335

336 *Proof:* We start from the left hand side of Eq. 5:
 337

$$\begin{aligned} \text{Tr}(AXQB^T) &= \text{Tr}(XQB^TA) \quad (\text{by trace rule: } \text{Tr}(M_1M_2) = \text{Tr}(M_2M_1)) \\ &\leq |\text{Tr}(XQB^TA)| \quad (\text{since } x \leq |x| \text{ for any scalar } x) \\ &\leq \sum_{i=1}^{\min(m,n)} \sigma_i(X) \sigma_i(QB^TA) \quad (\text{by Von Neumann's trace inequality [9]}) \end{aligned} \quad (6)$$

338 By singular value decomposition, we know that the non-zero singular values of a
 339 matrix (say M) are the square roots of the non-zero eigenvalues of MM^* , where M^*
 340 denotes conjugate transpose. Since QB^TA is a real matrix, $(QB^TA)^*$ is just $(QB^TA)^T$.
 341

342 Let q be the rank of matrix QB^TA , where $q \leq r$. The nonzero $\sigma_i(QB^TA)$ val-
 343 ues are the square roots of the non-zero eigenvalues of $(QB^TA)(QB^TA)^T$, which is
 344 QB^TAATBQ^T . Since $AA^T = \mathbb{I}$ and $BB^T = \mathbb{I}$, QB^TAATBQ^T is simply a diagonal
 345 matrix whose elements are $\rho_1^2, \dots, \rho_q^2$. As a result, the nonzero $\sigma_i(QB^TA)$'s are $\rho_1, \dots,$
 346 ρ_q (for $i \in [1, q]$), and the rest ($[q+1, n]$) are zeros.
 347

432 Therefore, putting this result into Eq. 6, we can obtain 486
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$$\text{Tr}(AXQB^T) \leq \sum_{i=1}^{\min(m,n)} \sigma_i(X)\sigma_i(QB^TA) \quad (\text{from Eq. 6})$$
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$$= \sum_{i=1}^q \sigma_i(X)\sigma_i(QB^TA) + \sum_{i=q+1}^{\min(m,n)} \sigma_i(X)\sigma_i(QB^TA)$$
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$$= \sum_{i=1}^q \sigma_i(X)\rho_i + \sum_{i=q+1}^{\min(m,n)} \sigma_i(X) \cdot 0$$
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$$\leq \sum_{i=1}^r \sigma_i(X)\rho_i \quad (\text{since } q \leq r \text{ and } \rho_i \geq 0).$$
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447 This proves Eq. 5. Moreover, following [12], $\text{Tr}(AXQB^T)$ attains maximum when 505
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$$A = (\sqrt{\rho_1}u_1, \dots, \sqrt{\rho_r}u_r, \mathbf{0})^T \text{ and } B = (\sqrt{\rho_1}v_1, \dots, \sqrt{\rho_r}v_r, \mathbf{0})^T.$$
 507

449 where $U_1\Sigma_1V_1$ is the singular value decomposition (SVD) of X ; $U_1 = (u_1, \dots, u_{\min(m,n)})$; 510
 450 $V_1 = (v_1, \dots, v_{\min(m,n)})$; and Σ_1 is a diagonal matrix [End of Proof]. 511

451 By setting X as Ψ_D and ρ_i in Q as 514
 452 515

$$\rho_i = \frac{\theta\sqrt{K+1}}{\sqrt{\sigma_i(\Psi_D)} + \varepsilon},$$
 516

453 where θ , K and ε are defined in paper, we can employ Eq. 5 and rewrite TWNN of 520
 454 Eq. 1 as: 521
 455 522
 456 523

$$\begin{aligned} 457 \|\Psi_D\|_{tw} &= \sum_{i=\lambda+1}^M w_i\sigma_i(\Psi_D) \quad (\text{by Eq. 1}) \\ 458 &= \sum_{i=1}^M \rho_i\sigma_i(\Psi_D) - \sum_{i=1}^{\lambda} \rho_i\sigma_i(\Psi_D) \\ 459 &\approx \sum_{i=1}^M \rho_i\sigma_i(\Psi_D) - \max_{AA^T=\mathbb{I}, BB^T=\mathbb{I}} \text{Tr}(A\Psi_DQB^T), \quad (\text{by Eq. 5}) \end{aligned} \quad (10)$$

460 From Eq. 7 in paper, $\rho_i = w_i$ when $i \geq \lambda + 1$. By replacing TWNN of Eq. 1 with 533
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540 Eq. 10, we can reformulate the optimization in Eq. 4 as
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$$\min_{\Psi_D} \sum_{i=1}^M \rho_i \sigma_i(\Psi_D) - \max_{AA^T = \mathbb{I}, BB^T = \mathbb{I}} \text{Tr}(A\Psi_D Q B^T) + \frac{\beta}{2} \|\Psi_I - \Psi_D - (\Psi_\eta)_t - \frac{1}{\beta} Y_t\|_F^2. \quad (11)$$

542 According to [12], we can solve Eq. 11 by an efficient two-step scheme to update
 543 Ψ_D , and (A, B) in an iterative manner. In the j -th ($j \in [1, J]$) iteration, the two-step
 544 scheme is described as:
 545

546
 547 **Step 1.1** Let Θ_j be $(\Psi_D)_{t+1}$ in the j -th iteration, and $[(U_2)_j(\Sigma_2)_j(V_2)_j] = \text{svd}(\Theta_j)$,
 548 where $(U_2)_j = (u_1, \dots, u_M)^T$ and $(V_2)_j = (v_1, \dots, v_M)^T$. Then we can estimate A_j
 549 and B_j using the following equation:
 550

$$A_j = (\sqrt{\rho_1} u_1, \dots, \sqrt{\rho_\lambda} u_\lambda, \mathbf{0})^T \text{ and } B_j = (\sqrt{\rho_1} v_1, \dots, \sqrt{\rho_\lambda} v_\lambda, \mathbf{0})^T. \quad (12)$$

551
 552 **Step 1.2** After obtaining A_j and B_j in Step 1.1, Θ_{j+1} at the $(j+1)$ -th iteration is
 553 computed as:
 554

$$\min_{\Psi_D} \sum_{i=1}^M \rho_i \sigma_i(\Psi_D) - \text{Tr}(A_j \Psi_D Q B_j^T) + \frac{\beta}{2} \|\Psi_I - \Psi_D - (\Psi_\eta)_t - \frac{1}{\beta} Y_t\|_F^2. \quad (13)$$

555 Now, we can employ the accelerated proximal gradient line search method (APGL) [12]
 556 to minimize Eq. 13 above. Let $f(\Psi_D) = -\text{Tr}(A\Psi_D Q B^T) + \frac{\beta}{2} \|\Psi_I - \Psi_D - (\Psi_\eta)_t -$
 557 $\frac{1}{\beta} Y_t\|_F^2$, and $e(\Psi_D) = \sum_{i=1}^M \rho_i \sigma_i(\Psi_D)$. For a given parameter $s > 0$, by introducing an
 558 auxiliary variable Z , APGL method constructs an approximation of Eq. 13 as:
 559

$$Q(\Psi_D, Z) = f(Z) + \langle \Psi_D - Z, \nabla f(Z) \rangle + \frac{1}{2s} \|\Psi_D - Z\|_F^2 + e(\Psi_D). \quad (14)$$

560
 561 Then, APGL method uses another iteration ($k \in [1, K_2]$) to iteratively update Ψ_D , Z
 562 and s . The initial value of Z and s are set as Θ_j and 1, respectively. Assuming that Γ_k
 563

648 is the k -th iteration for computing $\Theta_{j+1}, \Gamma_{k+1}$ in the $(k+1)$ -th iteration is computed as:
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$$\begin{aligned} \Gamma_{k+1} &= \arg \min_{\Psi_D} Q(\Psi_D, Z_k) \quad (\text{from Eq. 14}) \\ &= \langle \Psi_D - Z_k, \nabla f(Z_k) \rangle + \frac{1}{2s_k} \|\Psi_D - Z_k\|_F^2 + g(\Psi_D) \\ &\quad (\text{by removing terms with } Z_k \text{ from Eq. 14}) \\ &= \arg \min_{\Psi_D} \frac{1}{2s_k} \|\Psi_D - (Z_k - s_k \nabla f(Z_k))\|_F^2 + \sum_{i=1}^M \rho_i \sigma_i(\Psi_D) \quad (15) \\ &\quad \left(\text{by adding } \frac{s_k (\nabla f(Z_k))^2}{2} \right) \\ &= \arg \min_{\Psi_D} \frac{1}{2s_k} \|\Psi_D - \Re_k\|_F^2 + \sum_{i=1}^M \rho_i \sigma_i(\Psi_D), \quad (\text{by calculating } \nabla f(Z_k)) \end{aligned}$$

where $\Re_k = Z_k + s_k(A_j^T B_j Q^T - \frac{\beta}{2}(\Psi_I - (\Psi_\eta)_t - \frac{1}{\beta}Y_t))$. Now, according to [3], the closed-formed solution of Eq. 15 is given by:

$$\begin{cases} (U_3, \Sigma_3, V_3) = \text{svd}(\Re_k) \\ (\Gamma)_{k+1} = U_3(\Omega(\Sigma_3))V_3^T, \end{cases} \quad (16)$$

where the singular value shrinkage operator $\Omega(\Sigma_3)_{ii}$ is:

$$\Omega(\Sigma_3)_{ii} = \max(\Sigma_{ii} - 2\rho_i s_k, 0), \quad (17)$$

where Σ_{ii} is the i -th largest singular values in the diagonal matrix Σ_3 .

Meanwhile, according to [4] [12], Z_{k+1} and s_{k+1} are computed as:

$$Z_{k+1} = \Gamma_{k+1} + \frac{s_k - 1}{s_k}(\Gamma_{k+1} - \Gamma_k), \text{ and } s_{k+1} = \frac{1 + \sqrt{1 + 4(s_k)^2}}{2}. \quad (18)$$

Subproblem 2: Update $(\Psi_\eta)_{t+1}$: Given $(\Psi_D)_{t+1}$ and Y_t , $(\Psi_\eta)_{t+1}$ is updated by the minimization below:

$$\min_{\Psi_\eta} \alpha \sum_{g \in \Psi_\eta} \|g\|_\infty + \frac{\beta}{2} \|\Psi_I - (\Psi_D)_{t+1} - \Psi_\eta - \frac{1}{\beta}Y_t\|_F^2, \quad (19)$$

where α and g are defined in Eq. 10 of our paper. According to [7], the solution of Eq. 19 is the proximal operator related with a structured sparsity-inducing norm,

756 **Algorithm 1** Our ADMM procedure to recover Ψ_D from Ψ_I

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Input: a patch group matrix Ψ_I from input image

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Input: a patch group matrix Ψ_I from input image

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Part 2. Additional Comparison Results

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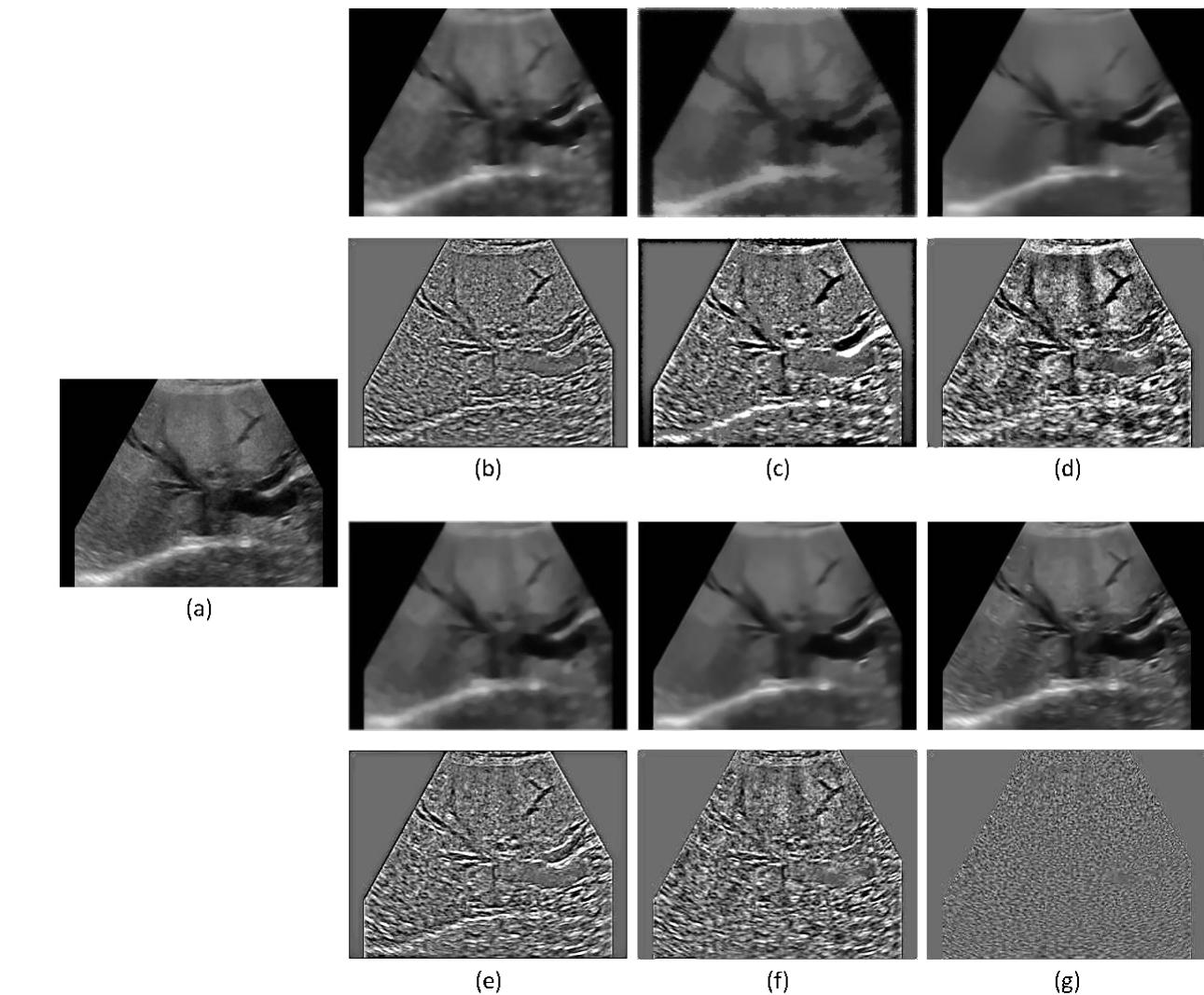


Figure 1: Additional comparison result #1. (a) input clinical ultrasound image with an inhomogeneous mass in the liver hilum. Top row of (b)-(g): despeckled results produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NLMLS [10], and our method, respectively. Bottom row of (b)-(g) shows the noise components of these despeckled results.

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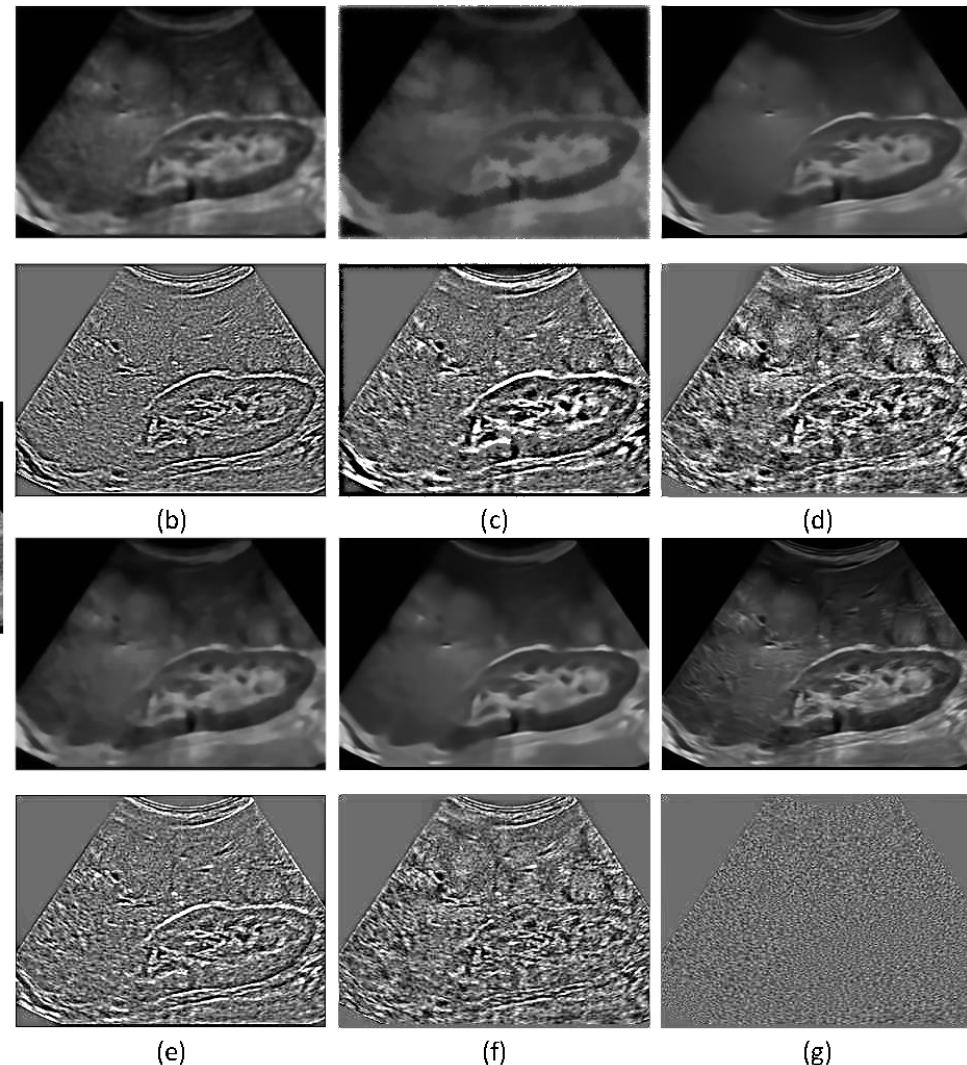


Figure 2: Additional comparison result #2. (a) input clinical ultrasound image with an enlarged hypoechoic inguinal lymph node. Top row of (b)-(g): despeckled results produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NLMLS [10], and our method, respectively. Bottom row of (b)-(g) shows the noise components of these despeckled results.

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Figure 3: Additional comparison result #3. (a) input clinical ultrasound image with multiple common bile duct stones. Top row of (b)-(g): despeckled results produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NLMLS [10], and our method, respectively. Bottom row of (b)-(g) shows the noise components of these despeckled results.

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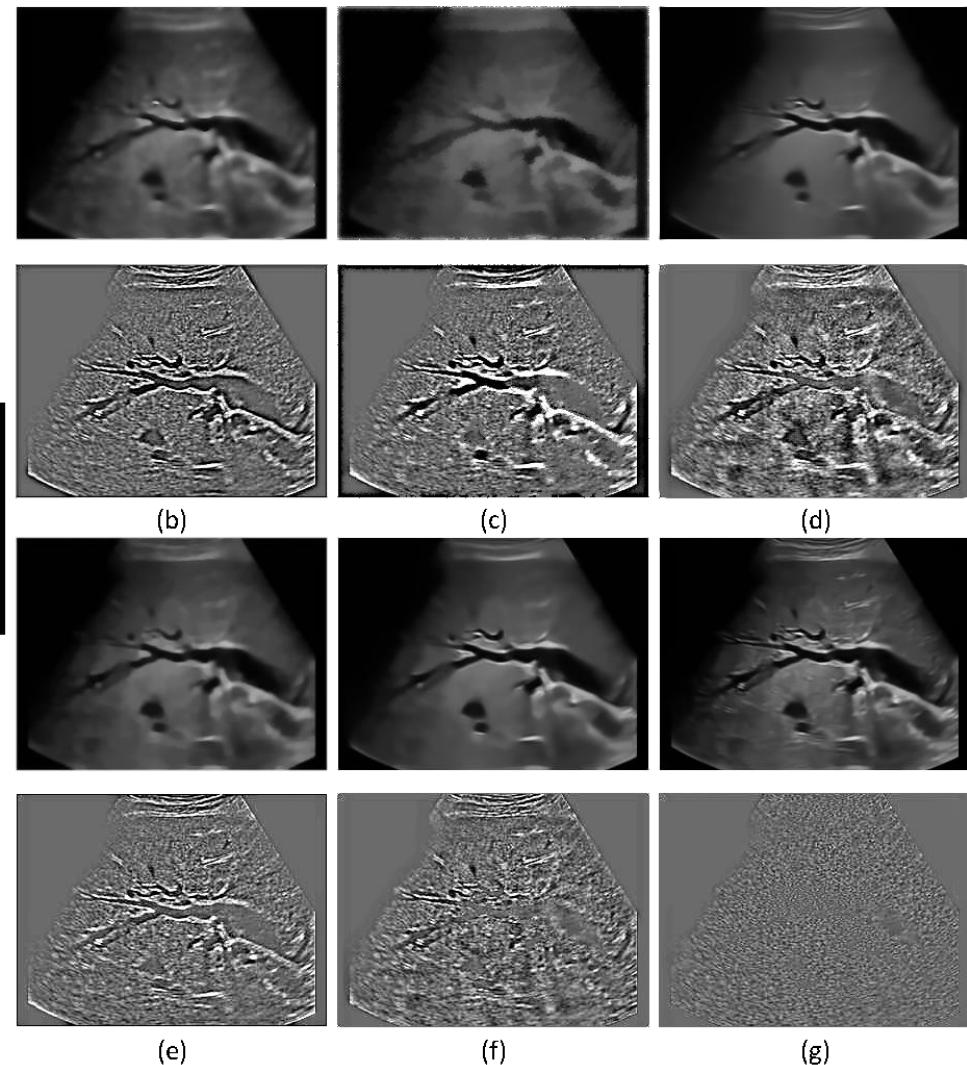
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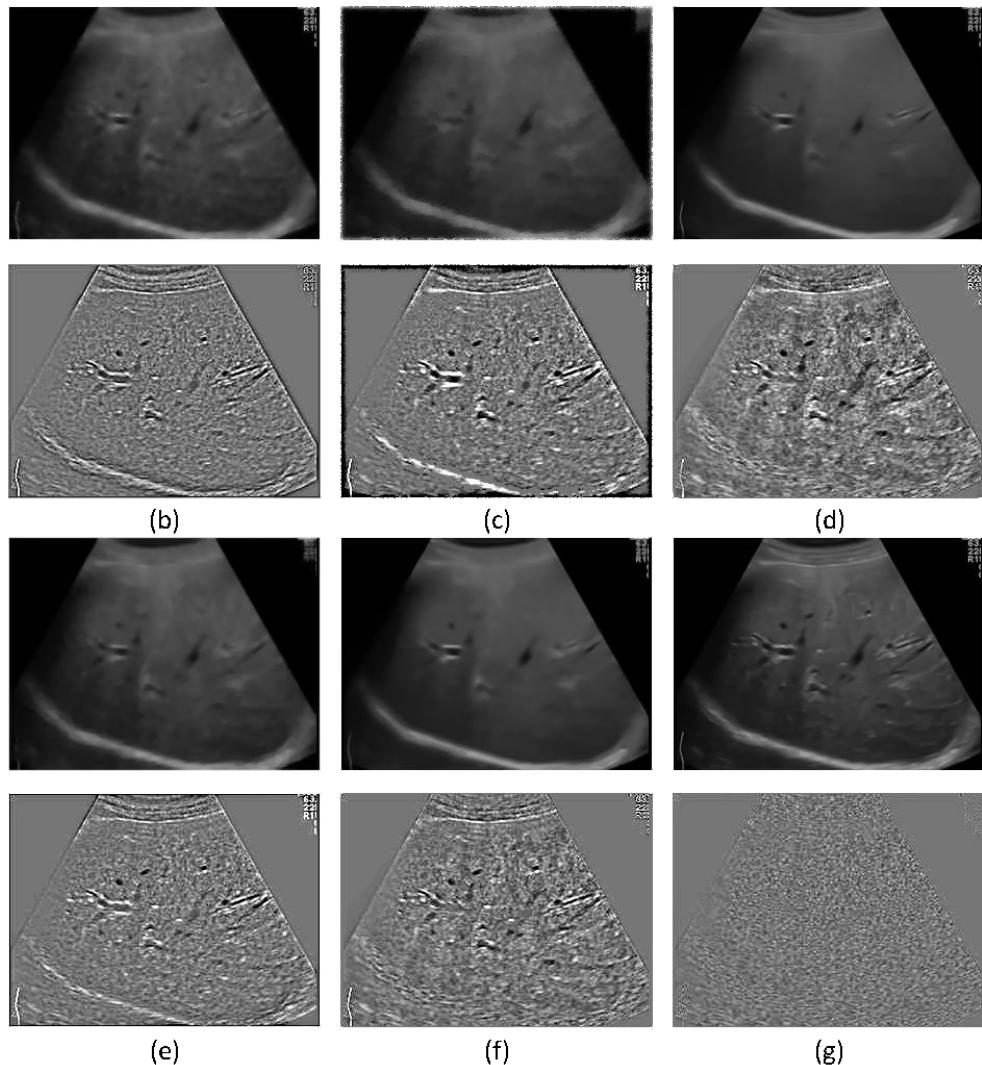
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Figure 4: Additional comparison result #4. (a) input clinical ultrasound image with dilated intrahepatic bile ducts. Top row of (b)-(g): despeckled results produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NLMLS [10], and our method, respectively. Bottom row of (b)-(g) shows the noise components of these despeckled results.



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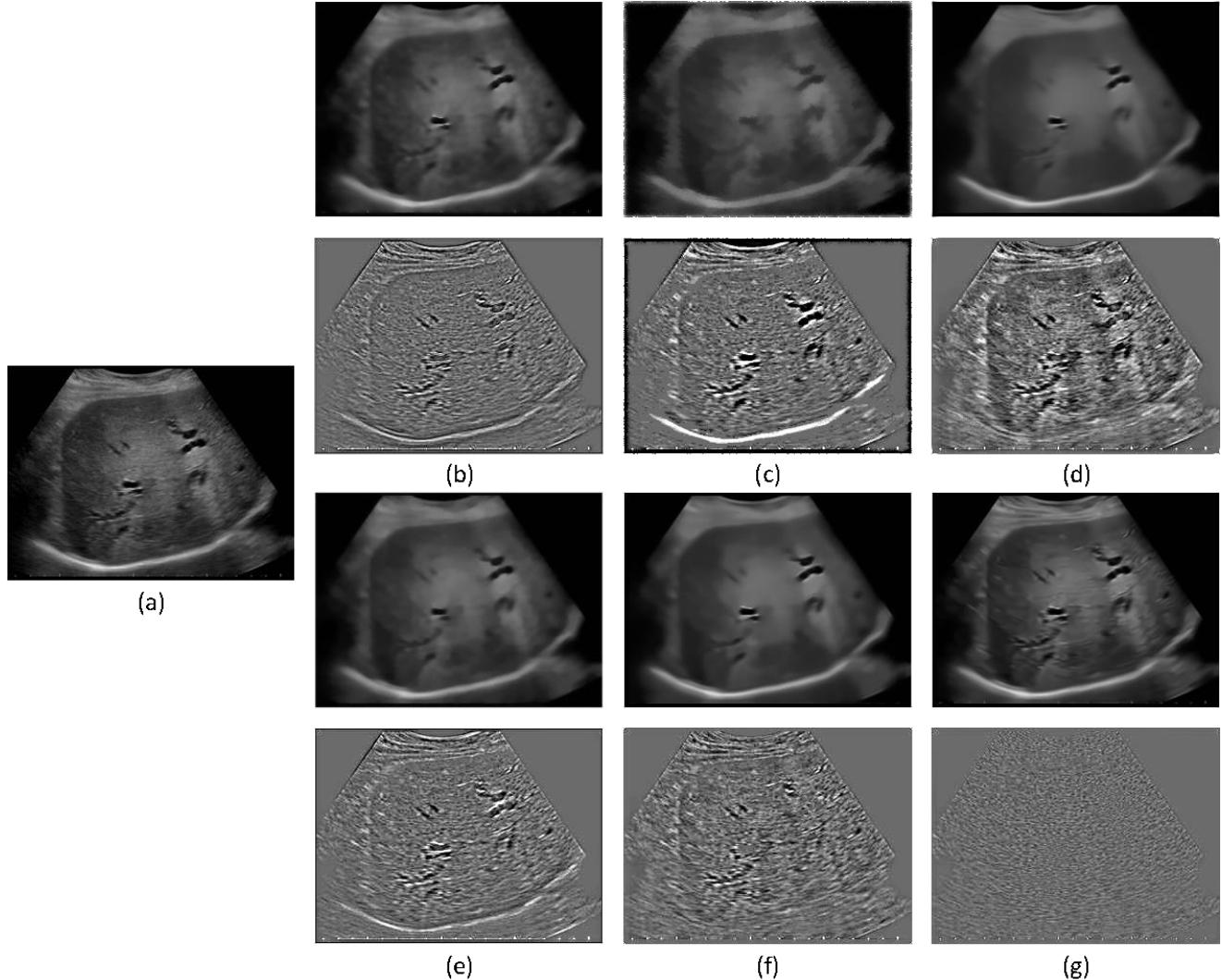


Figure 5: Additional comparison result #5. (a) input clinical ultrasound image with dilatated bile ducts.. Top row of (b)-(g): despeckled results produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NLMLS [10], and our method, respectively. Bottom row of (b)-(g) shows the noise components of these despeckled results.

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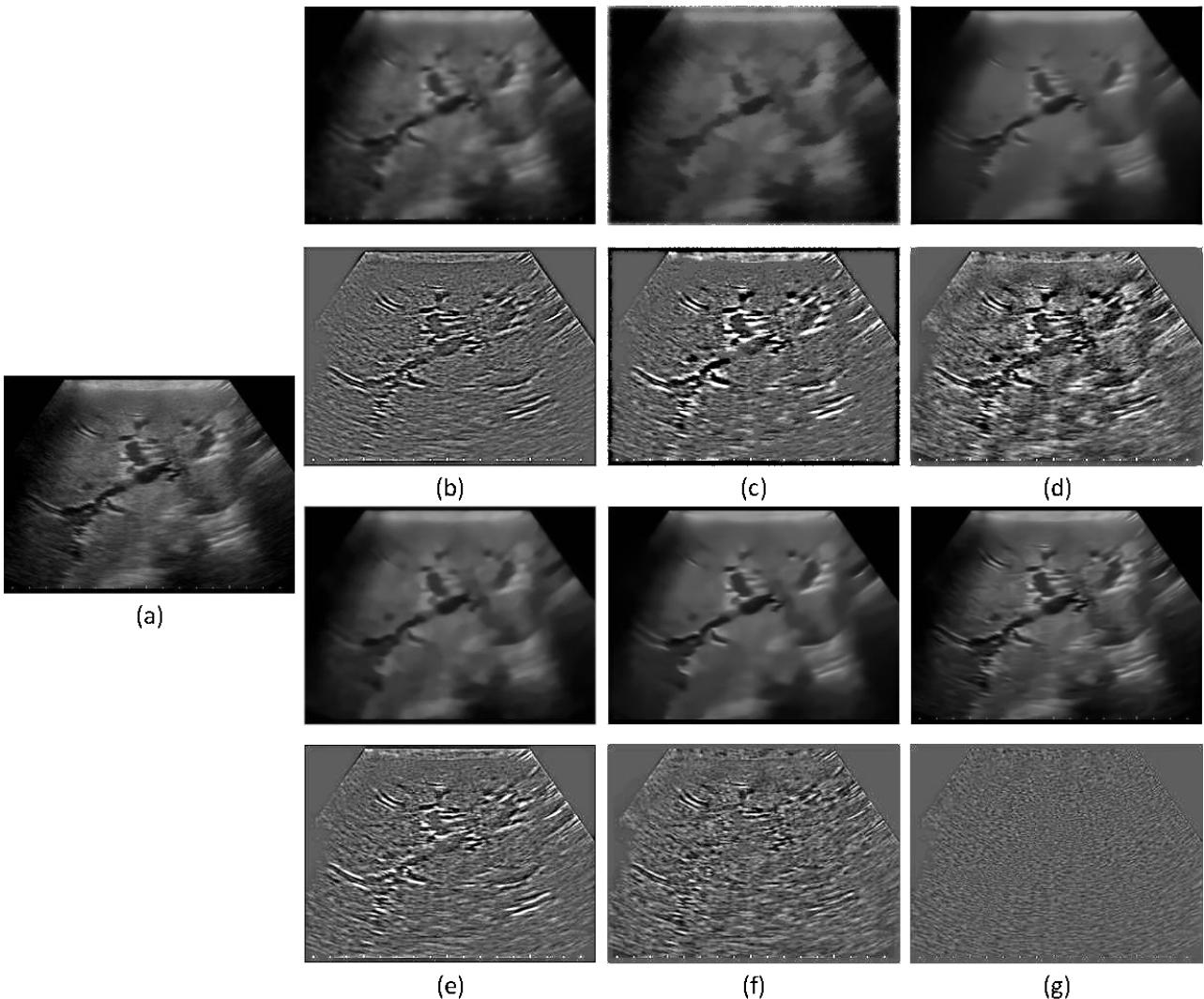


Figure 6: Additional comparison result #6. (a) input clinical ultrasound image with dilated bile ducts. Top row of (b)-(g): despeckled results produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NLMLS [10], and our method, respectively. Bottom row of (b)-(g) shows the noise components of these despeckled results.

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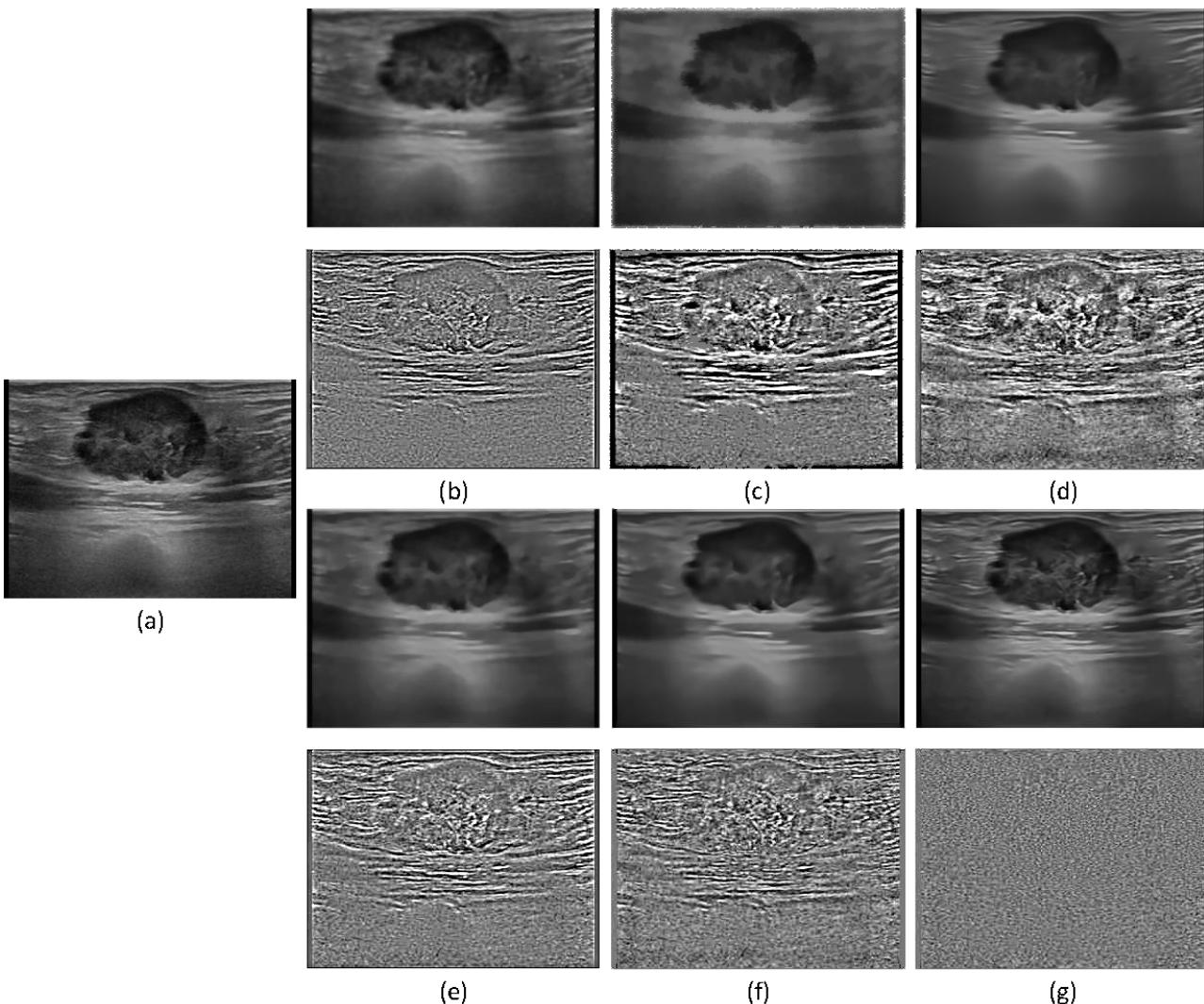
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Figure 7: Additional comparison result #7. (a) input clinical ultrasound image with an enlarged hypoechoic inguinal lymph node. Top row of (b)-(g): despeckled results produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NLMLS [10], and our method, respectively. Bottom row of (b)-(g) shows the noise components of these despeckled results.



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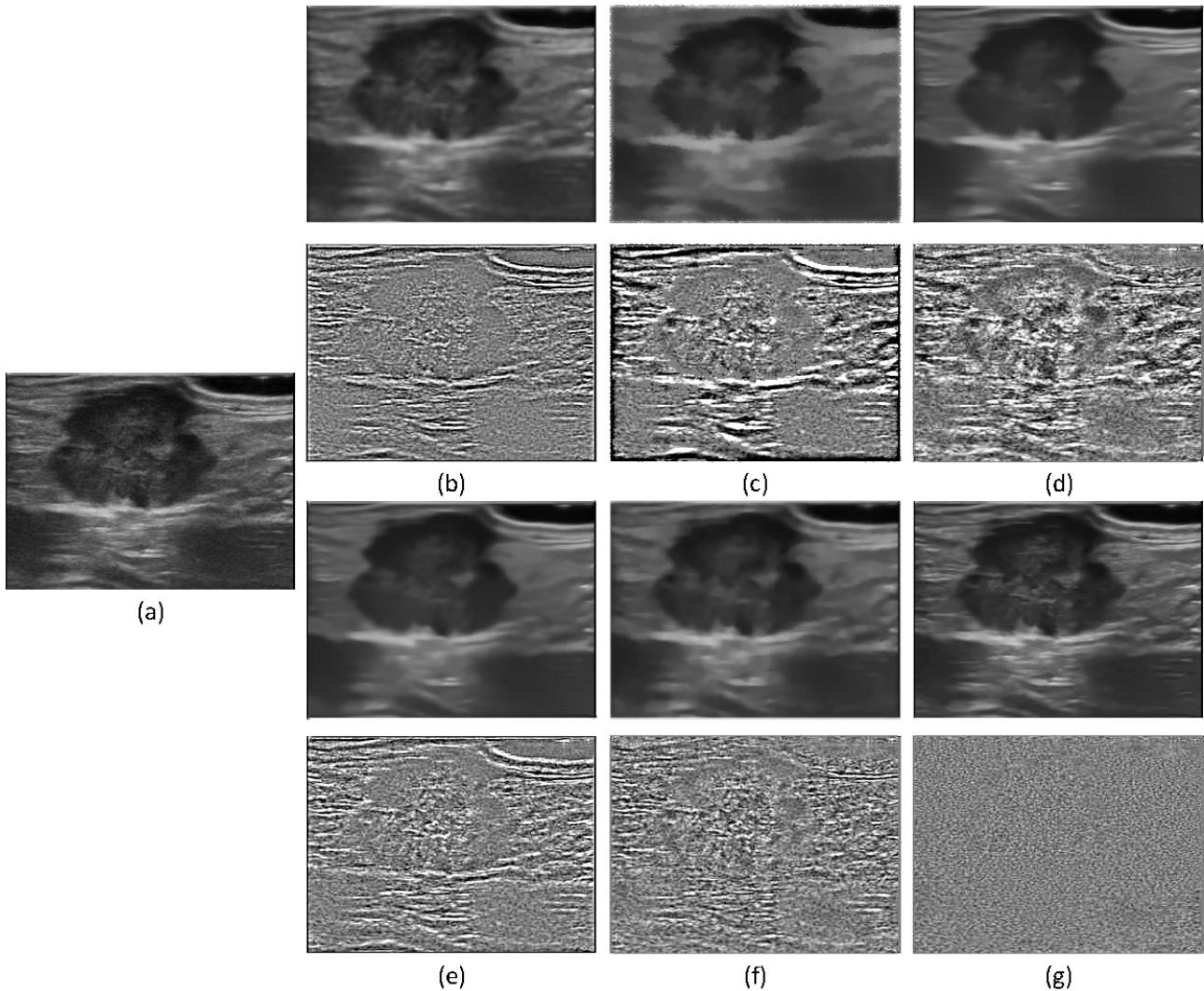


Figure 8: Additional comparison result #8. (a) input clinical ultrasound image with an enlarged hypoechoic inguinal lymph node. Top row of (b)-(g): despeckled results produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NLMLS [10], and our method, respectively. Bottom row of (b)-(g) shows the noise components of these despeckled results.

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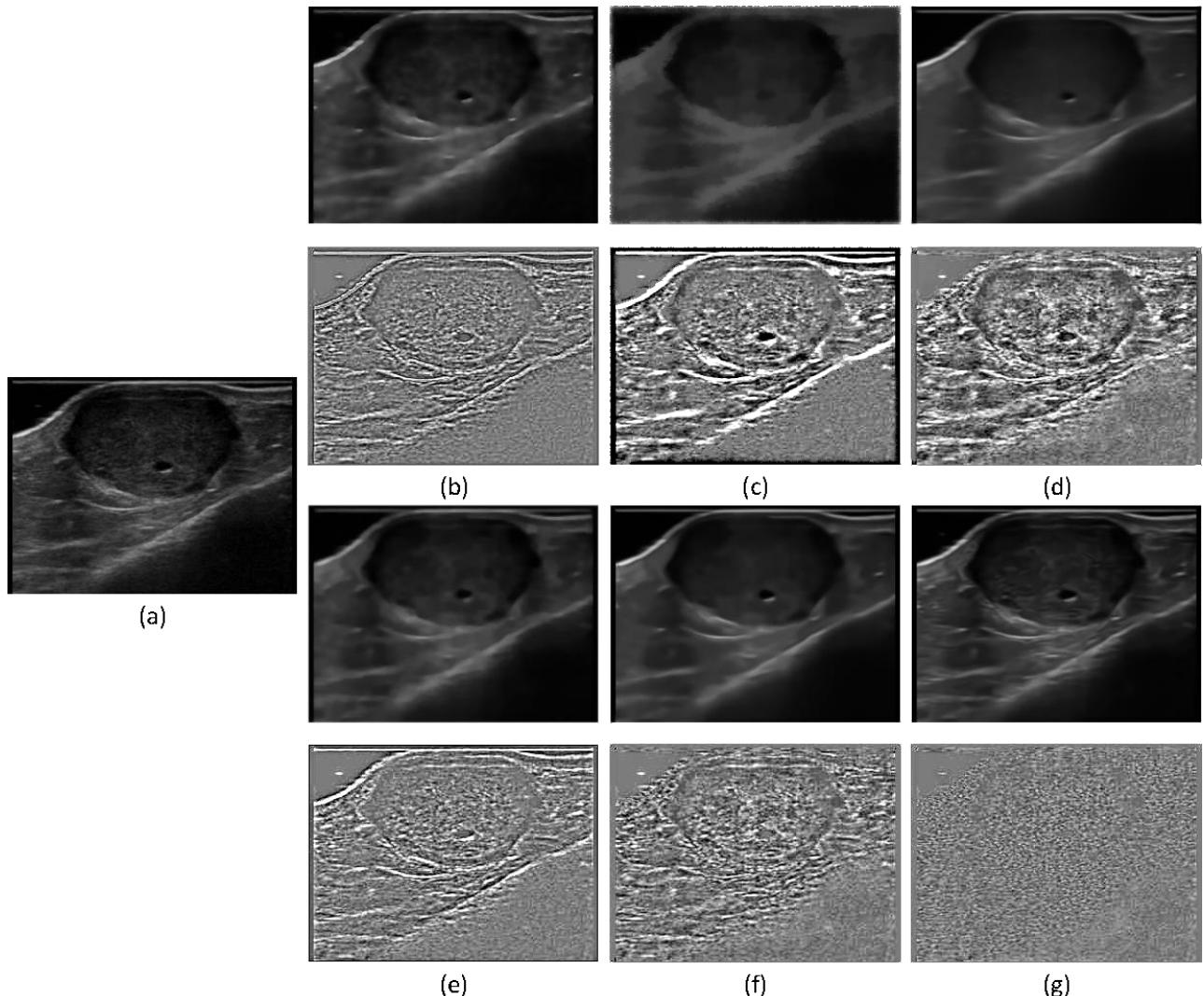


Figure 9: Additional comparison result #9. (a) input clinical ultrasound image with a highly vascularized compressible mass in the lower leg. Top row of (b)-(g): despeckled results produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NMLMS [10], and our method, respectively. Bottom row of (b)-(g) shows the noise components of these despeckled results.

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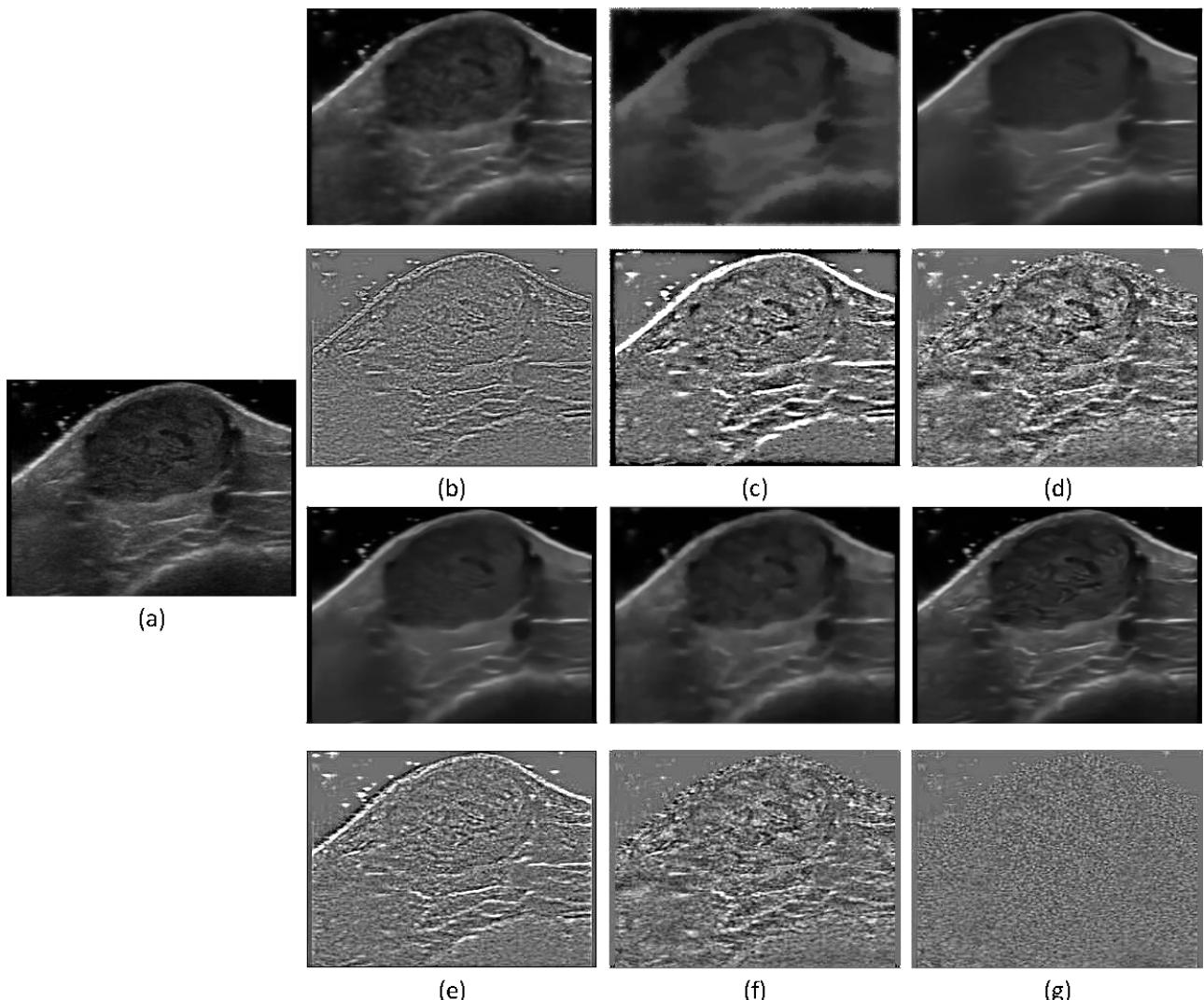
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Figure 10: Additional comparison result #10. (a) input clinical ultrasound image with a highly vascularized compressible mass in the lower leg. Top row of (b)-(g): despeckled results produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NMLMS [10], and our method, respectively. Bottom row of (b)-(g) shows the noise components of these despeckled results.



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Part 3. Additional Ultrasound Speckle Reduction Results

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Input images

Output results

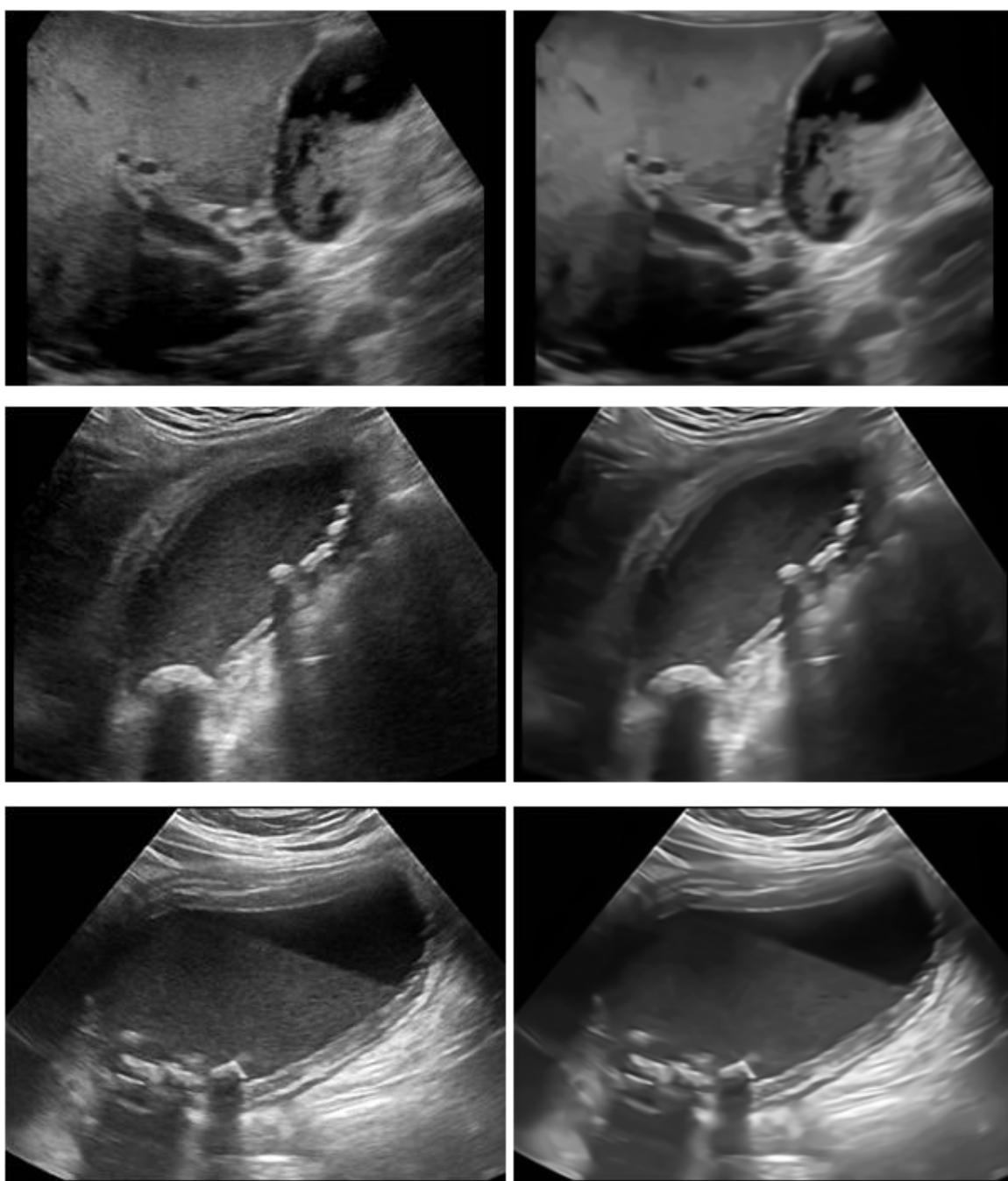


Figure 11: Additional ultrasound speckle reduction results produced from our method.

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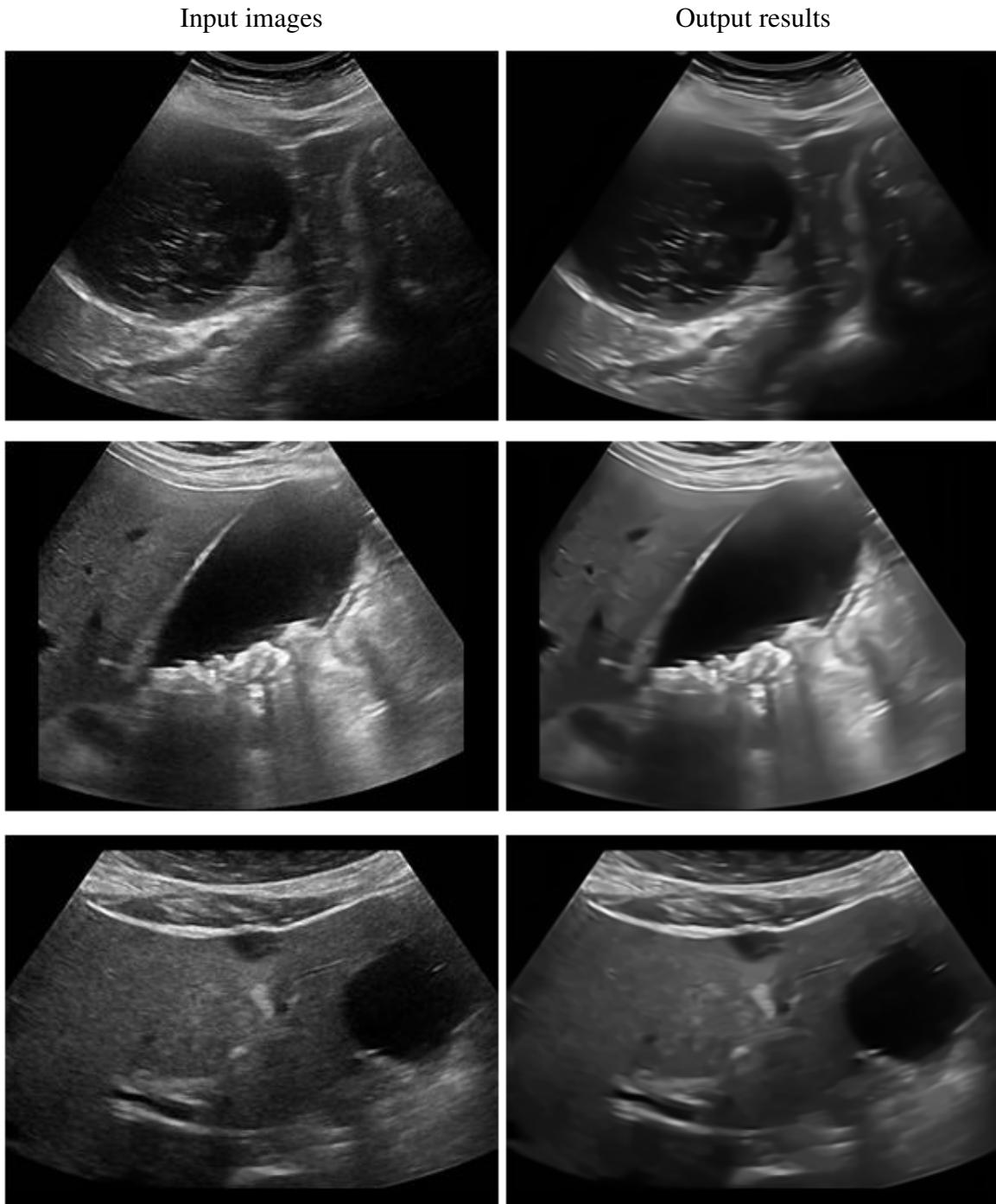
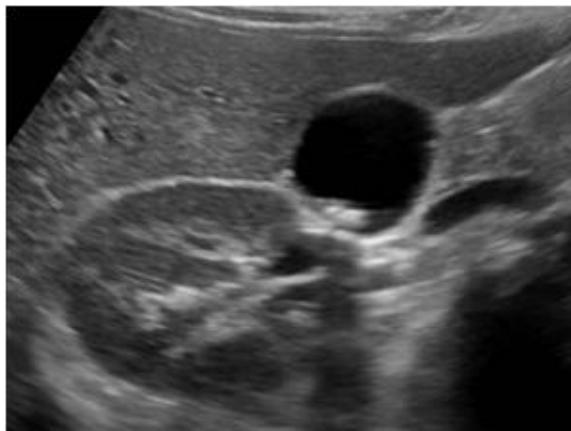


Figure 12: Additional ultrasound speckle reduction results produced from our method.

Input images



Output results

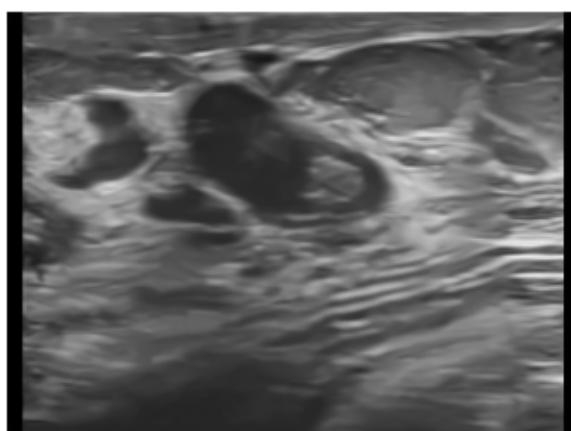
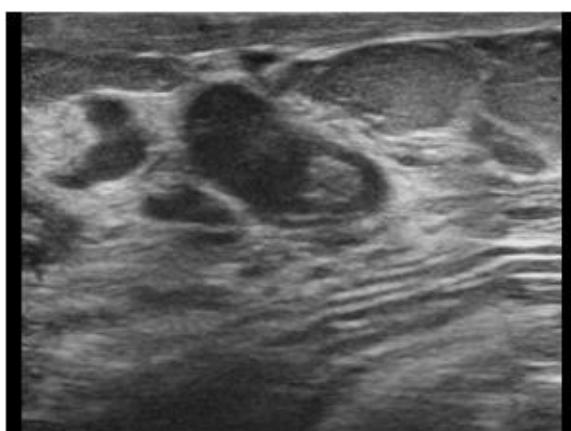
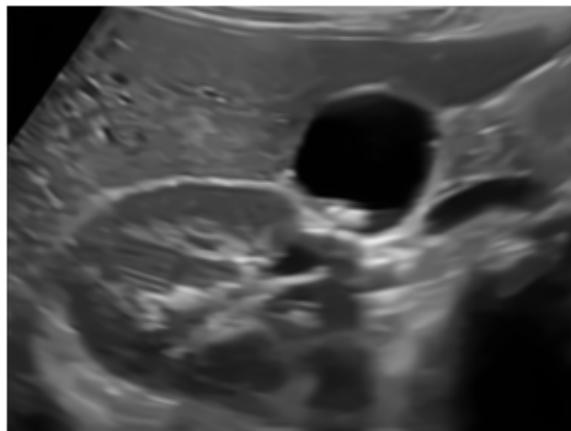
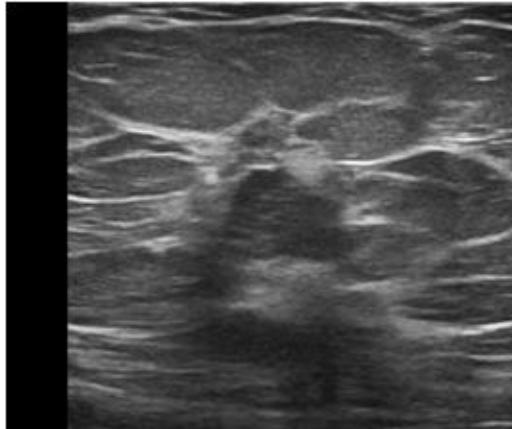


Figure 13: Additional ultrasound speckle reduction results produced from our method.

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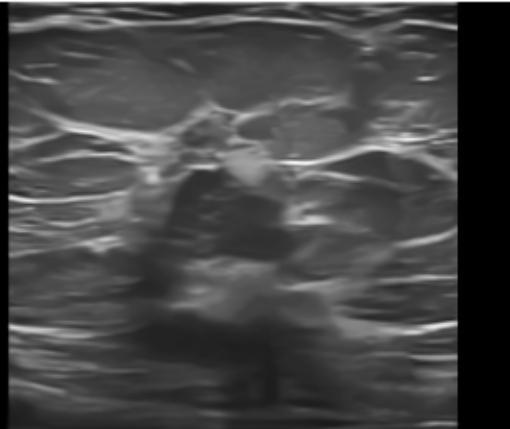
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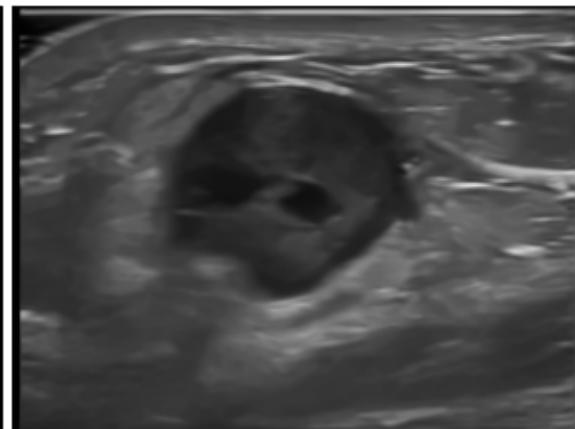
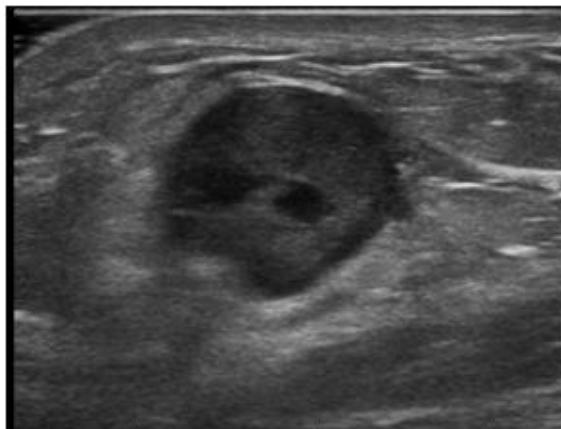


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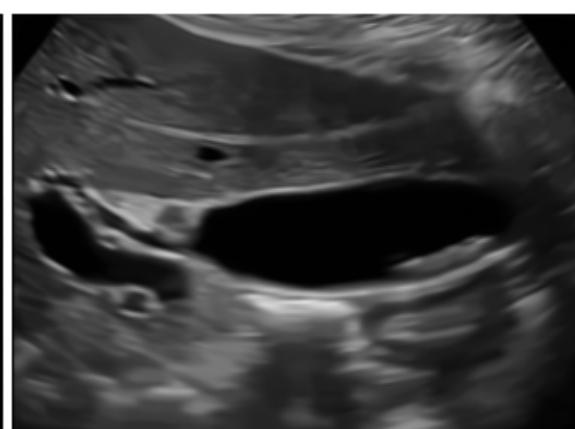
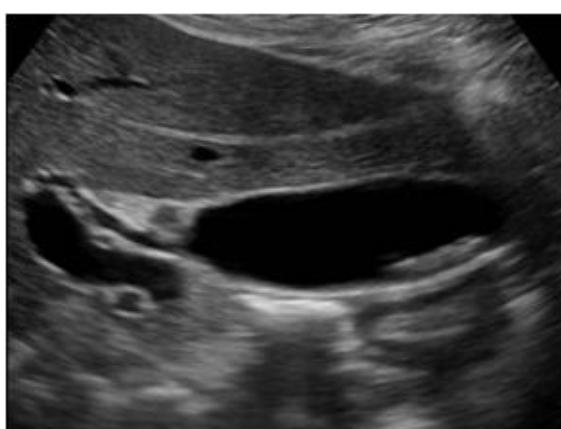
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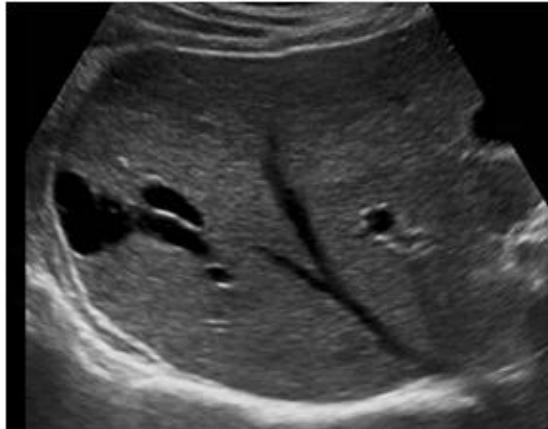
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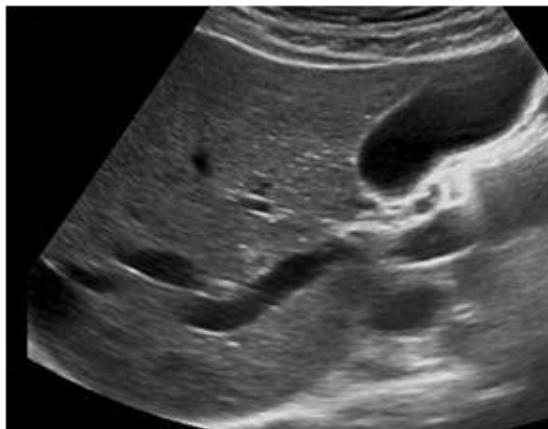
Figure 14: Additional ultrasound speckle reduction results produced from our method.

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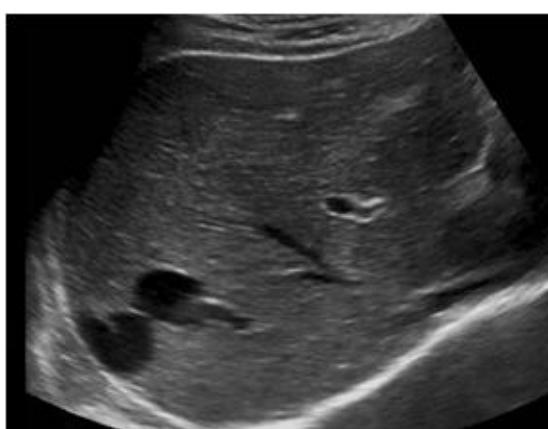
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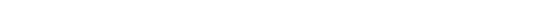
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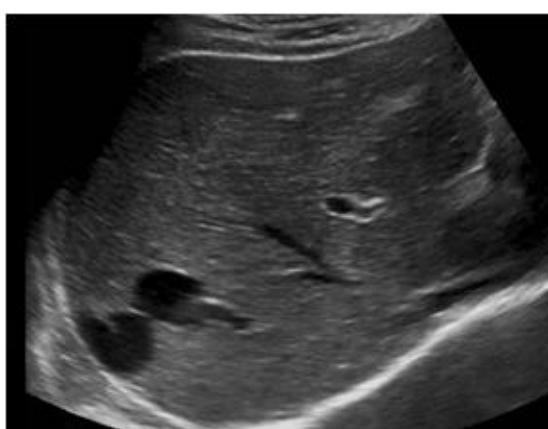
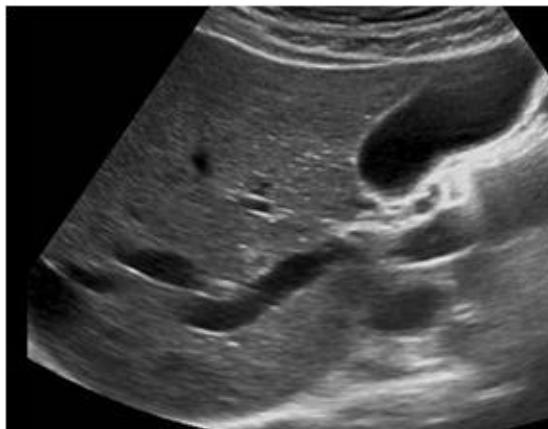
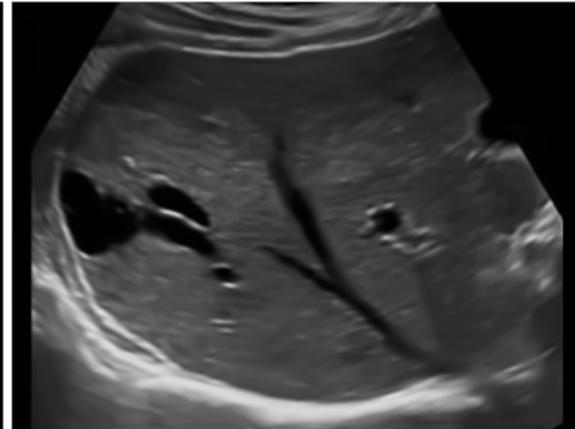


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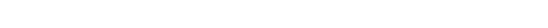


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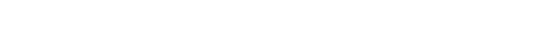
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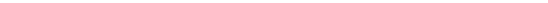
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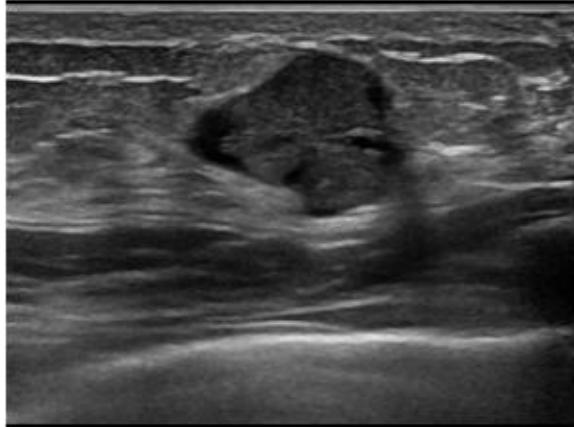
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Figure 15: Additional ultrasound speckle reduction results produced from our method.

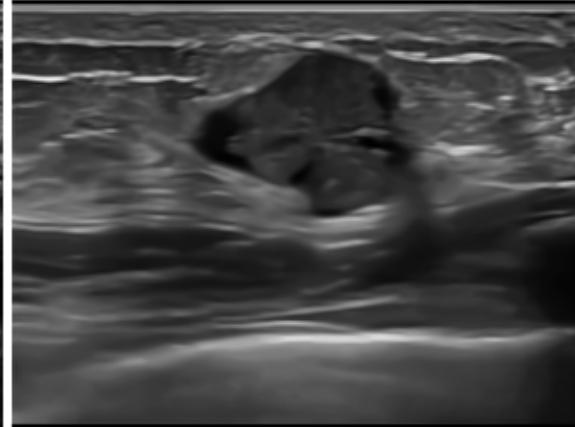
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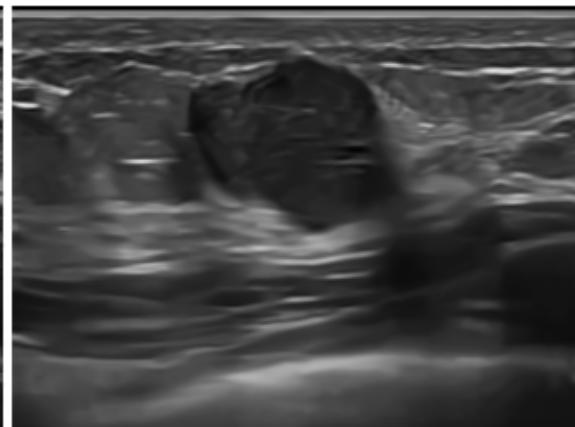
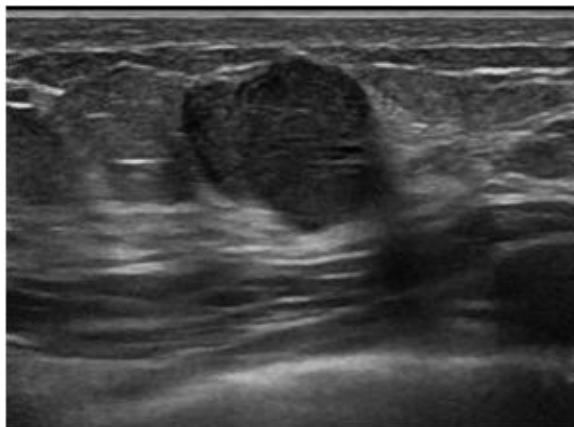


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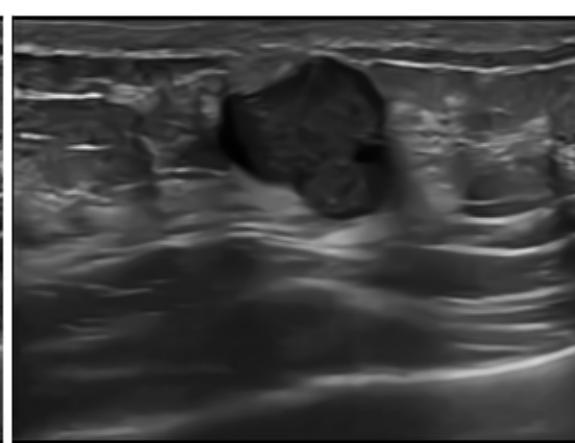
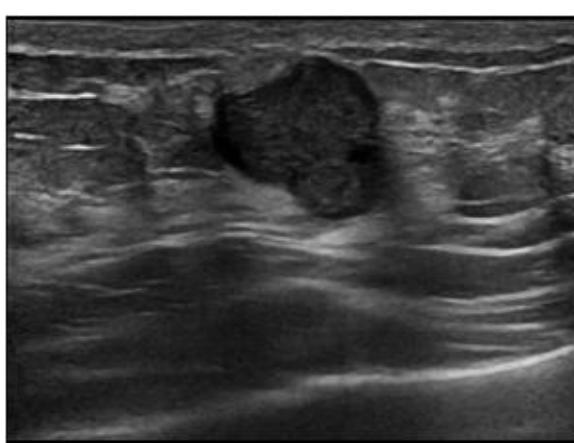
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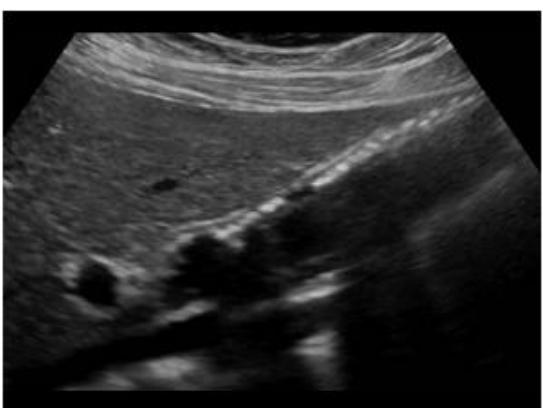
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Figure 16: Additional ultrasound speckle reduction results produced from our method.

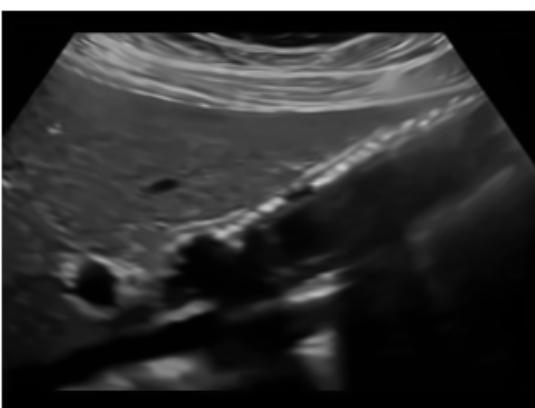
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Input images

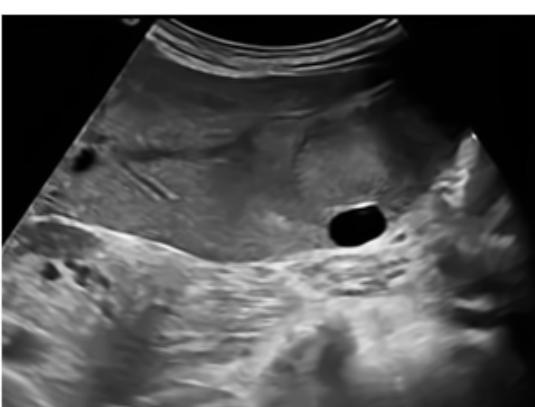


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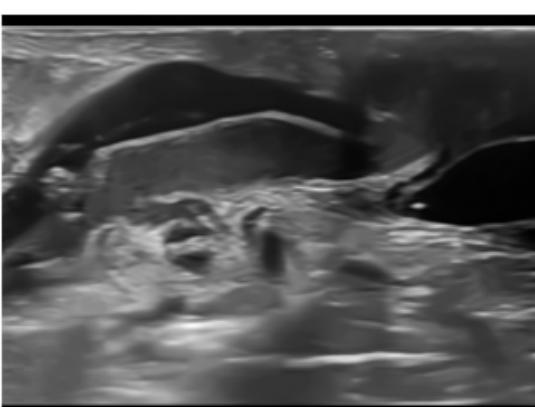
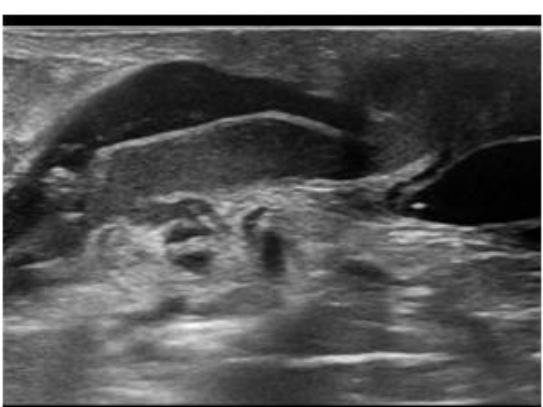
Output results



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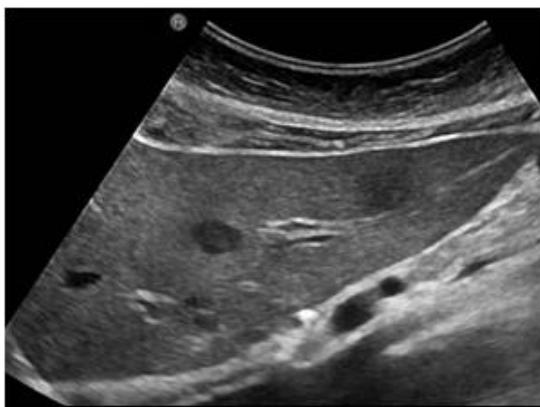
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Figure 17: Additional ultrasound speckle reduction results produced from our method.

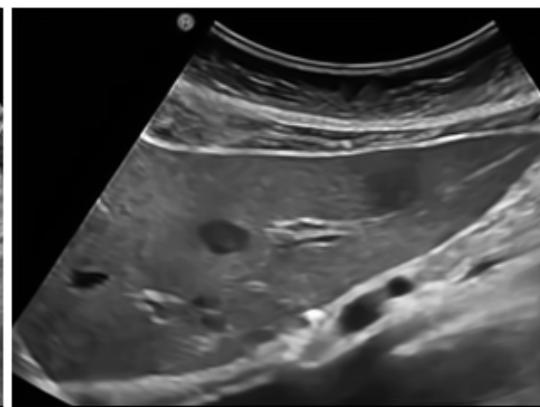
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Output results



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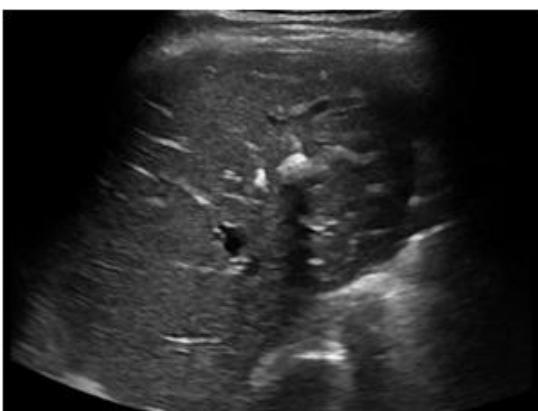
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Figure 18: Additional ultrasound speckle reduction results produced from our method.

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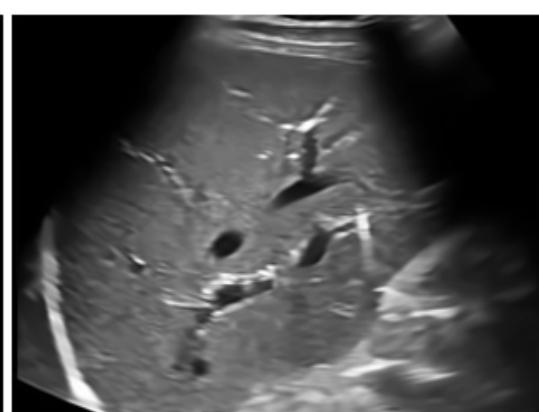


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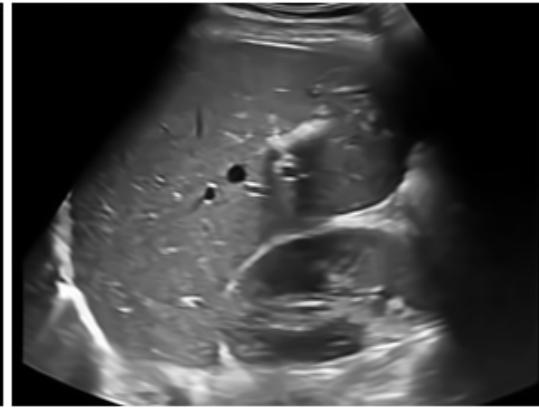
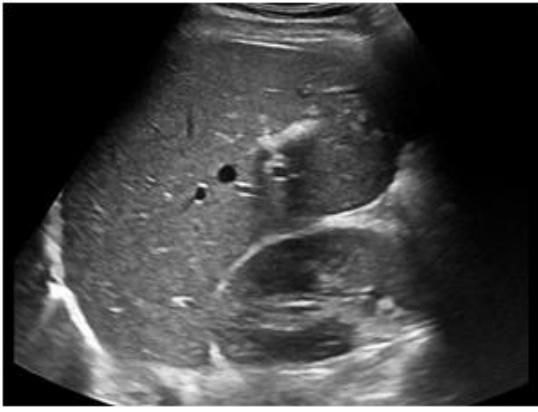
Output results



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Figure 19: Additional ultrasound speckle reduction results produced from our method.

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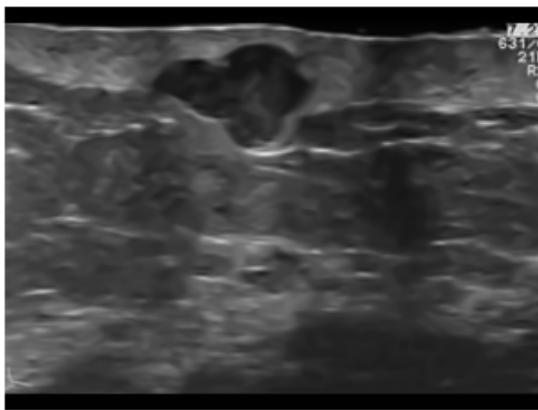
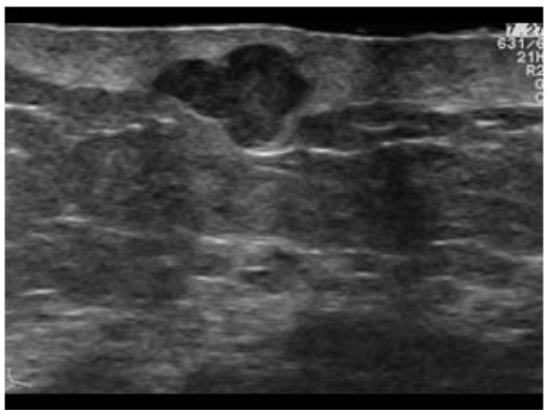


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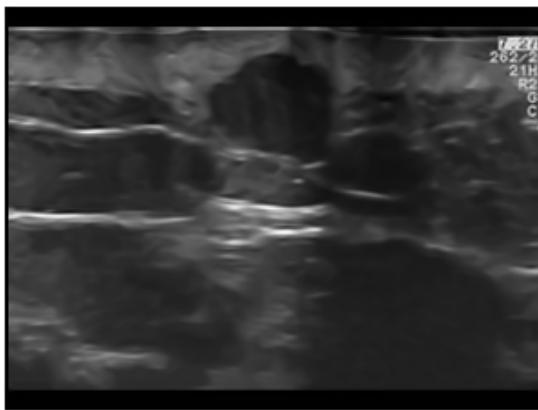
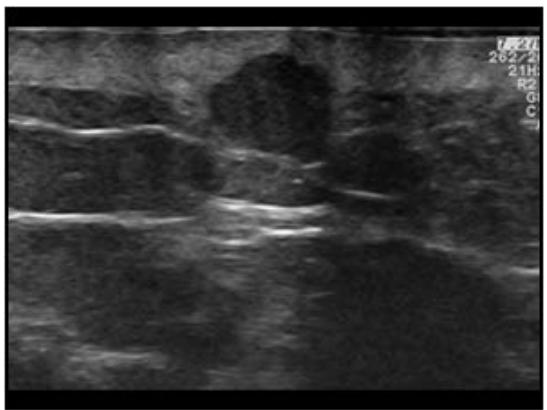
Output results



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Figure 20: Additional ultrasound speckle reduction results produced from our method.

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Part 4. Additional Segmentation results

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Figure 21: Additional segmentation comparison #1. (a) top row: segmentation result on the raw input breast ultrasound image with a benign fibroadenoma; bottom row: zoom-in view. Top row of (b)-(g): segmentation results on despeckled ultrasound images produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NLMLS [10], and our method, respectively; bottom row of (b)-(g): associated zoom-in views. Blue color: the ground truth delineated by clinical doctors; and Red color: segmentation results produced on different inputs.

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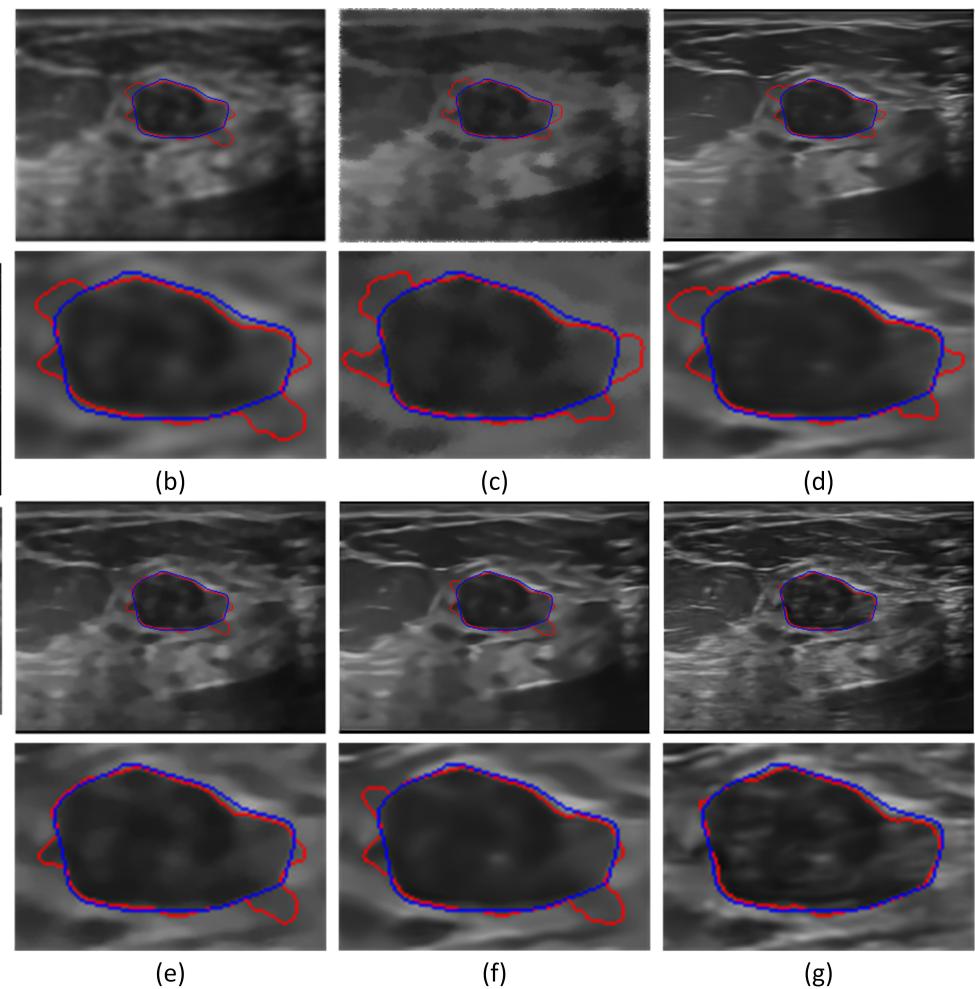
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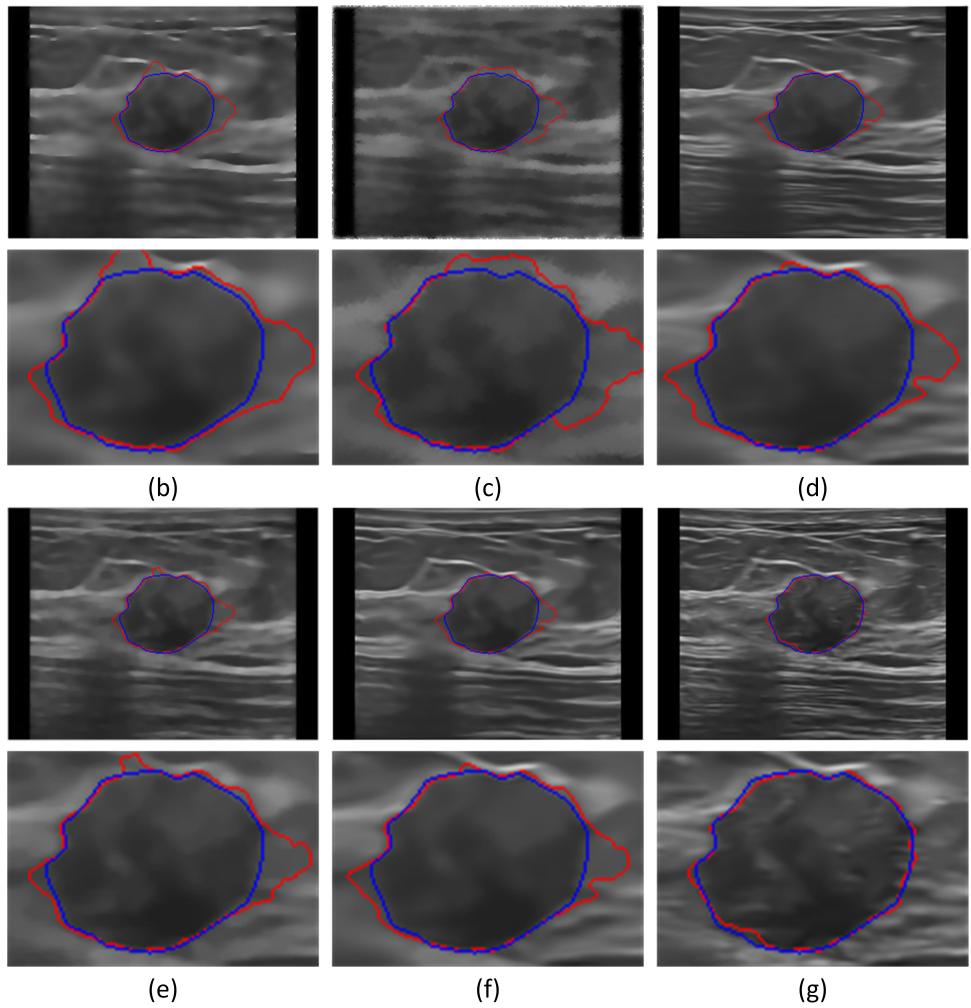
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Figure 22: Additional segmentation comparison #2. (a) top row: segmentation result on the raw input breast ultrasound image with a carcinoma tumor; bottom row: zoom-in view. Top row of (b)-(g): segmentation results on despeckled ultrasound images produced from SRAD [11], SBF [8], OBNLM [1], ADLG [2], NLMLS [10], and our method, respectively; bottom row of (b)-(g): associated zoom-in views. Blue color: the ground truth delineated by clinical doctors; and Red color: segmentation results produced on different inputs.

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