1 Appendix

1.1 Derivation of the upper bound for the entropy of a logistic mixture model

Let random variable X follows a distribution modeled by a mixture of logistic distributions with the probability density function defined by

$$p(x) = \sum_{i=1}^{K} \pi_i p_i(x), \tag{1}$$

where $\sum_i \pi_i = 1$ and $\pi_i > 0$. Let

$$p_i(x) = \frac{e^{-(x-\mu_i)/s_i}}{s_i \left(1 + e^{-(x-\mu_i)/s_i}\right)^2}, \tag{2}$$

where μ_i and s_i are the mean and scale parameters of the i-th component in the mixture. The cumulative distribution function (CDF) for the logistic distribution is simply a sigma function defined by

$$c_i(x) = \frac{1}{1 + e^{-(x-\mu_i)/s_i}}. \tag{3}$$

For a discrete input x in L levels, the probability value of $X = x$ is given by $P(X = x) = c(x + 1/2) - c(x - 1/2)$ for $x \in \{2, \cdots L-1\}$, $p(1) = c(1 + 1/2)$ and $p(L) = 1 - c\,(L - 1/2)$.

There is no closure form to calculate the entropy of a logistic mixture model defined by Eqs. 1 and 2. Although the entropy for discrete case can be calculated numerically, the calculation is pretty expensive. In our proposal, we propose to use an upper bound, given by the following proposition, as the estimation of the entropy.

Proposition 1. *The entropy of a random variable X that follows the mixture of logistic distributions defined by Eqs. 1 and 2 has an upper bound such that*

$$H(X) \leq -\sum_{i=1}^{K} \pi_i \log \pi_i + \sum_{i=1}^{K} \pi_i H_i(X), \tag{4}$$

where $H(X)$ is the entropy of variable X and $H_i(X)$ is the entropy of a variable that follows the logistic distribution function defined by Eq. 2.

Proof.

$$\begin{aligned} H(X) &= -\sum_x p(x) \log p(x) \\ &= -\sum_x \left(\sum_i \pi_i p_i(x) \log p(x) \right) \end{aligned}$$

$$
\begin{aligned}
&\leq -\sum_x \left(\sum_i \pi_i p_i(x) \log\left(\pi_i p_i(x)\right) \right), \text{ given } p(x) \geq \pi_i p_i(x) \\
&= -\sum_x \left(\sum_i \pi_i p_i(x) \left(\log \pi_i + \log p_i(x)\right) \right) \\
&= -\sum_i \sum_x \pi_i \log \pi_i p_i(x) - \sum_i \sum_x \pi_i p_i(x) \log p_i(x) \\
&= -\sum_i \pi_i \log \pi_i \sum_x p_i(x) - \sum_i \pi_i \sum_x p_i(x) \log p_i(x) \\
&= -\sum_i \pi_i \log \pi_i + \sum_i \pi_i H_i(X)
\end{aligned}
$$

□

Next, we use the differential entropy to approximate the true entropy $H_i(X)$ for a discrete case. Given proposition 1 and the mixture mode defined by Eqs. 1 and 2, we get the upper bound defined by

$$
-\sum_i \pi_i \log \pi_i + \sum_i \pi_i (\ln s_i + 2) \tag{5}
$$