Multi-Scale Weighted Nuclear Norm Image Restoration: Supplementary Material

1. We provide a detailed derivation of the z-update step (Equations (9)-(11)).
2. We report the deblurring results for the individual images over the NCSR dataset (corresponding to Table 2).
3. We report the inpainting results for the individual images over the NCSR dataset (corresponding to Table 5).
4. We report the deblurring results obtained with RED-WNNM and with our method using a single scale and using multiple scales, for the individual images over the Set 5 dataset (corresponding to Table 6).
5. We show additional visual results from the deblurring experiments on the BSD datasets.

1. Derivation of the z update step

Retaining only the terms that depend on z in (6), leads to the optimization problem

$$\min_z \frac{\mu_1}{2}\|z-x\|^2 + \frac{\mu_2}{2}\sum_{i\in\Omega}\|Z_i - \mathcal{R}_i\{z\}\|_{\mathrm{F}}^2. \tag{S1}$$

Note the jth column of $\mathcal{R}_i\{z\}$ is given by $R_{N_i^j}z$. Now, since the square Frobenius norm of a matrix equals the sum of square Euclidean norms of its columns, (S1) can be equivalently written as

$$\min_z \frac{\mu_1}{2}\|z-x\|^2 + \frac{\mu_2}{2}\sum_{i\in\Omega}\sum_{j=1}^{k}\|Z_i^j - R_{N_i^j}z\|^2. \tag{S2}$$

Setting the gradient w.r.t. z to zero, we obtain the equation

$$\mu_1(z-x) + \mu_2\sum_{i\in\Omega}\sum_{j=1}^{k}R_{N_i^j}^T(Z_i^j - R_{N_i^j}z) = 0 \tag{S3}$$

Gathering the terms that depend on z, this equation reads

$$\left(\mu_1 I + \mu_2\sum_{i\in\Omega}\sum_{j=1}^{k}R_{N_i^j}^T R_{N_i^j}\right)z = \mu_1 x + \mu_2\sum_{i\in\Omega}\sum_{j=1}^{k}R_{N_i^j}^T Z_i^j, \tag{S4}$$

where I is the identity matrix. Defining

$$W = \sum_{i\in\Omega}\sum_{j=1}^{k}R_{N_i^j}^T R_{N_i^j}, \quad \tilde{z} = \sum_{i\in\Omega}\sum_{j=1}^{k}R_{N_i^j}^T Z_i^j \tag{S5}$$

We get

$$\left(\mu_1 I + \mu_2 W\right)z = \mu_1 x + \mu_2\tilde{z}. \tag{S6}$$

Therefore,

$$z = \left(\mu_1 I + \mu_2 W\right)^{-1}\left(\mu_1 x + \mu_2\tilde{z}\right), \tag{S7}$$

demonstrating Equation (9).

2. Additional experimental results

2.1. Deblurring

Table S1 is a detailed version of Table 2 in the paper. It reports the deblurring results obtained for each of the individual images in the NCSR dataset.

	Image	Input	EPLL	IDD-BM3D	NCSR	RED+TNRD	IRCNN	Our
Gaussian blur, $\sigma_n = \sqrt{2}$	Barbara	23.75	25.41	27.08	28.01	26.76	27.00	27.42
	Boats	25.90	31.26	31.64	31.37	31.55	31.91	**32.03**
	C.man	23.39	27.39	28.10	28.28	28.37	28.52	**28.73**
	House	27.90	33.63	34.03	33.59	33.74	34.24	34.72
	Leaves	20.80	28.59	29.83	29.52	29.89	30.37	**30.54**
	Lena	26.17	31.13	31.41	31.14	31.39	31.61	31.54
	Monarch	23.42	30.34	30.90	30.37	31.04	31.29	**31.47**
	Parrot	25.60	30.92	31.44	31.53	31.57	31.30	**31.88**
	Peppers	25.12	30.12	29.96	29.95	30.51	**31.04**	30.92
	Starfish	24.49	29.52	30.16	30.02	30.56	30.41	**31.33**
	Average	24.655	29.831	30.455	30.379	30.539	30.769	**31.059**
Gaussian blur, $\sigma_n = 2$	Barbara	23.72	24.71	25.99	26.55	25.52	26.15	25.54
	Boats	25.85	30.73	31.17	31.23	31.04	31.41	**31.47**
	C.man	23.36	26.98	27.68	27.96	27.95	28.06	**28.22**
	House	27.82	33.14	33.56	33.36	33.21	33.79	**34.08**
	Leaves	20.78	27.84	29.18	29.26	29.17	29.64	**29.89**
	Lena	26.11	30.61	30.91	31.00	30.92	31.14	**31.20**
	Monarch	23.39	29.71	30.32	30.30	30.48	30.79	**30.95**
	Parrot	25.55	30.38	30.85	31.27	30.95	30.80	**31.28**
	Peppers	25.08	29.68	29.64	30.00	30.03	**30.68**	30.52
	Starfish	24.45	28.92	29.55	29.84	29.94	29.78	**30.61**
	Average	24.611	29.268	29.883	30.077	29.921	30.223	**30.375**
Uniform blur, $\sigma_n = \sqrt{2}$	Barbara	22.42	26.70	27.99	28.07	27.27	27.98	**28.62**
	Boats	22.27	30.01	31.21	31.03	31.12	31.26	**31.37**
	C.man	20.76	27.08	28.54	28.62	28.75	**29.08**	28.96
	House	24.08	33.13	34.45	34.27	33.76	34.13	**34.82**
	Leaves	16.95	25.60	27.90	28.19	28.13	**28.94**	28.83
	Lena	22.89	28.95	29.71	29.93	30.02	**30.33**	30.06
	Monarch	19.65	27.56	29.04	29.09	29.58	29.90	**29.96**
	Parrot	22.53	28.48	29.81	30.32	30.17	**30.51**	30.48
	Peppers	21.97	28.85	29.62	29.60	30.12	**30.13**	29.99
	Starfish	21.23	27.08	28.05	28.26	28.73	28.68	**29.07**
	Average	21.476	28.345	29.633	29.737	29.766	30.096	**30.216**
Uniform blur, $\sigma_n = 2$	Barbara	22.40	25.90	27.08	27.29	26.47	27.15	27.61
	Boats	22.25	29.02	30.22	30.45	30.31	30.36	30.33
	C.man	20.74	26.21	27.62	27.79	27.81	**28.26**	27.92
	House	24.04	32.25	33.63	33.70	33.12	33.32	**34.20**
	Leaves	16.95	24.41	26.82	27.39	26.77	27.79	**27.86**
	Lena	22.87	28.18	28.93	29.25	29.14	**29.50**	29.34
	Monarch	19.64	26.53	28.18	28.47	28.56	29.03	**29.09**
	Parrot	22.50	27.50	28.88	29.44	29.19	**29.61**	29.55
	Peppers	21.95	28.13	28.90	29.06	29.35	**29.44**	29.33
	Starfish	21.21	26.19	27.16	27.64	27.77	27.73	**28.20**
	Average	21.455	27.433	28.741	29.048	28.849	29.219	**29.343**

Table S1. **Deblurring comparison on Set NCSR.** Our method is compared to the state-of-the-art deblurring methods on Set NCSR with four different degradations. The best results are shown in bold.

2.2. Inpainting

Table S2 is a detailed version of Table 5 in the paper. It reports the inpainting results obtained for each of the individual images in the NCSR dataset.

	Image	EPLL	FoE	GSR	LINC	Our
25% blank pixels	Barbara	34.35	29.59	**39.25**	38.23	39.16
	Boats	35.47	34.68	38.05	37.37	**38.40**
	C.man	30.13	28.00	31.05	**31.23**	31.16
	House	37.10	37.03	40.40	39.83	**40.52**
	Leaves	31.47	28.72	34.49	33.59	**35.25**
	Lena	34.03	32.44	35.80	35.50	**35.97**
	Monarch	31.69	29.25	33.03	32.73	**33.62**
	Parrot	34.11	31.15	36.42	36.01	**36.69**
	Peppers	32.04	30.89	32.29	32.69	**32.83**
	Starfish	31.53	31.03	33.59	32.17	**34.13**
	Average	33.193	31.277	35.436	34.935	**35.772**
50% blank pixels	Barbara	40.60	32.56	**44.45**	43.30	44.37
	Boats	41.24	39.71	44.19	43.05	**44.50**
	C.man	34.81	30.19	34.55	35.43	**35.78**
	House	41.70	41.36	45.17	44.50	**45.69**
	Leaves	38.04	31.87	41.57	39.51	**42.47**
	Lena	39.77	35.88	41.54	40.72	**41.68**
	Monarch	37.03	32.90	36.88	37.69	**39.32**
	Parrot	39.47	34.42	41.92	40.83	**42.34**
	Peppers	36.10	33.08	35.79	36.62	**36.73**
	Starfish	37.18	35.54	39.44	37.38	**40.03**
	Average	38.594	34.750	40.552	39.905	**41.291**
75% blank pixels	Barbara	27.54	24.74	**33.83**	-	32.78
	Boats	29.70	29.09	31.64	-	**31.77**
	C.man	25.53	23.38	**26.21**	-	26.15
	House	32.46	31.49	**35.36**	-	35.15
	Leaves	24.65	22.23	27.71	-	**28.11**
	Lena	29.06	28.04	30.55	-	**30.78**
	Monarch	26.17	23.08	28.21	-	**28.46**
	Parrot	28.57	26.74	**30.20**	-	30.18
	Peppers	27.96	26.67	28.57	-	**29.01**
	Starfish	26.79	26.36	27.80	-	**28.15**
	Average	27.843	26.183	30.007	-	**30.055**

Table S2. **Inpainting comparison on Set NCSR.** Our method is compared to the state-of-the-art inpaining methods on Set NCSR with three different missing pixel ratios. The best results are shown in bold.

2.3. The effect of multiple scales

Table S3 is a detailed version of Table 6 in the paper. It reports the deblurring results for the individual images over Set 5 obtained with and without using multiple scales in our algorith, and with the RED-WNNM algorithm.

2.4. Additional visual results

Figures S1-S3 below show several additional deblurring results for images from the BSD dataset. Figure S1 shows the full image from the example of Fig. 2 in the paper. Figures S2 and S3 depict results for two additional images.

	image	Input	RED WNNM	Our w/o MS	Our w/ MS
Gaussian blur, $\sigma_n = \sqrt{2}$	Baby	30.10	35.00	**35.22**	35.21
	Bird	28.81	**36.81**	36.54	36.56
	Butterfly	21.48	29.22	28.81	**30.20**
	Head	28.93	32.97	**33.01**	33.01
	Woman	25.88	32.35	32.22	**32.70**
	Average	27.04	33.27	33.17	**33.54**
Gaussian blur, $\sigma_n = 2$	Baby	29.96	34.55	34.73	**34.74**
	Bird	28.71	**36.15**	35.822	35.70
	Butterfly	21.46	28.62	28.23	**29.73**
	Head	28.83	32.68	**32.73**	32.70
	Woman	25.83	31.86	31.72	**32.23**
	Average	26.96	32.78	32.65	**33.02**
Uniform blur, $\sigma_n = \sqrt{2}$	Baby	26.31	33.20	**33.22**	33.14
	Bird	24.64	33.90	33.94	**34.14**
	Butterfly	17.74	27.09	28.08	**28.83**
	Head	26.16	31.79	31.79	**31.81**
	Woman	22.14	30.27	30.82	**31.23**
	Average	23.40	31.25	31.57	**31.83**
Uniform blur, $\sigma_n = 2$	Baby	26.25	32.57	**32.61**	32.53
	Bird	24.60	32.91	33.00	**33.24**
	Butterfly	17.74	25.84	27.02	**27.98**
	Head	26.10	31.35	31.36	**31.41**
	Woman	22.11	29.22	29.95	**30.47**
	Average	23.36	30.38	30.79	**31.13**

Table S3. **The effect of multiple scales.** We compare deblurring performance on Set5 for RED with WNNM as its denoising engine, our method without multiple scales, and our method with multiple scales (1 and 0.75).

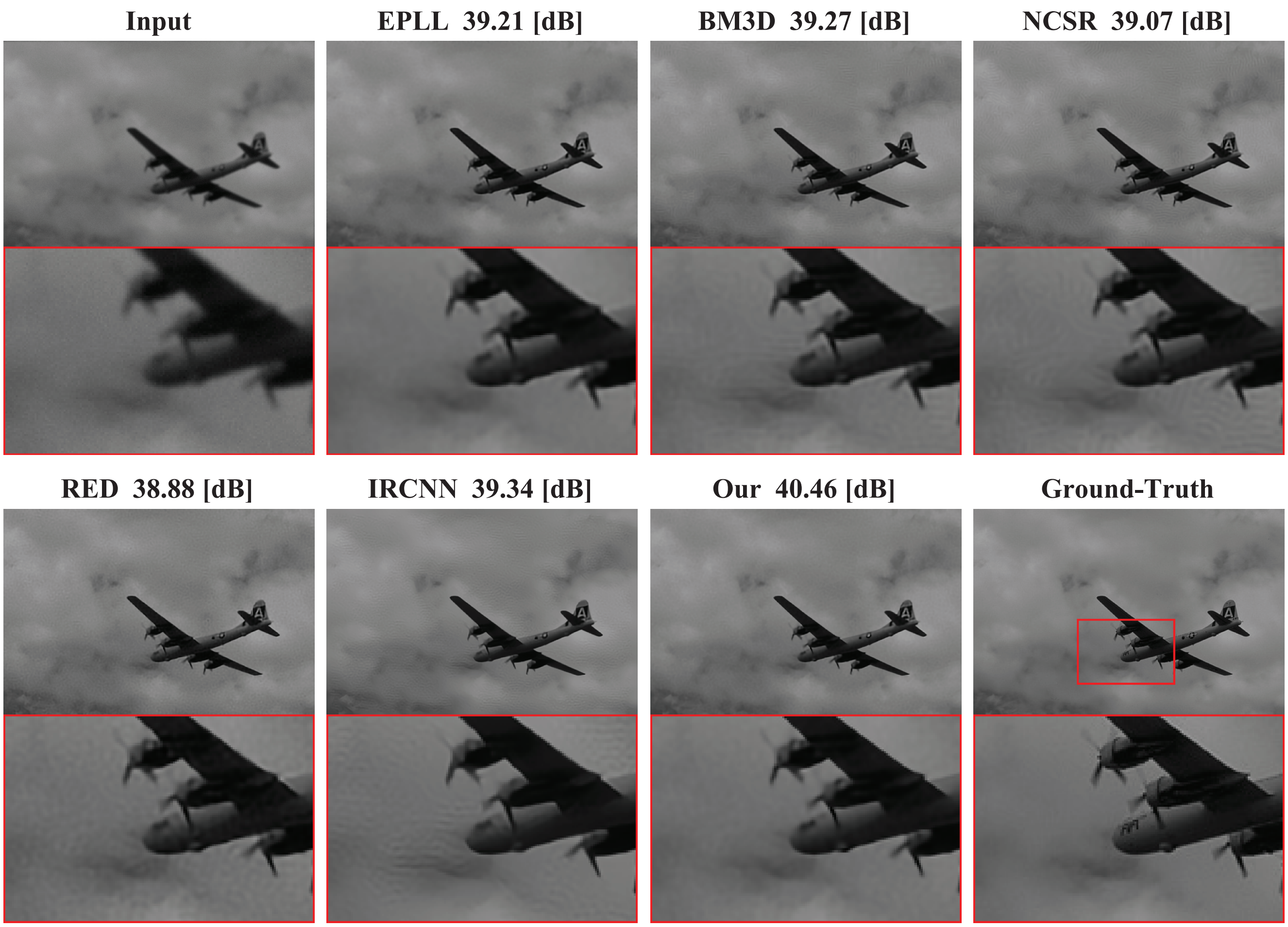

Figure S1. **Visual comparison of deblurring algorithms.** A degraded input image from the BSD dataset is shown on the top left. It suffers from Gaussian blur with standard deviation 1.6 and additive noise with $\sigma_n = 2$. As can be seen, while all state-of-the-art deblurring methods produce artifacts in the reconstruction, our algorithm produces sharp results without annoying distortions. Its precision is also confirmed by the very high PSNR it attaines w.r.t. the other methods.

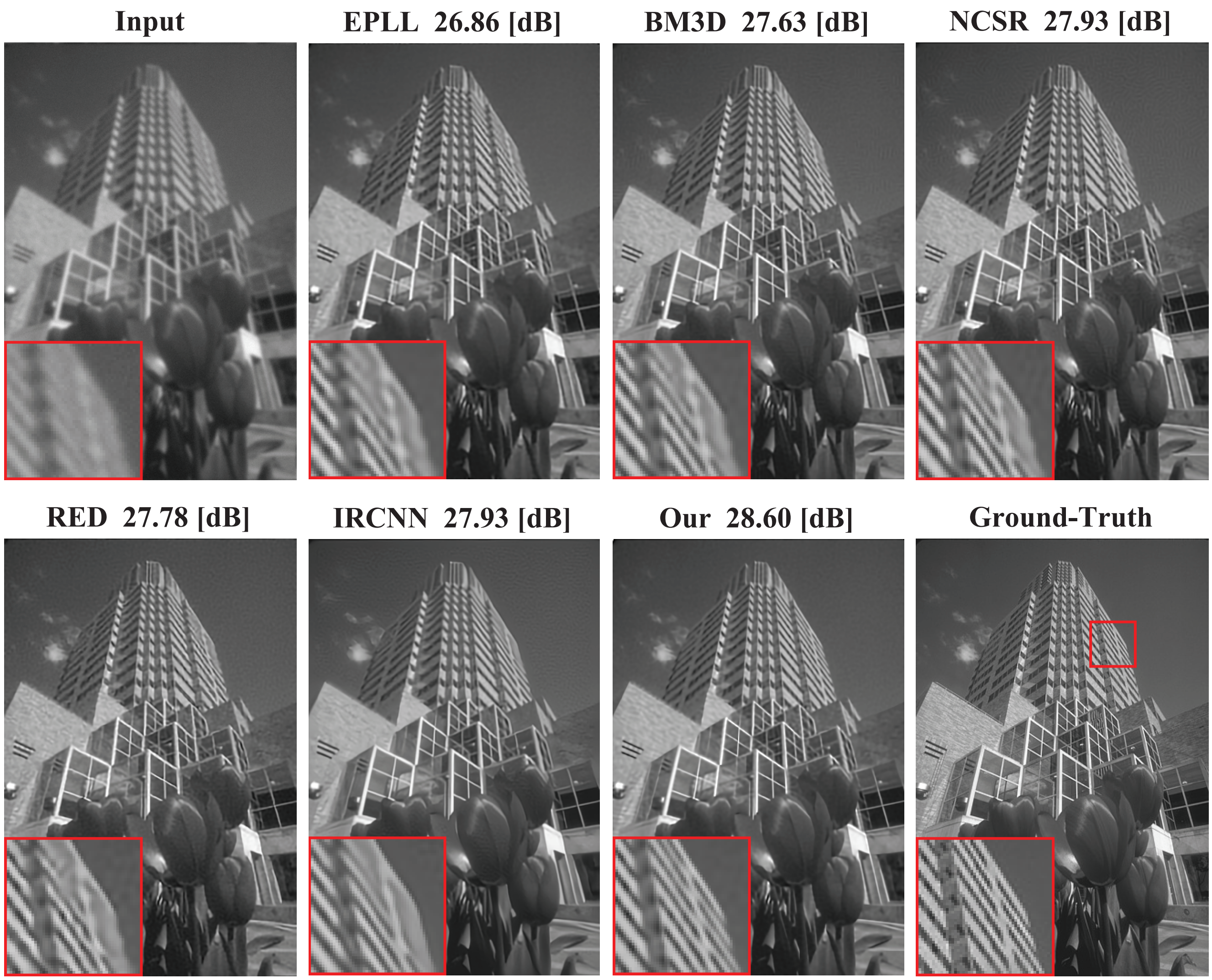

Figure S2. **Visual comparison of deblurring algorithms.** A degraded input image from the BSD dataset is shown on the top left. The blur is Gaussian with standard deviation 1.6 and the noise level is $\sigma_n = 2$. As can be seen, while all state-of-the-art deblurring methods fail to reproduce the lines on the building, our algorithm manages to restore them successfully. This is despite the very little information left after the degradation. This precision is also confirmed by the high PSNR value our algorithm attains.

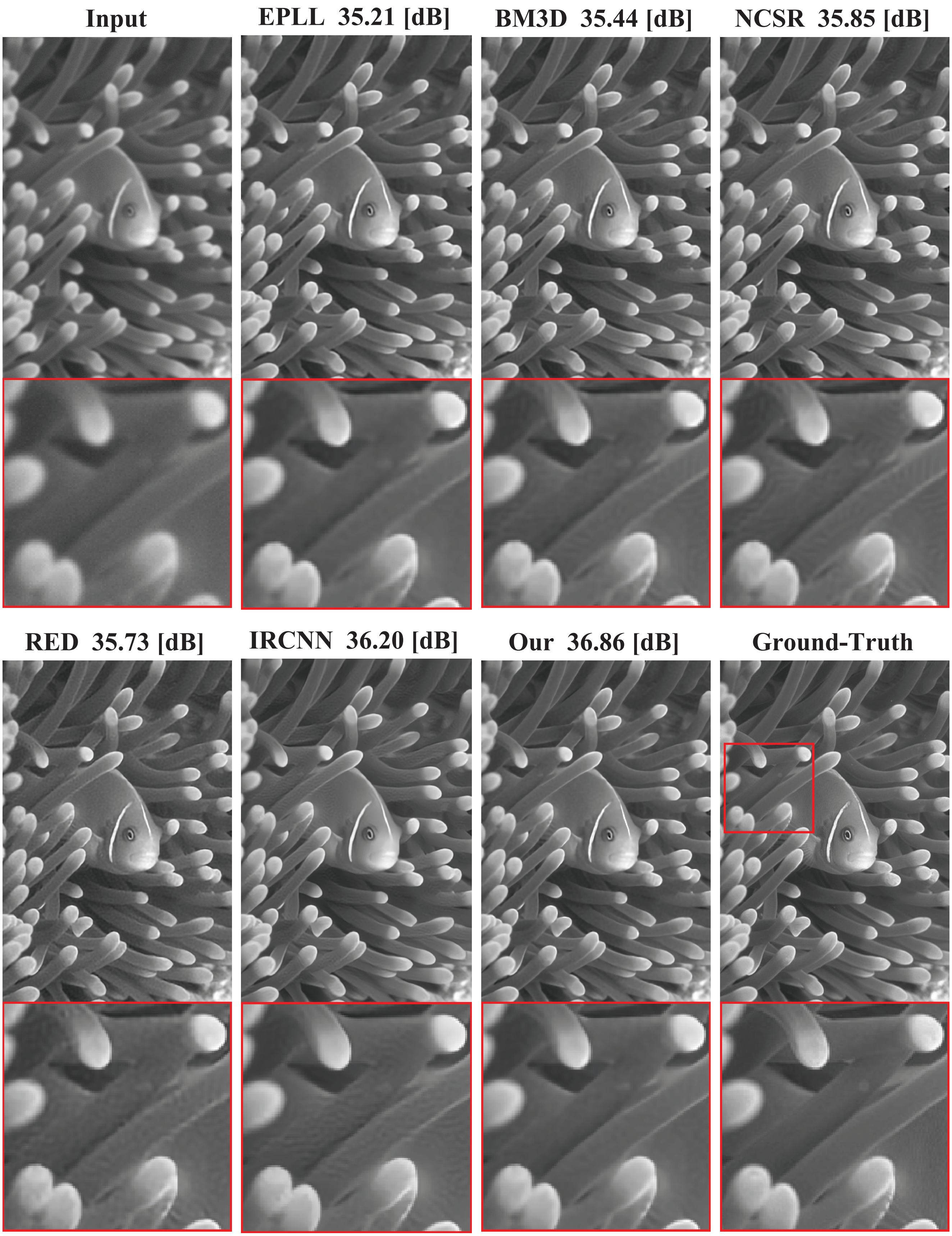

Figure S3. **Visual comparison of deblurring algorithms.** A degraded input image from the BSD dataset is shown on the top left. The blur is Gaussian with standard deviation 1.6 and the noise level is $\sigma_n = 2$. Here, again, all existing deblurring methods produce artifacts in the reconstruction, while our algorithm produces sharp and clean results without annoying distortions.