

Supplementary Material: Depth from Camera Motion and Object Detection

Depth from Motion Parallax & Detection

Building off of the model from Section 3.1, if there is x - or y -axis camera motion between observations, we can solve for object depth using corresponding changes in bounding box location (e.g., Box_1 to Box_3 in Figure 2).

To start, we account for any incidental depth-based changes in scale by reformulating (6) as

$$Z_j = Z_i \left(\frac{w_i}{w_j} \right). \quad (21)$$

Next, we use (21) in (4) to relate bounding box center coordinate x_j to corresponding 3D object coordinate X_j as

$$\begin{aligned} x_j &= \frac{f_x X_j}{Z_j} + c_x = \frac{f_x X_j}{Z_i \left(\frac{w_i}{w_j} \right)} + c_x \\ \implies (x_j - c_x) \left(\frac{w_i}{w_j} \right) &= \frac{f_x X_j}{Z_i}. \end{aligned} \quad (22)$$

Given a static object, changes in lateral object position X_j occur only from changes in camera position C_{Xi} (2). Thus,

$$X_j - X_i = -(C_{Xj} - C_{Xi}). \quad (23)$$

Finally, using (22) and (23), we can solve for Z_i by comparing two observations i, j with motion parallax as

$$\begin{aligned} (x_j - c_x) \left(\frac{w_i}{w_j} \right) - (x_i - c_x) &= \frac{f_x (X_j - X_i)}{Z_i} \\ \implies Z_i &= \frac{f_x (C_{Xi} - C_{Xj})}{(x_j - c_x) \left(\frac{w_i}{w_j} \right) - (x_i - c_x)}. \end{aligned} \quad (24)$$

Notably, (24) can also be derived using vertical motion as

$$Z_i = \frac{f_y (C_{Yi} - C_{Yj})}{(y_j - c_y) \left(\frac{w_i}{w_j} \right) - (y_i - c_y)}, \quad (25)$$

and scale measure $\frac{h_i}{h_j}$ can replace $\frac{w_i}{w_j}$ in (24) or (25). Also, if there is no z -axis camera motion (i.e., $Z_i = Z_j$), then $\frac{w_i}{w_j} = \frac{h_i}{h_j} = 1$ and we can simplify (24) and (25) as

$$Z_i = \frac{f_x (C_{Xi} - C_{Xj})}{x_j - x_i} = \frac{f_y (C_{Yi} - C_{Yj})}{y_j - y_i}. \quad (26)$$

Comparison of Depth Estimation Cues

We provide ODMD Test Set results in Table 3 to compare solutions using different depth estimation cues. We

Table 3. ODMD Results for Various Depth Estimation Cues.

Analytical Depth Estimation Cue	Object Depth Method	Mean Percent Error (20)				
		Norm.	Perturb		Robot	All Sets
			Camera Motion	Object Detect.		
Learning-based Methods						
Full x, y, z Motion	DBOX _{NS} (14)	0.5	3.9	6.4	12.5	5.8
Analytical Methods						
Optical Expansion	Box _{LS} (9)	0.0	4.5	21.6	21.2	11.8
Motion Parallax	Z_n (24)-(25)	0.0	33.9	51.6	65.6	37.8
Optical Expansion	Z_n (8)	0.0	5.2	80.9	124.1	52.5

evaluate three different analytical solutions that use single cues and DBOX_{NS}, which uses full x, y, z motion.

For the motion parallax solution, we use the average of the lateral (24) and vertical (25) motion parallax solutions, using scale measure $\frac{w_i}{w_j}$ in (24) and $\frac{h_i}{h_j}$ for (25). Notably, this is a two-observation solution, so we use the end point observations of each example, i.e., $i = n = 10$ and $j = 1$. For comparison, we similarly evaluate a two-observation, optical expansion-based solution, which uses the average of (8) when using $\frac{w_i}{w_j}$ and $\frac{h_i}{h_j}$ for the end point observations.

Motion parallax performs the best overall for the two-observation solutions in Table 3. The optical expansion solution performs surprising well with camera motion perturbations but much worse on the test sets with object detection errors (i.e., Perturb Object Detection and Robot). Both solutions are perfect on the error-free Normal Set.

The Box_{LS} solution, which uses optical expansion over all n observations, significantly outperforms both two-observation solutions, especially on test sets with object detection errors. Thus, for applications with real-world detection (e.g., the Robot Set), we find that incorporating many observations is more beneficial than choosing between optical expansion or motion parallax with fewer observations.

DBOX_{NS}, using all n observations and full x, y, z motion, performs the best overall and on all test sets with any kind of input errors. Admittedly, some error mitigation likely results from DBOX_{NS} using a probabilistic learning-based method. Still, DBOX_{NS} trains on ideal data without any input errors, so DBOX_{NS} predictions are based on an ideal model, just like the analytical methods. Accordingly, we postulate that DBOX_{NS}'s improvement over Box_{LS} is primarily the result of learning full x, y, z motion features, which are more reliable than a single depth cue (e.g., DBOX_p^z in Table 1).

Although our analytical model in Section 3.1 and current methods focus on x, y, z camera motion, adding rotation as an additional depth estimation cue is an area of future work. Nonetheless, our state-of-the-art results on the ODMS Driving Set in Table 2 do include examples with camera rotation from vehicle turning [15, Section 5.2] (Figure 6, center). Finally, as a practical consideration for robotics applications, motion planners using our current approach can simply incorporate rotation after estimating depth.

Camera-based Constraints on Generated Data

When generating new ODMD training data in Section 3.3, we consider the full camera model to ensure that generated 3D objects and their bounding boxes are within the camera's field of view. To derive this constraint, we first note that the center of a bounding box (1) is within view if $x_i \in [0, W_I], y_i \in [0, H_I]$, where W_I, H_I are the image width and height. Using (4), we represent these constraints for 3D camera-frame coordinates X_i, Y_i as

$$0 \leq x_i = \frac{f_x X_i}{Z_i} + c_x \leq W_I, \quad 0 \leq y_i = \frac{f_y Y_i}{Z_i} + c_y \leq H_I$$

$$\implies \frac{-c_x Z_i}{f_x} \leq X_i \leq \frac{(W_I - c_x) Z_i}{f_x} \quad (27)$$

$$\implies \frac{-c_y Z_i}{f_y} \leq Y_i \leq \frac{(H_I - c_y) Z_i}{f_y}. \quad (28)$$

We also consider constraints based on the maximum object size s_{\max} and camera movement range $\Delta \mathbf{p}_{\max}$ (15). We use $\Delta \mathbf{p}_{\max}$ by defining it in terms of its components parts as

$$\Delta \mathbf{p}_{\max} := [\Delta C_{X_{\max}}, \Delta C_{Y_{\max}}, \Delta C_{Z_{\max}}]^\top. \quad (29)$$

Then, using s_{\max} , $\Delta \mathbf{p}_{\max}$, and the initial object position $[X_1, Y_1, Z_1]^\top$ (16), we update the constraints in (27) as

$$\frac{-c_x(Z_1 - \Delta C_{Z_{\max}})}{f_x} \leq X_1 - \Delta C_{X_{\max}} - \frac{s_{\max}}{2} \leq$$

$$X_1 + \Delta C_{X_{\max}} + \frac{s_{\max}}{2} \leq \frac{(W_I - c_x)(Z_1 - \Delta C_{Z_{\max}})}{f_x}, \quad (30)$$

and, equivalently for height, update constraints in (28) as

$$\frac{-c_y(Z_1 - \Delta C_{Z_{\max}})}{f_y} \leq Y_1 - \Delta C_{Y_{\max}} - \frac{s_{\max}}{2} \leq$$

$$Y_1 + \Delta C_{Y_{\max}} + \frac{s_{\max}}{2} \leq \frac{(H_I - c_y)(Z_1 - \Delta C_{Z_{\max}})}{f_y}, \quad (31)$$

where $Z_1 - \Delta C_{Z_{\max}}$ accounts for camera approach to the object, $\Delta C_{X_{\max}}$ and $\Delta C_{Y_{\max}}$ account for lateral and vertical camera movement, and $\frac{s_{\max}}{2}$ accounts for object width and height. Because (30)-(31) use the maximum camera movement range and object size, they guarantee, first, (27)-(28) are satisfied for all n object positions $[X_i, Y_i, Z_i]^\top$ and, second, all corresponding bounding boxes are in view.

Given X_1, Y_1 , we can find the lower bound for the initial object depth ($Z_{1\min}$) by replacing Z_1 with $Z_{1\min}$ in (30) and

(31) to find

$$Z_{1\min} \geq \Delta C_{Z_{\max}} + \max \left(\left(\frac{f_x}{c_x} \right) \left(\frac{s_{\max}}{2} + \Delta C_{X_{\max}} - X_1 \right), \right.$$

$$\left. \left(\frac{f_y}{c_y} \right) \left(\frac{s_{\max}}{2} + \Delta C_{Y_{\max}} - Y_1 \right), \right.$$

$$\left(\frac{f_x}{W_I - c_x} \right) \left(\frac{s_{\max}}{2} + \Delta C_{X_{\max}} + X_1 \right),$$

$$\left. \left(\frac{f_y}{H_I - c_y} \right) \left(\frac{s_{\max}}{2} + \Delta C_{Y_{\max}} + Y_1 \right) \right). \quad (32)$$

In other words, given an object's center position and maximum size, the minimum viewable depth is constrained by the closest image boundary after camera movement. Note that there is no equivalent upper bound for $Z_{1\max}$.

Given Z_1 , similar to (32), we can find the lower and upper bounds for X_1, Y_1 in (16) using (30) and (31) to find

$$X_{1\min} \geq \left(\frac{c_x}{f_x} \right) (\Delta C_{Z_{\max}} - Z_1) + \Delta C_{X_{\max}} + \frac{s_{\max}}{2}$$

$$Y_{1\min} \geq \left(\frac{c_y}{f_y} \right) (\Delta C_{Z_{\max}} - Z_1) + \Delta C_{Y_{\max}} + \frac{s_{\max}}{2}$$

$$X_{1\max} \leq \left(\frac{W_I - c_x}{f_x} \right) (Z_1 - \Delta C_{Z_{\max}}) - \Delta C_{X_{\max}} - \frac{s_{\max}}{2}$$

$$Y_{1\max} \leq \left(\frac{H_I - c_y}{f_y} \right) (Z_1 - \Delta C_{Z_{\max}}) - \Delta C_{Y_{\max}} - \frac{s_{\max}}{2}. \quad (33)$$

When generating ODMD training data in Section 3.3, we cannot select the $Z_{1\min}$ constraint simultaneously with the $X_{1\min}, Y_{1\min}, X_{1\max}, Y_{1\max}$ constraints in (33). Alternatively, we choose a $Z_{1\min}$ value greater than the lower bound for $X_1, Y_1 = 0$ in (32), then randomly select $Z_1 \sim \mathcal{U}[Z_{1\min}, Z_{1\max}]$ for each training example. Once Z_1 is randomly determined, we use (33) to find

$$X_{1\min}(Z_1) = \left(\frac{c_x}{f_x} \right) (\Delta C_{Z_{\max}} - Z_1) + \Delta C_{X_{\max}} + \frac{s_{\max}}{2}$$

$$Y_{1\min}(Z_1) = \left(\frac{c_y}{f_y} \right) (\Delta C_{Z_{\max}} - Z_1) + \Delta C_{Y_{\max}} + \frac{s_{\max}}{2}$$

$$X_{1\max}(Z_1) = \left(\frac{W_I - c_x}{f_x} \right) (Z_1 - \Delta C_{Z_{\max}}) - \Delta C_{X_{\max}}$$

$$- \frac{s_{\max}}{2}$$

$$Y_{1\max}(Z_1) = \left(\frac{H_I - c_y}{f_y} \right) (Z_1 - \Delta C_{Z_{\max}}) - \Delta C_{Y_{\max}}$$

$$- \frac{s_{\max}}{2}, \quad (34)$$

which is the exact solution we use in (16). Notably, in absence of making adjustments for the specific object size or camera movement range of each example, (34) provides the

Table 4. Detailed ODMD Results.

Object Depth Method	Percent Error (20)				
	Mean	Median	Range		Standard Deviation
			Minimum	Maximum	
Normal Set					
DBox _p	1.73	0.96	0.0002	48.21	2.90
DBox _{Abs}	1.11	0.82	0.0004	21.10	1.19
DBox _{NS}	0.54	0.38	0.0001	8.68	0.63
Box _{LS}	0.00	0.00	0.0000	0.00	0.00
DBox _p ^z	12.89	8.54	0.0062	80.74	13.23
Perturb Camera Motion Set					
DBox _p	2.45	1.86	0.0008	23.61	2.28
DBox _{Abs}	2.05	1.55	0.0002	19.45	1.96
DBox _{NS}	3.91	2.93	0.0021	47.94	3.82
Box _{LS}	4.47	3.13	0.0007	43.02	4.57
DBox _p ^z	12.48	8.42	0.0025	74.18	12.18
Perturb Object Detection Set					
DBox _p	2.54	1.54	0.0020	45.94	3.39
DBox _{Abs}	1.75	1.26	0.0007	19.51	1.81
DBox _{NS}	6.35	1.98	0.0005	415.68	19.56
Box _{LS}	21.60	8.90	0.0003	158.04	28.27
DBox _p ^z	15.00	9.83	0.0189	296.31	16.93
Robot Set					
DBox _p	11.17	8.31	0.0022	253.02	13.94
DBox _{Abs}	13.29	9.44	0.0024	223.76	14.90
DBox _{NS}	12.47	8.11	0.0092	656.85	25.03
Box _{LS}	21.23	12.17	0.0010	262.48	26.92
DBox _p ^z	21.96	14.64	0.0099	342.40	26.39

Table 5. Detailed ODMS Results.

Object Depth Method	Percent Error (20)				Standard Deviation
	Mean	Median	Range		
			Minimum	Maximum	
Normal Set					
DBox _p ^z	11.82	8.17	0.0049	167.80	12.42
Box _{LS}	13.66	10.49	0.0025	137.39	11.97
DBox _{NS} ^z	9.20	6.69	0.0048	146.79	9.55
DBox _{Abs} ^z	21.31	11.98	0.0119	451.54	33.95
Perturb Set					
DBox _p ^z	20.34	15.25	0.0008	220.46	19.73
Box _{LS}	36.62	27.76	0.0050	141.85	30.06
DBox _{NS} ^z	31.55	19.95	0.0205	644.55	48.03
DBox _{Abs} ^z	25.49	15.12	0.0033	265.11	30.68
Robot Set					
DBox _p ^z	11.45	6.29	0.0061	418.41	23.81
Box _{LS}	17.62	9.15	0.0011	390.12	34.22
DBox _{NS} ^z	39.25	5.97	0.0082	8778.45	310.94
DBox _{Abs} ^z	20.36	10.28	0.0033	358.86	32.80
Driving Set					
DBox _p ^z	24.84	18.99	0.0323	213.93	22.83
Box _{LS}	33.29	26.50	0.1783	294.91	31.10
DBox _{NS} ^z	37.31	21.43	0.0108	613.14	55.75
DBox _{Abs} ^z	53.13	55.89	0.0878	296.88	26.65

greatest range of initial positions that also guarantees the object is in view for all n observations. Finally, (34) is linear, so we vectorize it for large batches of training examples.

Detailed ODMD and ODMS Results

We provide more comprehensive and detailed ODMD and ODMS results in Tables 4 and 5. Specifically, we provide a more precise mean percent error (20) and include the

Table 6. Detailed ODMD Results (Absolute Error).

Object Depth Method	Absolute Error (35)				
	Mean	Median	Range		Standard Deviation
			Minimum	Maximum	
Normal Set (cm)					
DBox _p	1.42	0.69	0.0002	46.57	2.62
DBox _{Abs}	0.87	0.59	0.0003	11.56	0.95
DBox _{NS}	0.41	0.28	0.0001	8.97	0.53
Box _{LS}	0.00	0.00	0.0000	0.00	0.00
DBox _p ^z	10.30	6.22	0.0040	77.13	11.68
Perturb Camera Motion Set (cm)					
DBox _p	1.93	1.38	0.0005	20.84	1.93
DBox _{Abs}	1.63	1.13	0.0001	17.17	1.65
DBox _{NS}	3.04	2.16	0.0024	41.03	3.12
Box _{LS}	3.44	2.37	0.0005	33.65	3.66
DBox _p ^z	9.92	6.26	0.0022	67.80	10.57
Perturb Object Detection Set (cm)					
DBox _p	2.06	1.11	0.0017	42.25	3.07
DBox _{Abs}	1.39	0.93	0.0005	15.91	1.55
DBox _{NS}	5.01	1.47	0.0004	281.55	15.07
Box _{LS}	17.58	7.08	0.0002	121.45	23.67
DBox _p ^z	11.81	7.12	0.0089	146.10	13.43
Robot Set (cm)					
DBox _p	8.08	5.79	0.0012	260.28	12.06
DBox _{Abs}	8.83	6.71	0.0018	55.83	7.87
DBox _{NS}	9.23	5.57	0.0045	579.77	23.51
Box _{LS}	14.49	8.63	0.0007	197.56	17.70
DBox _p ^z	14.65	10.29	0.0089	161.98	14.69

Table 7. Detailed ODMS Results (Absolute Error).

Object Depth Method	Absolute Error (35)				
	Mean	Median	Range		Standard Deviation
			Minimum	Maximum	
Normal Set (cm)					
DBox _p ^z	3.65	2.70	0.0020	30.77	3.66
Box _{LS}	4.74	3.43	0.0006	55.77	4.81
DBox _{NS} ^z	2.98	2.14	0.0008	82.58	3.74
DBox _{Abs} ^z	5.57	3.98	0.0059	50.57	5.81
Perturb Set (cm)					
DBox _p ^z	7.19	4.46	0.0003	77.16	8.06
Box _{LS}	15.17	8.30	0.0016	79.01	16.45
DBox _{NS} ^z	12.21	5.77	0.0091	295.98	23.27
DBox _{Abs} ^z	6.68	5.20	0.0004	37.75	5.65
Robot Set (cm)					
DBox _p ^z	3.34	1.78	0.0013	88.89	5.94
Box _{LS}	5.21	2.58	0.0005	79.71	10.54
DBox _{NS} ^z	12.06	1.71	0.0019	1634.35	84.61
DBox _{Abs} ^z	5.64	3.04	0.0010	70.20	7.93
Driving Set (m)					
DBox _p ^z	3.63	1.86	0.0031	37.95	5.00
Box _{LS}	5.08	2.35	0.0142	58.07	8.07
DBox _{NS} ^z	5.03	2.37	0.0004	105.97	8.43
DBox _{Abs} ^z	9.05	5.82	0.0046	57.60	9.24

median, range, and standard deviation for each test set.

We also provide ODMD and ODMS results for the absolute error in Tables 6 and 7, which we calculate for each example as

$$\text{Absolute Error} = |Z_n - \hat{Z}_n|, \quad (35)$$

where Z_n and \hat{Z}_n are ground truth and predicted object depth at final camera position \mathbf{p}_n . Notably, we use percent

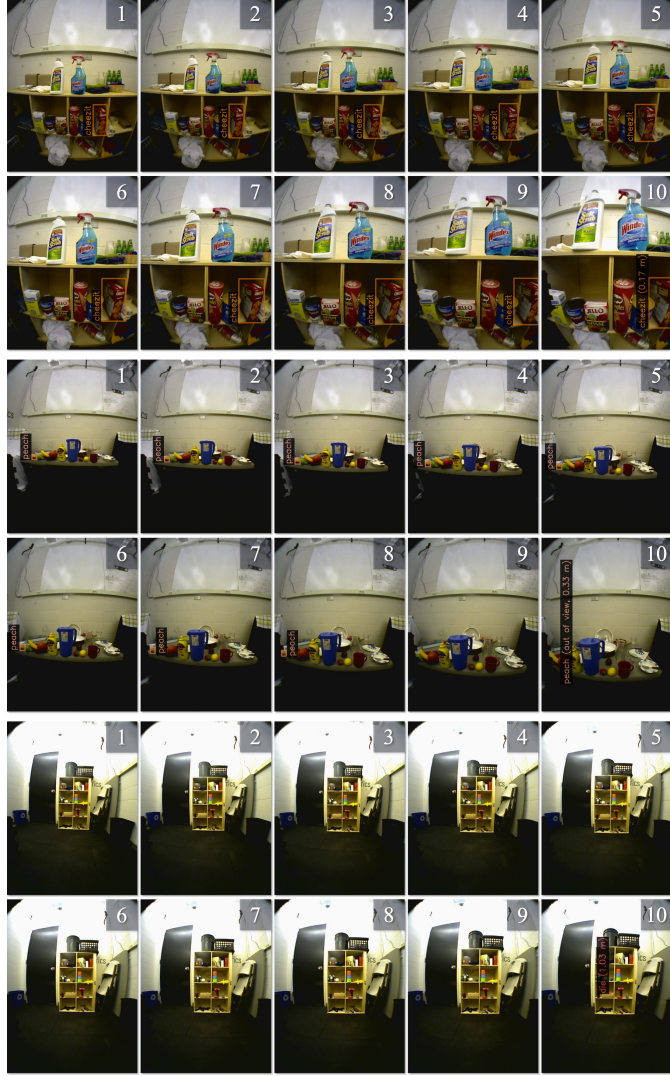


Figure 8. **ODMD Robot Test Set Examples** (best viewed in electronic format). Every two rows show a ten-observation example progressing from left to right, and we show the ground truth object depth in the final image. In the cheezit example (top two rows), the camera perspective changes and the detected object partially leaves view (observations 8-10), causing a distortion to the bounding box shape relative to earlier observations. In the peach example (middle two rows), the detected object completely leaves view during the final two observations (9-10), providing no bounding box information at the prediction location. Finally, for the 16 mm die example (bottom), the camera starts far away from the small object (1), which is not detected until the camera is closer in the final observation (10).

error (20) in the paper to provide a consistent comparison across domains (and examples) with markedly different object depth distances. For example, the 0.10 m absolute error from Figure 7 is a much better result for a camera phone application than it would be for robot grasping.

Object Motion Considerations

The ODMS Driving Set includes moving objects [[15], Section 5.2]. On the other hand, our analytical model in Section 3.1 assumes static objects. Nonetheless, DBox_p^z achieves the current state-of-the-art result on the ODMS Driving Set in Table 2. We attribute DBox_p^z 's success to

training with camera movement perturbations (18). Note that training with these perturbations improves robustness to input errors for the relative distance changes between the camera and object, whether caused by camera motion errors *or* unintended motion of the object itself. In general, objects that move much less than the camera are not an issue.

ODMD Robot Test Set Examples

For the ODMD Robot Test Set, we intentionally select challenging objects and settings that make object detection and depth estimation difficult. To illustrate this point, we show a few example challenges in Figure 8.