1 Further Training Details

This section provides further training details. For a fair comparison, we follow the settings in iCaRL [1]. On CIFAR-100, the learning rate starts from 2.0 and is divided by 5 after 49 and 63 epochs. Moreover, 70 epochs are run in each incremental training phase. Likewise, on ImageNet the learning rate starts from 2.0 and is divided by 5 after 20, 30, 40 and 50 epochs. Each training phase has 60 epochs. On both datasets, the optimizer is SGD and the batch size is 128.

2 Rectified Cosine Normalization

This section conducts a simulation to justify our rectified cosine normalization. It is reported empirically in the main manuscript that using rectified cosine normalization for training can better encourage the separation between classes.

Specifically, in evaluating the binary cross-entropy loss $\mathcal{L}_{bce}(x_k)$, our rectified cosine normalization computes the activation $a_{ijk}$ of an image $x_k$ for class $i$ to be $a_{ijk} = \bar{W}_i^T \bar{F}_k$, while the cosine normalization in [2] uses $a_{ijk} = \bar{w}_i^T \bar{f}_k$. The bar indicates $l_2$-normalization, $W_i = (w_i, b_i)$ is formed by the concatenation of the classification weight vector $w_i$ and an additional learnable bias $b_i$; likewise, $F_k = (f_k, 1)$ (referred to as the augmented feature representation) concatenates the feature $f_k$ (referred to as the original feature representation) of the image $x_k$ and a constant bias 1. Due to the $l_2$-normalization, the activation $a_{ijk}$ is in the range of $[-1, 1]$. We thus introduce a learnable $\eta$ to control the curvature of the sigmoid function $\sigma(\cdot)$:

$$\sigma(a_{ijk}) = \frac{1}{1 + \exp(-\eta a_{ijk})}.$$  \hfill (1)

Fig. 1 illustrates the benefits of our rectified cosine normalization, taking the 2-dimensional cases as examples. In these examples, each class has only one trainable data $f_k$ (or $F_k$) and the corresponding class embedding $w_k$ (or $W_k$), where $f_k, w_k \in \mathbb{R}^2$ and $F_k, W_k \in \mathbb{R}^3$. They are learned via the steepest descent algorithm. Notably, $f_k$ (or $F_k$) is considered a parameter to be solved, with the aim of simulating the ideal case where the feature extractor has enough capacity. The first row presents the results of training with rectified cosine normalization.

* Both authors contributed equally to the paper
Fig. 1. Comparison of our rectified cosine normalization (denoted as rectified-CN) and the typical cosine normalization (denoted as CN) with different number of classes. Dots in various colors represent the learned feature vector $f_k$ for class $k$, and the line segment of the same color denotes the corresponding weight vector $w_k$.

in the augmented feature space $F_k = (f_k, 1)$ with $f_k = (x, y) \in R^2$. Both the learned features $F_k$ and the class embeddings $W_i = (w_i, b_i)$ are projected onto the $xy$-plane for visualization and comparison. The second row corresponds to the results of using typical cosine normalization, for which both $f_k$ and $w_i$ are learned directly in the original feature space without augmentation. It is clear to see that in the cases with 3 and 4 classes, both rectified cosine normalization and typical cosine normalization work well. The features $f_k$ point in the same direction as their class embeddings $w_k$, as expected. We note that minimizing the $L_{bce},(x_k)$ (equation (6) in the main manuscript) requires that (1) $F_k$ should be separated from the class embeddings $W_i$ not of the same class (i.e. $i \neq k$) by at least 90 degrees and that (2) the angle between $F_k$ and $W_k$ of the same class should be smaller than 90 degrees and preferably be 0 degrees. These apply to $f_k$ and $w_i$ learned directly in the original feature space. When the number of classes is small, these requirements are easy to fulfill, be it in the 2-dimensional original feature space or in the 3-dimensional augmented feature space. However, become difficult to hold simultaneously in the 2-dimensional feature space when the number of classes increases beyond 4. Intriguingly, in those cases, the feature vectors of different classes may collapse into few modes, putting much emphasis on the first requirement due mainly to the excessive amount of negative examples (i.e. for a given $f_k$, the number of class embeddings $w_i, i \neq k$ not of the same class is much larger than the case $i = k$). In contrast, both requirements can still hold in the 3-dimensional augmented feature space. We observe that this is achieved by setting all $b_i$’s to negative values and having $F_k$ and $W_k$ (respectively, $W_i, i \neq k$) point in approximately similar directions (respectively, in significantly different directions) when projected onto the xy-plane. The larger
degree of freedom in adapting the class embeddings $W_I$ to the two requirements in the higher dimensional augmented feature space explains the generally better separation between feature vectors $f_k$ with our rectified cosine normalization in the lower dimensional original feature space.

3 Detail Formulation of Our Evaluation

We follow the common metrics in prior works [1,3,2] to evaluate the performance of incremental learning; that is, incremental accuracy, average incremental accuracy. Furthermore, we additionally propose a metric called phase accuracy. The details of these metrics are described as follows.

Incremental Accuracy. Let $A_{j,i}$ be the average accuracy of classes learned in phase $i$ after training the network sequentially to phase $j$, where $i < j$. Incremental accuracy is defined as $A_j = \sum_{i=1}^{j} A_{j,i}$, which is the accuracy for classifying all the seen classes at the end of training phase $j$. It is the most commonly-used metric, but has the limitation of showing only the accuracy over the seen classes as a whole without giving any detail of how the model performs on separate groups of classes (learned incrementally in each phase).

Average Incremental Accuracy. Average incremental accuracy accumulates the incremental accuracy, obtained from each training phase, up to the current phase $j$ and then takes the average, i.e. $\frac{1}{j} \sum_{i=1}^{j} A_i$.

Phase Accuracy. A model that learns well incrementally should present a balanced distribution of $A_{j,i}, i \in \{1, ..., j\}$. Thus, we propose phase accuracy to evaluate at the end of the entire incremental training to present the classification accuracy on separate groups of classes (a group represents a phase in this thesis). It provides a breakdown look at whether the model would favor some groups of classes over the others as a result of catastrophic forgetting.

4 Adding 5 New Classes in Each Phase

Here we conduct more experiments based on almost the same setting as the one in the main manuscript (i.e. CIFAR-100 and ImageNet datasets, two training scenarios, memory size of 1000 and 2000, as well as the three evaluation metrics), but now the number of new classes in each incremental phase is set to 5 (e.g. in total 20 phases while training from scratch).

4.1 Incremental Accuracy Comparison

The quantitative evaluation results in terms of incremental accuracy are provided in Fig. 2, while Table 1 particularly summarizes the results at the end of the entire incremental-training.

Similar to the observations we have in the main manuscript, on CIFAR-100 and ImageNet, when learning from scratch, our model outperforms all the
baselines with memory size 1000 and 2000. Also, we find again that Hou19 [2] and Mnemonics [5] are unable to perform well when the feature extractor is learned from scratch. When starting from 50 classes, ours performs comparably to Mnemonics [5] and outperforms all the other baselines on ImageNet. On CIFAR-100, our model performs comparably to the other baselines with memory size 2000 and achieves the second highest incremental accuracy with memory size 1000. When starting from 50 classes, ours outperforms all the other baselines, except Mnemonics [5], on ImageNet. On CIFAR-100, our model performs comparably to the other baselines with memory size 2000 and achieves the second highest incremental accuracy with memory size 1000.

### 4.2 Phase Accuracy Comparison

Fig. 3 presents the phase accuracy for different methods with the training scenario “from 50”, where the baselines perform more closely to our method in terms
Fig. 3. Phase accuracy comparison: (a)(b) are results on CIFAR-100 and (c)(d) on ImageNet. $M$ is the memory size, and the model is pre-trained with 50 classes. The mean absolute deviation in phase accuracy is shown in parentheses.

Table 2. Comparison of forgetting measure under the setting that learns 10 new classes in each phase.

<table>
<thead>
<tr>
<th>Training scenario</th>
<th>Dataset</th>
<th>CIFAR</th>
<th>ImageNet</th>
<th>CIFAR</th>
<th>ImageNet</th>
<th>CIFAR</th>
<th>ImageNet</th>
<th>CIFAR</th>
<th>ImageNet</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Memory size</td>
<td>1K</td>
<td>2K</td>
<td>1K</td>
<td>2K</td>
<td>1K</td>
<td>2K</td>
<td>1K</td>
<td>2K</td>
</tr>
<tr>
<td>iCaRL [1]</td>
<td></td>
<td>24.6</td>
<td>20.6</td>
<td>22.3</td>
<td>20.6</td>
<td>24.6</td>
<td>20.4</td>
<td>24.1</td>
<td>20.4</td>
</tr>
<tr>
<td>End-to-End [3]</td>
<td></td>
<td>41.6</td>
<td>32.9</td>
<td>39.1</td>
<td>31.0</td>
<td>38.2</td>
<td>29.1</td>
<td>37.2</td>
<td>28.4</td>
</tr>
<tr>
<td>BIC [4]</td>
<td></td>
<td>37.3</td>
<td>28.6</td>
<td>36.1</td>
<td>31.4</td>
<td>33.5</td>
<td>26.4</td>
<td>33.9</td>
<td>27.3</td>
</tr>
<tr>
<td>Hou19 [2]</td>
<td></td>
<td>27.3</td>
<td>16.8</td>
<td>38.7</td>
<td>28.8</td>
<td>17.2</td>
<td>11.1</td>
<td>20.4</td>
<td>13.4</td>
</tr>
<tr>
<td>Mnemonics [5]</td>
<td></td>
<td>39.9</td>
<td>33.1</td>
<td>38.4</td>
<td>31.7</td>
<td>22.8</td>
<td>18.9</td>
<td>15.9</td>
<td>14.0</td>
</tr>
<tr>
<td>Ours</td>
<td></td>
<td>25.0</td>
<td>22.3</td>
<td>24.6</td>
<td>23.6</td>
<td>17.7</td>
<td>16.1</td>
<td>18.4</td>
<td>17.8</td>
</tr>
</tbody>
</table>

of incremental accuracy. Shown in parentheses is the mean absolute deviation from the average of each method’s phase accuracy. The smaller the deviation, the more balanced the classification accuracy is on different classes. From the figure, our scheme is seen to achieve the minimum mean absolute deviation in phase accuracy on CIFAR-100 and comparable mean absolute deviation to iCaRL [1] and Mnemonics [5] on ImageNet.

5 Forgetting Measure

In this section, we present the results of forgetting measure [6] on every class at the end of training, in order to understand to what degree the competing methods forget the previously learned classes. Generally, the lower the forgetting measure, the less the catastrophic forgetting. Table 2 summarizes the results for the setting that learns 10 new classes in each training phase. It shows that our method has low forgetting measure and performs comparably to iCaRL [1], Hou19 [2] and Mnemonics [5].

Table 3 further provides the results for the setting of learning 5 new classes in each phase. In this setting, the model needs to be updated more frequently and tends to forget more easily what it has learned. On CIFAR-100, ours achieves the lowest forgetting measure, when learning from scratch. Moreover, it has the
Table 3. Comparison of forgetting measure under the setting that learns 5 new classes in each phase

<table>
<thead>
<tr>
<th>Training scenario</th>
<th>from scratch</th>
<th>from 50 classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dataset</td>
<td>CIFAR</td>
<td>ImageNet</td>
</tr>
<tr>
<td>Memory size</td>
<td>1K</td>
<td>2K</td>
</tr>
<tr>
<td>iCaRL [1]</td>
<td>34.5</td>
<td>29.4</td>
</tr>
<tr>
<td>End-to-End [3]</td>
<td>55.6</td>
<td>44.2</td>
</tr>
<tr>
<td>BIC [4]</td>
<td>50.4</td>
<td>40.9</td>
</tr>
<tr>
<td>Hou19 [2]</td>
<td>61.7</td>
<td>51.6</td>
</tr>
<tr>
<td>Mnemonics [5]</td>
<td>47.4</td>
<td>39.4</td>
</tr>
<tr>
<td>Ours</td>
<td><strong>28.0</strong></td>
<td><strong>23.9</strong></td>
</tr>
</tbody>
</table>

Fig. 4. Comparison of discrepancy measure $\mathcal{D}(p^*_i|k, p_i|k)$ on CIFAR-100 for memory sizes $M = 1K, 2K$ and training scenario “FS” and “from 50,” with the mean absolute deviation in phase accuracy shown in parentheses. The bars in the top row are evaluated by five random orderings of classes.

second lowest forgetting measure with the training scenario “from 50”. On ImageNet, iCaRL [1] performs comparably to ours, when learning from scratch. Since Hou19 [2] and Mnemonics [5] are unable to perform well when learning from scratch, they suffer from serious catastrophic forgetting, showing large forgetting measure. When learning from 50 classes, ours performs comparably to Mnemonics [5] and outperforms the other baselines.

To sum up, our method has the ability to preserve well the knowledge of old classes under various settings, while showing high incremental accuracy in learning new classes and more balanced phase accuracy.

6 More Ablation Experiments

6.1 Symmetric vs. Asymmetric Discrepancy Measure $\mathcal{D}(p^*_i|k, p_i|k)$

We argue in Sec. 3.1 of the main manuscript that an $a^*_i|k$-symmetric discrepancy measure is desirable. That is, $\mathcal{D}(p^*_i|k, p_i|k)$ is preferably symmetric with respect
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Fig. 5. Comparison of iCaRL with/without rectified cosine normalization and our method on CIFAR-100 for memory sizes $M = 1K, 2K$ and training scenarios "FS" and "from 50," with the mean absolute deviation in phase accuracy shown in parentheses. The bars in the top row are evaluated by five random orderings of classes.

6.2 iCaRL with vs. without Rectified Cosine Normalization

This section investigates whether our rectified cosine normalization could also benefit iCaRL [1]. Compared in Fig. 5 are iCaRL [1] with (denoted as Norm-iCaRL) and without (denoted as iCaRL) rectified cosine normalization. Note that the other design aspects of iCaRL remain the same. As a benchmark, our scheme is also presented. A comparison between Norm-iCaRL and iCaRL in the top row of Fig. 5 reveals that rectified cosine normalization does help improve the incremental accuracy of iCaRL [1], reducing largely the performance gap between ours and iCaRL [1]. However, without our weighted-Euclidean regularization, both Norm-iCaRL and iCaRL [1] exhibit higher variations in phase accuracy than ours (cp. the bottom row of Fig. 5).
References