

— Supplementary Material —

RPSRNet: End-to-End Trainable Rigid Point Set Registration Network using Barnes-Hut 2^D -Tree Representation

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This supplementary document summarizes several relevant details for deeper understanding about our RPSRNet in three main Secs. 1, 2 and 3. The Sec. 1 lists down the parameter settings and training protocols of all benchmark methods alongside our proposed RPSRNet. We also show the effects of different batch size and scaled loss learning on the prediction accuracy of the network. The next Sec. 2 gives insights to our BH 2^D -tree convolution operation using linearized node-indexing. In the end, Sec. 3 provides more insightful results and evaluation of our method on ModelNet40 [12] and KITTI [5] datasets.

1. Evaluation Method and Training Details.

We train the registration networks of DCP [10], PRNet [11], and PointNetLK [1] methods all from scratch (w/o using their pre-trained models) following standard training protocols from the respective studies. Both DCP and its extended version PRNet, and PointNetLK take 250 epochs to train the network with a learning rate of 10^{-3} and using ADAM optimizer. During experiments, we find that PRNet has inherent issues of ill-conditioned feature embedding matrix computations which causes random crashes during training. Hence, we do not evaluate PRNet. While a batch size 10 is set for the DCP, PointNetLK is trained with batch size 32 and internal alignment iterations 10. The optimal hyper-parameters for our RPSRNet are – *required number of epochs*: 250, *batch size*: 16 (see the reason in Sec. 1.1), *learning rate*: 0.0001, *loss dissipation factor* (β): 0.2, *rotational/translational scale* (σ_R/σ_t): -5.0 / -2.5 for KITTI dataset and -9.0 / -10.0 for ModelNet40 dataset, and a *dropout*: 0.2. On the other hand, the maximum number of iterations for all the unsupervised iterative alignment methods ICP [3], CPD [7], FilterReg [4], FGR [13], and GA [6], is set to 200. The mean $\mu = 0.0$ and Gaussian spread $\sigma = 0.05$ are set for FilterReg and CPD (data-based automatic initialization of σ is also possible). FGR method uses Fast Point Feature Histogram (FPFH) [9, 8] descriptor of the input source and target point clouds and use them in their global optimization step to estimate rigid transforma-

tion. Hence, it takes a search radius to localize descriptor space for every point. We set the *search-radius* parameter to 0.15. To evaluate the GA, we set its astrophysical parameter settings as – *Gravitational constant* (G): $6.67 \cdot 10^{-3}$, *force softening length* (ϵ): 0.05, *time step* (Δt): 0.05, and *energy dissipation rate*: 0.3.

1.1. Importance of Batch-Size and Scaled Loss

On KITTI [5] and ModelNet40 [12] Datasets					
Batch Sizes	SLL	(K2-w)	(K1-w/o)	(M1-seen)	(M2-unseen)
	1	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$	$\varphi_{\text{rmse}}, \Delta t_{\text{rmse}}$
8	✗	1.4394, 2.19	1.481, 1.53	5.82, 0.04452	5.878, 0.0468
	✓	0.9747, 0.87	0.9234, 0.84	4.3111, 0.0221	4.667, 0.0281
16	✗	1.5823, 2.24	1.4645, 1.39	6.1468, 0.0472	6.2004, 0.0423
	✓	0.9889, 0.81	0.9651, 0.7	4.588, 0.02377	5.221, 0.0252
32	✗	1.5452, 2.25	1.3855, 1.44	7.006, 0.0536	8.613, 0.0542
	✓	0.9407 , 1.03	0.9025 , 0.7	4.1318 , 0.0189	4.203 , 0.0195

Table 1. Importance of different batch sizes and scaled loss learning (SLL) in RPSRNet: Our method using scaled loss learning (SLL) with 32 input samples in a batch reports minimum rotational and translation RMSE on both the KITTI and ModelNet40 datasets. There is a notable exception on Δt_{rmse} while using the batch size 16 for KITTI (K2-w) setup.

We investigate thoroughly how different batch sizes, scaled version of our loss function (Sec.3.3 of main matter) and pre-processing step of removing ground points impact on the final prediction accuracies. The evaluation Table 1 shows that with scaling, there are notable improvements on translation error for KITTI [5] dataset. The similar improvement on rotational error is clear for ModelNet40 [12] dataset. For the experiments in the main matter, we select batch size 16.

2. BH 2^D -tree Convolution and Node Indexing

This section describes the compact storage mechanism and input signal processing of our BH 2^D -tree representation of point cloud. To this end, we have already saved several lists of attributes of the tree nodes at every depth d :

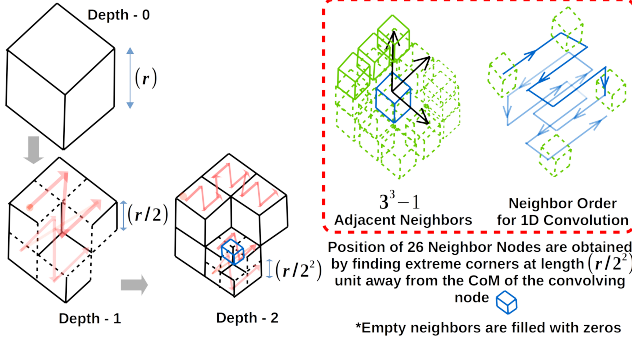


Figure 2. **Neighbors of a non-empty node at depth d .** 26 adjacent nodes $\kappa_d(n_{d,l})$ of $n_{d,l}$ (always at the current depth of convolving node) are the ones which are at the extreme end by $\frac{r}{2^d}$ length from $n_{d,l}$. The zero-fill operation sets zero values to the neighbors which are found empty. A linear array stores the neighbors contagiously following the order shown at right. Therefore, our 1D convolution kernel size is 26.

Hierarchical Pooling. Pooling is a converse operation of convolution where we fetch the information from child nodes to the parent nodes or in other words from nodes at the higher depth of our BH-tree to the lower depth, e.g., from $d = 3$ 2. After a convolution, the parent-child information needs to be reconstructed for Max-Pooling. Lets observe the parent-child relationship using L_d . The j^{th} child $C_j(P_i)$ of a non-empty parent node $P_i = n_{d,i} = N_d[i]$ can be retrieved from the array $N_{(d+1)}$, using the formula:

$$C_j(P_i) = N_{d+1}[L_{d+1}[i \cdot 8 + j]] \quad (1)$$

Since we store the labels at every depth and do not recompute the neighbors at runtime, we first perform a *child-fill* operation which is to fill attributes' values of the empty neighbor nodes by zero at the current depth. Therefore our input-tensor will have the size of $2^3 \cdot |N_d|$ as of the label array L_d . For the *neighbor-fill* and *child-fill* operations, we write custom functions in CUDA/C++ for Pytorch-binding.

3. Insightful Results and Evaluations.

We present more insightful registration results for KITTI [5] LiDAR odometry and ModelNet40 [12] datasets (see Table. 2, Fig. 3, and Fig. 4 respectively). The quantitative results on KITTI dataset give important few remarks about other methods:

1. Only on seq-00 and seq-05, FGR and ICP wins the contest of lowest translation error. Although, their difference from RPSRNet is significantly small.
2. In the (K2) setup, all methods record higher transformation errors, especially on translational part, on most of the sequences.
3. On seq-01, GA [6], FGR [13], ICP [3] FilterReg [4], PointNetLK [1], and our RPSRNet¹ (single internal iteration) report 39%, 119%, 6%, 171%, 86% and 6.90%

increase in the translational error when evaluated on (K2) setup compared to the same on (K1) setup.

On the other hand, Fig. 3 and 4 show registration outcomes from competing methods on some randomly selected samples. The final alignments applying different methods show that most methods struggle to find globally optimal transformation when input data is largely incomplete.

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On KITTI [5] Dataset									
Seq.	CPD [7]	GA* [6]	FGR [13]	ICP [3]	FilterReg [4]	DCP-v2 [10]	PointNetLK [1]	RPSRNet ¹ (ours)	RPSRNet ³ (ours)
	$\varphi_{rmse}, \Delta t_{rmse}$	$\varphi_{rmse}, \Delta t_{rmse}$	$\varphi_{rmse}, \Delta t_{rmse}$	$\varphi_{rmse}, \Delta t_{rmse}$	$\varphi_{rmse}, \Delta t_{rmse}$	$\varphi_{rmse}, \Delta t_{rmse}$	$\varphi_{rmse}, \Delta t_{rmse}$	$\varphi_{rmse}, \Delta t_{rmse}$	$\varphi_{rmse}, \Delta t_{rmse}$
00	4.99, 1.12	4.82, 1.09	4.77, 0.82	4.72, 0.80	4.77, 0.80	4.87, 1.07	5.44, 1.27	4.48, 1.01	3.30 , 0.81
	<i>4.91, 1.39</i>	<i>4.78, 0.93</i>	<i>4.78, 0.83</i>	<i>4.70, 0.84</i>	<i>4.92, 0.88</i>	<i>4.78, 0.80</i>	<i>6.11, 1.27</i>	<i>4.56, 0.73</i>	<i>3.31, 1.0</i>
01	3.18, 1.54	2.99, 1.74	3.06, 1.10	3.06, 2.27	3.02, 0.89	3.0, 0.95	4.50, 1.33	2.79, 1.28	2.13 , 0.48
	<i>3.15, 1.88</i>	<i>3.06, 2.42</i>	<i>2.88, 2.42</i>	<i>2.83, 2.41</i>	<i>2.77, 2.41</i>	<i>1.80, 0.68</i>	<i>5.22, 1.18</i>	<i>3.04, 1.38</i>	<i>2.0, 1.03</i>
02	3.87, 1.28	3.67, 0.98	3.71, 1.02	3.63, 1.0	3.42, 0.69	3.52, 0.76	4.21, 1.0	3.69, 0.73	2.66 , 0.6
	<i>3.0, 1.11</i>	<i>3.71, 1.09</i>	<i>3.66, 0.9</i>	<i>3.66, 0.91</i>	<i>3.81, 1.1</i>	<i>3.52, 0.53</i>	<i>5.77, 0.96</i>	<i>3.73, 1.10</i>	<i>2.88, 1.11</i>
03	0.38, 0.88	0.34, 0.72	0.31, 0.59	0.14, 0.56	0.14, 0.49	0.24, 0.66	0.90, 0.81	0.14, 0.72	0.10 , 0.41
	<i>0.23, 0.58</i>	<i>0.18, 0.43</i>	<i>0.20, 0.74</i>	<i>0.13, 0.91</i>	<i>0.35, 0.78</i>	<i>0.18, 0.3395</i>	<i>2.1, 1.01</i>	<i>0.16, 0.85</i>	<i>0.08, 0.68</i>
04	2.58, 1.09	2.64, 1.12	2.65, 1.06	2.64, 1.07	1.97, 0.89	2.20, 1.37	3.88, 1.31	2.74, 0.55	1.11 , 0.50
	<i>2.77, 0.91</i>	<i>2.64, 0.85</i>	<i>2.64, 1.09</i>	<i>2.65, 1.11</i>	<i>2.11, 1.18</i>	<i>2.07, 1.01</i>	<i>4.86, 1.27</i>	<i>2.28, 0.38</i>	<i>1.19, 0.21</i>
05	3.81, 0.79	3.41, 0.7135	3.29, 0.4142	2.95, 0.75	3.16, 0.62	2.01, 0.42	4.09, 0.79	3.09, 0.87	1.91 , 0.71
	<i>3.0, 0.69</i>	<i>3.30, 0.64</i>	<i>3.27, 0.80</i>	<i>2.8, 1.12</i>	<i>3.89, 0.99</i>	<i>3.15, 0.63</i>	<i>4.2, 1.12</i>	<i>3.35, 1.06</i>	<i>1.11, 0.91</i>
06	4.67, 0.96	4.13, 0.85	4.04, 0.87	3.64, 1.20	4.04, 0.88	3.02, 0.69	3.08, 0.99	4.01, 0.64	3.0 , 0.50
	<i>2.85, 1.28</i>	<i>1.55, 1.02</i>	<i>4.13, 1.21</i>	<i>3.26, 1.32</i>	<i>4.03, 1.29</i>	<i>3.74, 0.48</i>	<i>4.87, 0.97</i>	<i>2.64, 0.48</i>	<i>3.94, 0.69</i>
07	4.89, 1.0	4.39, 0.78	4.46, 0.91	4.41, 1.03	4.11, 0.99	4.48, 1.22	6.04, 1.44	4.06, 0.81	3.58 , 0.61
	<i>4.31, 0.71</i>	<i>4.34, 0.77</i>	<i>4.46, 0.88</i>	<i>4.37, 0.95</i>	<i>4.22, 0.99</i>	<i>4.45, 1.58</i>	<i>8.21, 1.81</i>	<i>4.45, 1.08</i>	<i>3.11, 1.07</i>

Table 2. Evaluation on KITTI [5] LiDAR sequences 00 to 07. The first row under each sequence reports the RMSE on angular and translational deviations from ground-truth measured on its validation samples in (**K1-w/o**) setup: without ‘ground points’. The next row under the same sequence reports the same error (with gray-colored and *italic* font-style for better distinction) on the same validation set in (**K2-w**) setup: with ‘ground points’. The lowest transformation errors achieved by a method is highlighted in **bold** or underlined **bold** font.

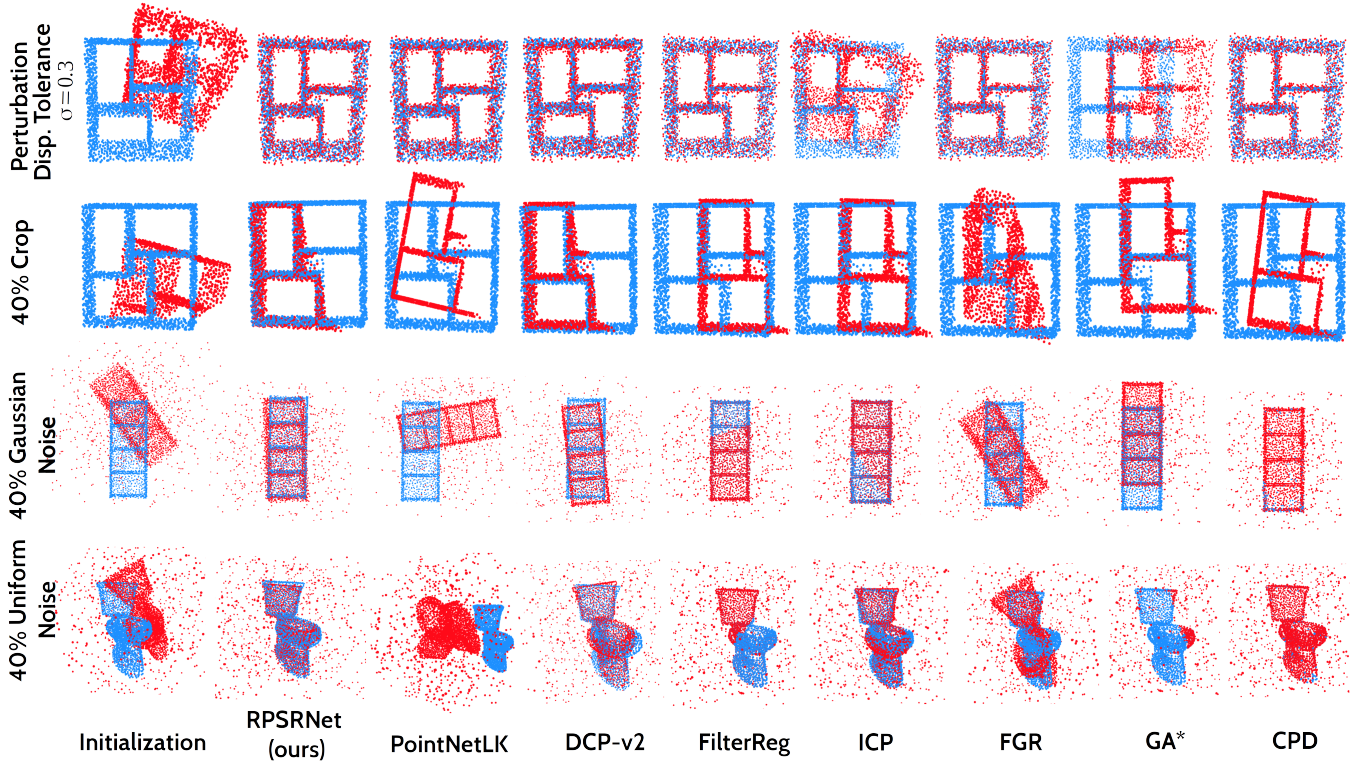


Figure 3. Registration outcomes on few samples of ModelNet40 dataset. The results show robustness of RPSRNet compared to state-of-the-art methods.

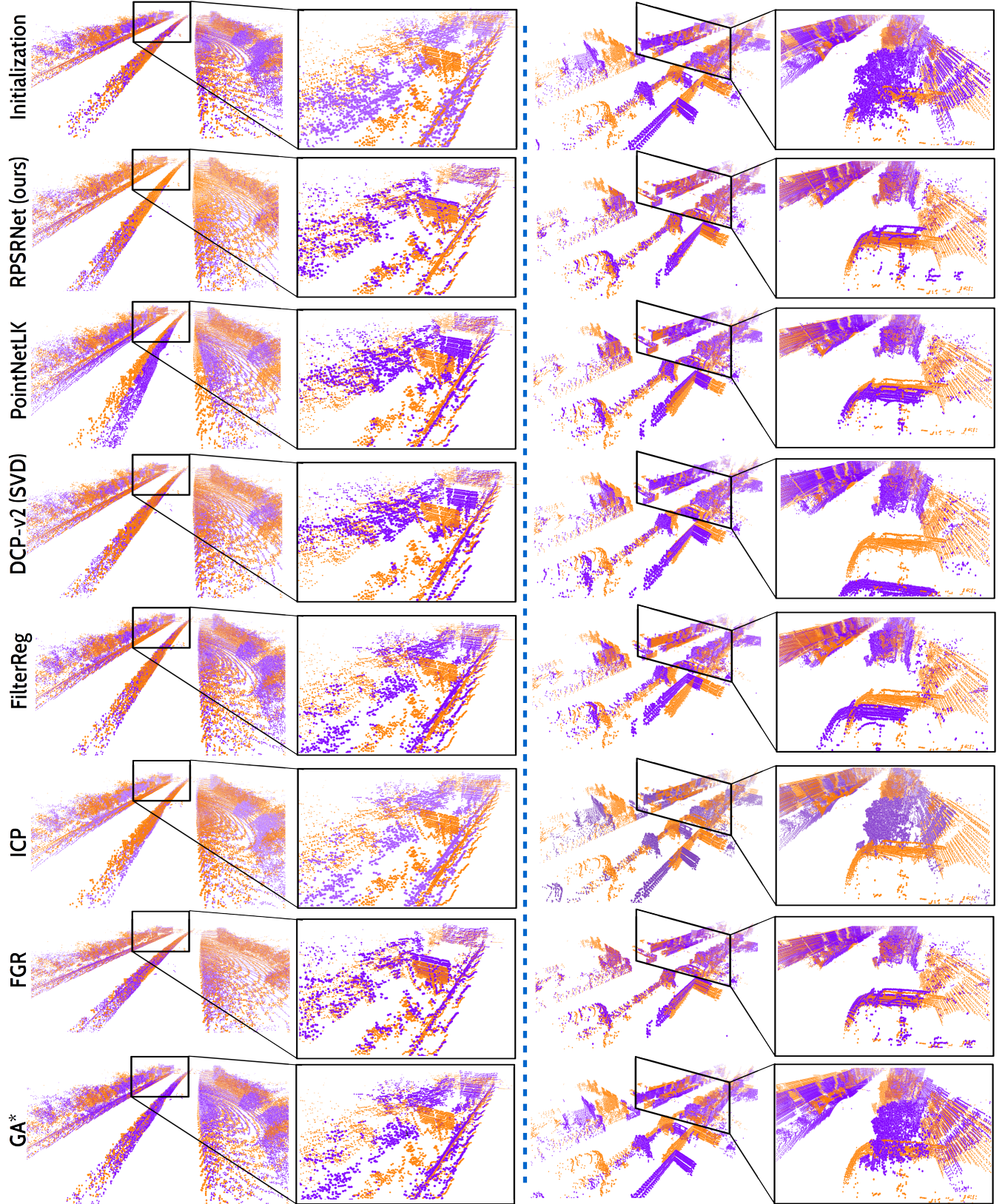


Figure 4. Results of our RPSRNet³ on some randomly selected samples from the validation set of KITTI [5, 2] dataset (*left column*: frame 000131 (as **Y**) and 000136 (as **X**) of seq-01 and *right column*: frame 001721 (as **Y**) and 001726 (as **X**) from the seq-05). Zoomed parts of each image highlights how other competing methods perform in aligning static objects of target scene.