Hybrid Rotation Averaging: A Fast and Robust Rotation Averaging Approach - Supplementary Material

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1. Derivation of Globally Optimal Rotation Averaging

Give a set of relative rotations $\{R_{ij}\}$, where $i, j \in [n]$, the aim of rotation averaging is to obtain the absolute rotations $\{R_i\}$ that satisfy the constraints

$$R_{ij} = R_j R_i^{-1} = R_j R_i^T \tag{1}$$

between absolute and relative rotations. Usually there are more edges than nodes in an undirected graph, so there are more constraints than unknowns. Rotation averaging can be formulated as

$$\min_{R_1, \cdots, R_n} \sum_{(i,j) \in E} d^p(R_{ij}, R_j R_i^T), \tag{2}$$

Chordal distances is popular as the distance measure in (2). Each residual along an edge of $\mathcal{G}$ will hence read

$$\begin{aligned} & \|R_j - R_{ij} R_i\|_F^2 \\ = & \|R_j\|_F^2 - 2\,\mathrm{tr}(R_j^T R_{ij} R_i) + \|R_i\|_F^2 \\ = & \, 6 - 2\,\mathrm{tr}(R_j^T R_{ij} R_i). \end{aligned} \tag{3}$$

The set of absolute rotations can be represented by

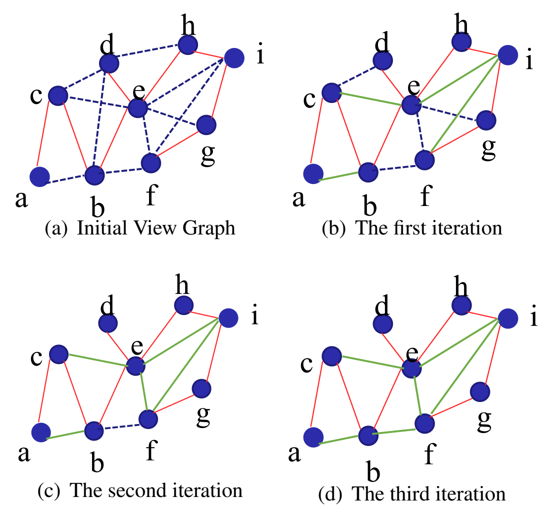
$$R = [R_1 \; R_2 \; \cdots \; R_n], \tag{4}$$

and the graph $\mathcal{G}$ can be represented by

$$G = \begin{bmatrix} 0 & a_{12}R_{12} & \cdots & a_{1n}R_{1n} \\ a_{21}R_{21} & 0 & \cdots & a_{2n}R_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}R_{n1} & a_{n2}R_{n2} & \cdots & 0 \end{bmatrix}, \tag{5}$$

where $a_{ij} = 1$ if the edge between views i and j exists, and $a_{ij} = 0$ otherwise. By combining Eqs. (3), (4) and (5), problem (2) can be rewritten as

$$\min_R \, (6 - 2\,\mathrm{tr}(R^T G R)) \Leftrightarrow \min_R \; -\mathrm{tr}(R^T G R). \tag{6}$$



(a) Initial View Graph (b) The first iteration (c) The second iteration (d) The third iteration

Figure 1. The procedure of fast view graph filtering, where the blue nodes represent images, the *red solid lines* represent the edges in the selected maximum spanning tree (MaxST), the *blue dotted lines* are edges that need to be validated, and the *green solid lines* are edges that passed the verification.

Eriksson *et al.* [1] proves that there is no duality gap between the primal problem and the corresponding dual problem if the maximum residuals stay below a certain threshold. For readers who are interested in the duality gap and global optimality proof, we kindly refer them to [1] for more details.

2. Fast View Graph Filtering

We present how the fast view graph filtering works in Fig. 1. In Fig. 1, the blue nodes represent images, the *red solid lines* represent the edges in the selected maximum spanning tree (MaxST), the *blue dotted lines* are edges that need to be validated, and the *green solid lines* are edges that passed the verification. In Fig. 1 (a), we deem the edges in the MaxST as valid. Then, we can collect all the weak triplets based on the MaxST: $(a, c, b), (b, c, e), (e, d, b), (e, d, h), (g, f, i), (h, i, e)$ before the first iteration.

In Fig. 1 (b), edges $(a, b), (c, e), (e, i), (f, i)$ passed the verification, edges $(d, b), (d, h)$ are deleted as they failed to pass the verification. Based on the first iteration, we also collect all the weak triplets $(c, e, d), (e, i, f), (e, i, g)$. In Fig. 1 (c), $(c, d), (e, g)$ are deleted due to it fails to pass the verification, (e, f) is marked by green solid line as it survives the validation. Fig. 1 (d), we only need to validate weak triplet (b, e, f), suppose (e, g) passed the verification and we mark it by green solid line.

References

[1] Anders Eriksson, Carl Olsson, Fredrik Kahl, and Tat-Jun Chin. Rotation averaging with the chordal distance: Global minimizers and strong duality. *IEEE Trans. Pattern Anal. Mach. Intell.*, 2020. 1