

Appendix

9. Proof of Proposition 1

Proof.

Feasibility of iterates We prove

$$\sum_{j \in \mathcal{J}_i} \lambda_i^j + \omega M_{ij} = c_i \quad (7)$$

just before line 7 in Algorithm 1. We do an inductive proof over the number of iterates w.r.t iterations t .

$t = 0$: Follows from $\bar{M} = 0$ and the uniform distribution of costs in line 1 of Algorithm 1.

$t > 0$: Let $\lambda(t-1)$, $M(t-1)$, $\bar{M}(t-1)$ be last iterations' Lagrange multipliers, min-marginals differences and (deferred) min-marginal differences. Also let $\lambda(t)$, $M(t)$ and $\bar{M}(t)$ be the same from current iteration just before line 7. Note that $\bar{M}(t) = M(t-1)$. It holds that

$$\sum_{j \in \mathcal{J}_i} \left[\lambda_i^j(t) + \omega M_{ij}(t) \right] = \sum_{j \in \mathcal{J}_i} \left[\lambda_i^j(t-1) - \omega M_{ij}(t) + \sum_{k \in \mathcal{J}_i} \left(\frac{\omega}{|\mathcal{J}_i|} \bar{M}_{ik}(t) \right) + \omega M_{ij}(t) \right] \quad (8a)$$

$$= \sum_{j \in \mathcal{J}_i} \left[\lambda_i^j(t-1) + \omega M_{ij}(t-1) \right] \quad (8b)$$

$$= c_i. \quad (8c)$$

Non-decreasing Lower Bound In order to prove that iterates have non-decreasing lower bound we will consider an equivalent lifted representation in which proving the non-decreasing lower bound will be easier.

Lifted Representation Introduce $\lambda_i^{j,\beta}$ for $\beta \in \{0, 1\}$ and the subproblems

$$E(\lambda^{j,1}, \lambda^{j,0}) = \min_{x \in \mathcal{X}_j} x^\top \lambda^{j,1} + (1-x)^\top \lambda^{j,0} \quad (9)$$

Then (D) is equivalent to

$$\max_{\lambda^1, \lambda^0} \sum_{j \in \mathcal{J}} E(\lambda^{j,1}, \lambda^{j,0}) \text{ s.t. } \sum_{j \in \mathcal{J}_i} \lambda_i^{j,\beta} = \beta \cdot c_i \quad (10)$$

We have the transformation from original to lifted λ

$$\lambda \mapsto (\lambda^1 \leftarrow \lambda, \lambda^0 \leftarrow 0) \quad (11)$$

and from lifted to original λ (except a constant term)

$$(\lambda^1, \lambda^0) \mapsto \lambda^1 - \lambda^0. \quad (12)$$

It can be easily shown that the lower bounds are invariant under the above mappings and feasible λ for (D) are mapped to feasible ones for (10) and vice versa.

The update rule line 6 in Algorithm 1 for the lifted representation can be written as

$$\lambda_i^{j,\beta} \leftarrow \lambda_i^{j,\beta} - \omega \cdot \min(m_{ij}^\beta - m_{ij}^{1-\beta}, 0) + \omega \cdot \min(\bar{m}_{ij}^\beta - \bar{m}_{ij}^{1-\beta}, 0) \quad (13)$$

It can be easily shown that (13) and line 6 in Algorithm 1 are corresponding to each other under the transformation from lifted to original λ .

Continuation of Non-decreasing Lower Bound Define

$$\lambda_i^{j,\beta} = \lambda_i^j - \omega \cdot \min(m_{ij}^\beta - m_{ij}^{1-\beta}, 0). \quad (14)$$

Then $E(\lambda'^{j,1}, \lambda'^{j,0}) = E(\lambda^{j,1}, \lambda^{j,0})$ are equal due to the definition of min-marginals. Define next

$$\lambda_i''^{j,\beta} = \lambda_i^j + \omega \cdot \min(\bar{m}_{ij}^\beta - \bar{m}_{ij}^{1-\beta}, 0). \quad (15)$$

Then $E(\lambda''^{j,1}, \lambda''^{j,0}) \geq E(\lambda'^{j,1}, \lambda'^{j,0})$ since $\lambda'' \geq \lambda'$. This proves the claim. Another side-benefit of the lifted update scheme (13) is that evaluating $E(\lambda^{j,1}, \lambda^{j,0})$ during the course of optimization in Algorithm 1 always gives a lower bound to the true dual objective calculated after accounting for deferred min-marginal differences. \square

10. Detailed results

10.1. Cell tracking

Table 3. Detailed results of FastDOG on *Cell tracking - small dataset*

	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
MEAN	-4387353	3	-4376244	10.34
DIC-C2DH-HeLA	-3435189	3.5	-3404116	9.43
drosophila	-12972321	1.7	-12971663	2.54
Fluo-C2DL-MS-01	-2702746	7	-2680328	26.49
Fluo-C2DL-MS-02	-2902893	1	-2899711	5.93
Fluo-N2DH-GOWT1-01	-6722103	0.3	-6719992	3.11
Fluo-N2DH-GOWT1-02	-8480162	0.8	-8477563	6.18
Fluo-N2DL-HELA	-4045972	6.5	-4034819	30.97
flywing-11	-98500	3.3	-98280	3.03
PhC-C2DH-U373-01	-1529164	3.7	-1519372	7.12
PhC-C2DH-U373-02	-984484	2.1	-956599	8.58

Table 4. Detailed results of FastDOG on *Cell tracking - large dataset*

	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
MEAN	-154874000	39	-154171050	71.49
flywing-100-1	-101971727	38.9	-101644067	57.78
flywing-100-2	-102464774	41	-102144340	52.43
flywing-245	-386543634	93.5	-383920619	160.53
PhC-C2DL-PSC-01	-96025589	13.1	-95881123	52.82
PhC-C2DL-PSC-02	-87364274	8.7	-87265102	33.89

10.2. Graph matching

Table 5. Detailed results of FastDOG on *Graph matching - hotel dataset*

	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
MEAN	-4293	0.2	-4293	0.01
energy-hotel-frame15frame22	-4598	0.2	-4598	0.01
energy-hotel-frame15frame29	-4540	0.1	-4540	0.01
energy-hotel-frame15frame36	-4481	0.2	-4481	0.01
energy-hotel-frame15frame43	-4377	0.2	-4377	0.01
energy-hotel-frame15frame50	-4294	0.2	-4294	0.01
energy-hotel-frame15frame57	-4244	0.2	-4244	0.01
energy-hotel-frame15frame64	-4172	0.2	-4172	0.01
energy-hotel-frame15frame71	-4135	0.2	-4135	0.01
energy-hotel-frame15frame78	-4036	0.2	-4036	0.01
energy-hotel-frame15frame85	-3985	0.2	-3985	0.01
energy-hotel-frame15frame92	-3898	0.2	-3898	0.01
energy-hotel-frame15frame99	-3860	0.2	-3860	0.01

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	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
energy-hotel-frame1frame15	-4498	0.2	-4498	0.01
energy-hotel-frame1frame22	-4438	0.1	-4438	0.01
energy-hotel-frame1frame29	-4368	0.2	-4368	0.01
energy-hotel-frame1frame36	-4306	0.2	-4306	0.01
energy-hotel-frame1frame43	-4194	0.2	-4194	0.01
energy-hotel-frame1frame50	-4125	0.2	-4125	0.01
energy-hotel-frame1frame57	-4064	0.2	-4064	0.01
energy-hotel-frame1frame64	-4021	0.2	-4021	0.01
energy-hotel-frame1frame71	-3969	0.2	-3969	0.01
energy-hotel-frame1frame78	-3874	0.2	-3874	0.01
energy-hotel-frame1frame85	-3817	0.2	-3817	0.01
energy-hotel-frame1frame8	-4570	0.1	-4570	0.01
energy-hotel-frame1frame92	-3728	0.3	-3728	0.01
energy-hotel-frame1frame99	-3691	0.4	-3691	0.01
energy-hotel-frame22frame29	-4615	0.1	-4615	0.01
energy-hotel-frame22frame36	-4527	0.2	-4527	0.01
energy-hotel-frame22frame43	-4428	0.2	-4428	0.01
energy-hotel-frame22frame50	-4343	0.2	-4343	0.01
energy-hotel-frame22frame57	-4302	0.2	-4302	0.01
energy-hotel-frame22frame64	-4219	0.2	-4219	0.01
energy-hotel-frame22frame71	-4188	0.2	-4188	0.01
energy-hotel-frame22frame78	-4109	0.2	-4109	0.01
energy-hotel-frame22frame85	-4063	0.2	-4063	0.01
energy-hotel-frame22frame92	-3979	0.2	-3979	0.01
energy-hotel-frame22frame99	-3956	0.2	-3956	0.01
energy-hotel-frame29frame36	-4605	0.1	-4605	0.01
energy-hotel-frame29frame43	-4493	0.1	-4493	0.01
energy-hotel-frame29frame50	-4408	0.2	-4408	0.01
energy-hotel-frame29frame57	-4373	0.2	-4373	0.01
energy-hotel-frame29frame64	-4295	0.2	-4295	0.01
energy-hotel-frame29frame71	-4253	0.2	-4253	0.01
energy-hotel-frame29frame78	-4167	0.2	-4167	0.01
energy-hotel-frame29frame85	-4118	0.2	-4118	0.01
energy-hotel-frame29frame92	-4037	0.2	-4037	0.01
energy-hotel-frame29frame99	-4007	0.2	-4007	0.01
energy-hotel-frame36frame43	-4571	0.1	-4571	0.01
energy-hotel-frame36frame50	-4489	0.2	-4489	0.01
energy-hotel-frame36frame57	-4451	0.2	-4451	0.01
energy-hotel-frame36frame64	-4373	0.2	-4373	0.01
energy-hotel-frame36frame71	-4326	0.2	-4326	0.01
energy-hotel-frame36frame78	-4249	0.2	-4249	0.01
energy-hotel-frame36frame85	-4192	0.2	-4192	0.01
energy-hotel-frame36frame92	-4124	0.2	-4124	0.01
energy-hotel-frame36frame99	-4094	0.2	-4094	0.01
energy-hotel-frame43frame50	-4563	0.1	-4563	0.01
energy-hotel-frame43frame57	-4532	0.2	-4532	0.01
energy-hotel-frame43frame64	-4450	0.2	-4450	0.01
energy-hotel-frame43frame71	-4422	0.2	-4422	0.01
energy-hotel-frame43frame78	-4351	0.2	-4351	0.01

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	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
energy-hotel-frame43frame85	-4295	0.2	-4295	0.01
energy-hotel-frame43frame92	-4221	0.2	-4221	0.01
energy-hotel-frame43frame99	-4190	0.2	-4190	0.01
energy-hotel-frame50frame57	-4566	0.1	-4566	0.01
energy-hotel-frame50frame64	-4517	0.2	-4517	0.01
energy-hotel-frame50frame71	-4463	0.1	-4463	0.01
energy-hotel-frame50frame78	-4400	0.2	-4400	0.01
energy-hotel-frame50frame85	-4342	0.2	-4342	0.01
energy-hotel-frame50frame92	-4260	0.2	-4260	0.01
energy-hotel-frame50frame99	-4240	0.2	-4240	0.01
energy-hotel-frame57frame64	-4567	0.2	-4567	0.01
energy-hotel-frame57frame71	-4508	0.2	-4508	0.01
energy-hotel-frame57frame78	-4475	0.2	-4475	0.01
energy-hotel-frame57frame85	-4398	0.2	-4398	0.01
energy-hotel-frame57frame92	-4344	0.2	-4344	0.01
energy-hotel-frame57frame99	-4332	0.2	-4332	0.01
energy-hotel-frame64frame71	-4578	0.1	-4578	0.01
energy-hotel-frame64frame78	-4545	0.2	-4545	0.01
energy-hotel-frame64frame85	-4481	0.1	-4481	0.01
energy-hotel-frame64frame92	-4413	0.2	-4413	0.01
energy-hotel-frame64frame99	-4385	0.2	-4385	0.01
energy-hotel-frame71frame78	-4550	0.2	-4550	0.01
energy-hotel-frame71frame85	-4552	0.2	-4552	0.01
energy-hotel-frame71frame92	-4469	0.1	-4469	0.01
energy-hotel-frame71frame99	-4413	0.2	-4413	0.01
energy-hotel-frame78frame85	-4589	0.1	-4589	0.01
energy-hotel-frame78frame92	-4545	0.1	-4545	0.01
energy-hotel-frame78frame99	-4534	0.2	-4534	0.01
energy-hotel-frame85frame92	-4578	0.1	-4578	0.01
energy-hotel-frame85frame99	-4528	0.1	-4528	0.01
energy-hotel-frame8frame15	-4572	0.2	-4572	0.01
energy-hotel-frame8frame22	-4491	0.2	-4491	0.01
energy-hotel-frame8frame29	-4424	0.2	-4424	0.01
energy-hotel-frame8frame36	-4379	0.2	-4379	0.01
energy-hotel-frame8frame43	-4262	0.2	-4262	0.01
energy-hotel-frame8frame50	-4179	0.2	-4179	0.01
energy-hotel-frame8frame57	-4131	0.2	-4131	0.01
energy-hotel-frame8frame64	-4060	0.2	-4060	0.01
energy-hotel-frame8frame71	-4021	0.2	-4021	0.01
energy-hotel-frame8frame78	-3931	0.2	-3931	0.01
energy-hotel-frame8frame85	-3877	0.2	-3877	0.01
energy-hotel-frame8frame92	-3802	0.3	-3802	0.01
energy-hotel-frame8frame99	-3762	0.3	-3762	0.02
energy-hotel-frame92frame99	-4593	0.1	-4593	0.01

Table 6. Detailed results of FastDOG on *Graph matching - house* dataset

	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
MEAN	-3778	0.4	-3778	0.01
energy-house-frame10frame100	-3720	0.6	-3720	0.01
energy-house-frame10frame95	-3809	0.4	-3809	0.01
energy-house-frame10frame96	-3786	0.4	-3786	0.01
energy-house-frame10frame97	-3748	0.4	-3748	0.01
energy-house-frame10frame98	-3766	0.4	-3766	0.01
energy-house-frame10frame99	-3728	0.5	-3728	0.01
energy-house-frame11frame100	-3739	0.5	-3739	0.01
energy-house-frame11frame101	-3748	0.4	-3748	0.01
energy-house-frame11frame96	-3809	0.4	-3809	0.01
energy-house-frame11frame97	-3748	0.4	-3748	0.01
energy-house-frame11frame98	-3781	0.4	-3781	0.01
energy-house-frame11frame99	-3736	0.4	-3736	0.02
energy-house-frame12frame100	-3768	0.5	-3768	0.01
energy-house-frame12frame101	-3775	0.5	-3775	0.01
energy-house-frame12frame102	-3783	0.5	-3783	0.01
energy-house-frame12frame97	-3780	0.5	-3780	0.01
energy-house-frame12frame98	-3807	0.5	-3807	0.01
energy-house-frame12frame99	-3766	0.5	-3766	0.01
energy-house-frame13frame100	-3749	0.5	-3749	0.01
energy-house-frame13frame101	-3773	0.4	-3773	0.01
energy-house-frame13frame102	-3775	0.4	-3775	0.01
energy-house-frame13frame103	-3749	0.4	-3749	0.01
energy-house-frame13frame98	-3798	0.4	-3798	0.01
energy-house-frame13frame99	-3754	0.4	-3754	0.01
energy-house-frame14frame100	-3785	0.5	-3785	0.01
energy-house-frame14frame101	-3796	0.4	-3796	0.01
energy-house-frame14frame102	-3806	0.4	-3806	0.01
energy-house-frame14frame103	-3769	0.4	-3769	0.01
energy-house-frame14frame104	-3761	0.5	-3761	0.01
energy-house-frame14frame99	-3788	0.4	-3788	0.01
energy-house-frame15frame100	-3784	0.5	-3784	0.01
energy-house-frame15frame101	-3796	0.4	-3796	0.01
energy-house-frame15frame102	-3798	0.4	-3798	0.01
energy-house-frame15frame103	-3774	0.4	-3774	0.01
energy-house-frame15frame104	-3762	0.4	-3762	0.01
energy-house-frame15frame105	-3745	0.5	-3745	0.01
energy-house-frame16frame101	-3804	0.4	-3804	0.01
energy-house-frame16frame102	-3815	0.4	-3815	0.01
energy-house-frame16frame103	-3787	0.3	-3787	0.01
energy-house-frame16frame104	-3774	0.4	-3774	0.01
energy-house-frame16frame105	-3752	0.4	-3752	0.01
energy-house-frame17frame102	-3820	0.3	-3820	0.01
energy-house-frame17frame103	-3799	0.3	-3799	0.01
energy-house-frame17frame104	-3774	0.4	-3774	0.01
energy-house-frame17frame105	-3756	0.4	-3756	0.01
energy-house-frame18frame103	-3821	0.4	-3821	0.01
energy-house-frame18frame104	-3794	0.4	-3794	0.01

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	LB ↑	LB time [s]	UB ↓	UB time [s]
energy-house-frame18frame105	-3777	0.4	-3777	0.01
energy-house-frame19frame104	-3799	0.4	-3799	0.01
energy-house-frame19frame105	-3767	0.4	-3767	0.01
energy-house-frame1frame86	-3833	0.3	-3833	0.01
energy-house-frame1frame87	-3808	0.3	-3808	0.01
energy-house-frame1frame88	-3758	0.3	-3758	0.01
energy-house-frame1frame89	-3776	0.4	-3776	0.01
energy-house-frame1frame90	-3710	0.4	-3710	0.01
energy-house-frame1frame91	-3761	0.4	-3761	0.01
energy-house-frame20frame105	-3772	0.4	-3772	0.01
energy-house-frame2frame87	-3837	0.3	-3837	0.01
energy-house-frame2frame88	-3807	0.3	-3807	0.01
energy-house-frame2frame89	-3807	0.3	-3807	0.01
energy-house-frame2frame90	-3766	0.4	-3766	0.01
energy-house-frame2frame91	-3791	0.4	-3791	0.01
energy-house-frame2frame92	-3753	0.4	-3753	0.01
energy-house-frame3frame88	-3808	0.4	-3808	0.01
energy-house-frame3frame89	-3815	0.4	-3815	0.01
energy-house-frame3frame90	-3761	0.4	-3761	0.01
energy-house-frame3frame91	-3808	0.4	-3808	0.01
energy-house-frame3frame92	-3769	0.5	-3769	0.01
energy-house-frame3frame93	-3763	0.5	-3763	0.01
energy-house-frame4frame89	-3826	0.4	-3826	0.01
energy-house-frame4frame90	-3772	0.4	-3772	0.01
energy-house-frame4frame91	-3813	0.4	-3813	0.01
energy-house-frame4frame92	-3769	0.5	-3769	0.01
energy-house-frame4frame93	-3770	0.5	-3770	0.01
energy-house-frame4frame94	-3781	0.4	-3781	0.01
energy-house-frame5frame90	-3757	0.4	-3757	0.01
energy-house-frame5frame91	-3801	0.4	-3801	0.01
energy-house-frame5frame92	-3759	0.4	-3759	0.01
energy-house-frame5frame93	-3764	0.4	-3764	0.01
energy-house-frame5frame94	-3765	0.4	-3765	0.01
energy-house-frame5frame95	-3773	0.4	-3773	0.01
energy-house-frame6frame91	-3824	0.4	-3824	0.01
energy-house-frame6frame92	-3779	0.4	-3779	0.01
energy-house-frame6frame93	-3780	0.4	-3780	0.01
energy-house-frame6frame94	-3787	0.4	-3787	0.01
energy-house-frame6frame95	-3794	0.5	-3794	0.01
energy-house-frame6frame96	-3770	0.5	-3770	0.01
energy-house-frame7frame92	-3764	0.5	-3764	0.01
energy-house-frame7frame93	-3768	0.4	-3768	0.01
energy-house-frame7frame94	-3771	0.4	-3771	0.01
energy-house-frame7frame95	-3780	0.5	-3780	0.01
energy-house-frame7frame96	-3755	0.5	-3755	0.01
energy-house-frame7frame97	-3712	0.5	-3712	0.01
energy-house-frame8frame93	-3787	0.5	-3787	0.01
energy-house-frame8frame94	-3794	0.5	-3794	0.01
energy-house-frame8frame95	-3800	0.5	-3800	0.01

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	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
energy-house-frame8frame96	-3781	0.5	-3781	0.01
energy-house-frame8frame97	-3738	0.5	-3738	0.01
energy-house-frame8frame98	-3759	0.6	-3759	0.01
energy-house-frame9frame94	-3806	0.3	-3806	0.01
energy-house-frame9frame95	-3811	0.4	-3811	0.01
energy-house-frame9frame96	-3797	0.4	-3797	0.01
energy-house-frame9frame97	-3748	0.4	-3748	0.01
energy-house-frame9frame98	-3770	0.5	-3770	0.01
energy-house-frame9frame99	-3726	0.5	-3726	0.01

Table 7. Detailed results of FastDOG on *Graph matching - worms* dataset

	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
MEAN	-48934	51.4	-48316	2.36
worm01-16-03-11-1745	-46447	38.2	-46209	5.98
worm02-16-03-11-1745	-50003	24.6	-49994	0.14
worm03-16-03-11-1745	-50566	26.5	-50525	0.96
worm04-16-03-11-1745	-49372	59.2	-48973	1.95
worm05-16-03-11-1745	-49914	40.6	-49224	2.83
worm06-16-03-11-1745	-50497	35.8	-50442	1.83
worm07-16-03-11-1745	-49748	50.5	-46552	5.36
worm08-16-03-11-1745	-49517	38.2	-49131	2.58
worm09-16-03-11-1745	-45431	176.3	-44986	1.85
worm10-16-03-11-1745	-46972	103	-41550	7.14
worm11-16-03-11-1745	-48868	76.1	-48582	0.86
worm12-16-03-11-1745	-51032	49.9	-50191	3.19
worm13-16-03-11-1745	-46243	66.6	-45661	2.44
worm14-16-03-11-1745	-47636	39.4	-46479	4.3
worm15-16-03-11-1745	-49573	35.6	-49540	0.25
worm16-16-03-11-1745	-48948	39.1	-48277	2.49
worm17-16-03-11-1745	-48175	48.2	-48003	2.72
worm18-16-03-11-1745	-48390	24	-48201	2.76
worm19-16-03-11-1745	-48966	44.1	-48694	1.47
worm20-16-03-11-1745	-49536	38	-49353	2.63
worm21-16-03-11-1745	-49893	68.2	-49801	1.4
worm22-16-03-11-1745	-48233	48	-47888	2.59
worm23-16-03-11-1745	-50020	34.9	-49955	1.94
worm24-16-03-11-1745	-49679	30.5	-49233	1.61
worm25-16-03-11-1745	-47312	101.6	-47138	1.3
worm26-16-03-11-1745	-47537	47.6	-45603	5.14
worm27-16-03-11-1745	-50075	41.2	-50042	0.33
worm28-16-03-11-1745	-49516	25.5	-49470	0.8
worm29-16-03-11-1745	-50110	66.1	-50052	0.81
worm30-16-03-11-1745	-49804	24.3	-49730	1.16

Table 8. Detailed results of FastDOG on *MRF - color-seg* dataset

	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
MEAN	308472094	11.1	308474165	3.32
colseg-cow3	455392703	12.6	455394996	4
colseg-cow4	451816564	19.9	451820311	5.78
colseg-garden4	18207016	0.9	18207187	0.17

Table 9. Detailed results of FastDOG on *MRF - color-seg-n4* dataset

	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
MEAN	20011	8.5	20017	0.19
clownfish-small	14817	4.2	14818	0.26
crops-small	11923	5.5	11926	0.31
fourcolors	69520	0.5	69529	0.04
lake-small	14311	5.3	14313	0.19
palm-small	12236	17.6	12239	0.3
penguin-small	8234	7.3	8239	0.11
pfau-small	24231	21.5	24255	0.22
snail	13104	1	13105	0.04
strawberry-glass-2-small	11725	13.7	11728	0.23

Table 10. Detailed results of FastDOG on *MRF - color-seg-n8* dataset

	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
MEAN	19990	13	19995	0.39
clownfish-small	14794	7.3	14795	0.41
crops-small	11853	9.4	11854	0.66
fourcolors	69550	0.7	69564	0.08
lake-small	14327	7.7	14329	0.31
palm-small	12253	26.7	12256	0.66
penguin-small	8258	11.8	8260	0.23
pfau-small	24008	31.9	24026	0.79
snail	13105	2.2	13106	0.09
strawberry-glass-2-small	11766	19.5	11768	0.32

Table 11. Detailed results of FastDOG on *MRF - object-seg* dataset

	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
MEAN	31317	45.3	31323	0.14
objseg-349	2369	62.8	2386	0.21
objseg-353	40634	103.5	40646	0.37
objseg-358	38019	12.9	38019	0.01
objseg-416	38160	20.7	38160	0.01
objseg-552	37402	26.8	37402	0.1

10.4. QAPLib

Table 12. Detailed results of FastDOG on QAPLib - small dataset

	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
MEAN	3747352	89.4	43303812	48.53
bur26a	5313365	58.3	5710342	20.91
bur26b	3712826	59.3	4002866	22.82
bur26c	5314069	59.6	5886359	23.79
bur26d	3710949	59.7	4200747	25.05
bur26e	5313329	58.3	5839758	22.21
bur26f	3710993	59.9	4278500	24.92
bur26g	9977748	61.2	11109652	22.59
bur26h	6970285	59.8	7795328	24.23
chr12a	7916	21.7	12480	4.02
chr12b	6059	22.5	10162	4.55
chr12c	9791	22.2	12610	3.79
chr15a	7727	26.6	10606	4.56
chr15b	3759	26.4	11240	6.41
chr15c	8658	26.3	10282	2.8
chr18a	6637	29.8	14568	7.1
chr18b	1514	30.5	1814	2.46
chr20a	2155	30.7	4406	5.33
chr20b	2224	10.1	4576	5.38
chr20c	8319	33.6	28882	8.4
chr22a	5898	36.1	7116	5.84
chr22b	6071	31.5	6902	3.85
chr25a	3077	38.9	5290	6.31
els19	9712306	37.3	23805836	14.55
esc16a	0	26.9	82	1.42
esc16b	0	28.4	310	2.43
esc16c	0	27.6	190	1.73
esc16d	0	27.5	24	0.63
esc16e	0	26.5	32	1.44
esc16f	0	25.9	0	0.01
esc16g	0	26.5	40	0.7
esc16h	299	30.3	1146	3.69
esc16i	0	26.6	22	0.41
esc16j	0	27.1	12	0.6
esc32a	0	56.2	368	4.55
esc32b	0	59.7	400	34.48
esc32c	0	60.8	850	10.38
esc32d	0	58.3	310	3.65
esc32e	0	48.3	2	0.19
esc32f	0	48.9	2	0.2
esc32g	0	47.4	8	1.03
esc32h	0	61.4	622	5.49
had12	1368	21.8	1814	3.28
had14	2027	24.1	3172	4.2
had16	2468	29.8	3998	6.51
had18	3425	34.5	5856	8.38

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	LB ↑	LB time [s]	UB ↓	UB time [s]
had20	4757	39.3	7634	10.15
kra30a	36471	88	144480	38.78
kra30b	36460	85.6	134300	37.48
kra32	7281	63.5	24532	13.79
lipa20a	2840	39	3852	6.49
lipa20b	16950	37.8	34066	13.07
lipa30a	10401	86.8	13825	18.69
lipa30b	82583	82.5	196200	41.01
lipa40a	23604	267.7	32767	135.8
lipa40b	244077	264.2	621993	140.02
lipa50a	50567	523.8	64059	388.58
lipa50b	481647	514.1	1569702	437.65
nug12	350	21.7	720	3.47
nug14	540	25.2	1312	4.55
nug15	598	27.5	1420	7.07
nug16a	768	29.7	2114	7.12
nug16b	655	29.5	1576	5.78
nug17	826	31.4	2198	7.81
nug18	903	34.5	2524	8
nug20	1158	40.4	3118	9.73
nug21	1034	42.7	3108	12.93
nug22	1201	45.3	5322	15.3
nug24	1448	51.6	4824	14.23
nug25	1515	55.2	5022	18.59
nug27	1873	56.4	7082	34.42
nug28	1926	59.7	6836	35.77
nug30	2239	85.6	8100	31.31
rou12	136732	22.4	303114	7.16
rou15	163890	28.3	436650	10.29
rou20	279928	39.2	920170	17.66
scr12	23784	21.7	41334	5.45
scr15	39380	25.8	79850	7.76
scr20	73910	35	188508	11.9
sko42	4665	312.7	20198	163.27
sko49	6254	497.2	29036	403.01
ste36a	5261	202.8	25616	98.78
ste36b	5261	200.2	100400	97.95
ste36c	5263021	200.2	19722744	125.68
tai10a	71410	19.3	162712	6.37
tai10b	971188	19.3	1235947	4.41
tai12a	120215	21.9	306868	7.37
tai12b	13788115	21.9	56755459	7.95
tai15a	154314	27.3	472096	10.77
tai15b	51183141	27.6	52878241	4.94
tai17a	184366	32	598290	13.21
tai20a	170523	39	889832	17.48
tai20b	24686130	40.1	195231805	21.36
tai25a	326908	56.4	1455224	27.28
tai25b	73310534	56.2	548164198	29.84

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	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
tai30a	440217	85.8	2197658	44.54
tai30b	48043384	85.6	1295280184	60.82
tai35a	425910	181.3	2947642	99.65
tai35b	31126323	185.1	504198900	135.18
tai40a	511712	266.8	3735636	148.4
tai40b	51867467	267.9	1016709833	210.71
tai50a	694972	518.8	5929038	373.43
tai50b	34412453	528.3	759613745	429.98
tho30	50192	82.8	193392	39.02
tho40	79120	264	328136	144.48
wil50	11341	523.5	55572	444.26

Table 13. Detailed results of `FastDOG` on *QAPLib* - large dataset

	LB \uparrow	LB time [s]	UB \downarrow	UB time [s]
MEAN	8924162	2713.6	137566865	4215.28
esc128	0	373.9	192	9.79
esc64a	0	121.9	224	3.91
lipa60a	85472	1203.7	110137	836.39
lipa60b	946111	1197.5	3262713	814.44
lipa70a	130413	2142.2	173573	1597.51
lipa70b	1673026	3539	6000823	3408
lipa80a	215340	3545	258341	2823.49
lipa80b	2242309	5967.7	10172855	6052
lipa90a	245528	3547	367338	4727.65
lipa90b	3828710	3547.3	16340078	4932.2
sko100a	26946	3514.4	177118	8309.74
sko100b	27213	3511.4	180416	8423.26
sko100c	26378	3514.8	173824	8265.25
sko100d	26524	3514.9	175804	8618.05
sko100e	26619	3514.6	176208	8642.08
sko100f	26315	3515.1	173978	8693.04
sko56	8628	968.2	41544	631.11
sko64	11139	1569.9	58882	1156.87
sko72	14205	2478.6	79146	1894.4
sko81	18262	3561.4	108628	3111.76
sko90	21746	3541.3	137230	5076.87
tai100a	1412256	3507.9	23975244	7753.14
tai100b	132097785	3514.7	1746517724	7731.76
tai60a	1230709	1209.9	8410894	774.26
tai60b	42883035	1240.6	997899804	819.49
tai64c	488264	186.7	2212584	39.62
tai80a	1242459	3563.6	15733802	2964.97
tai80b	69799747	3563.6	1156222051	3622.35
wil100	45565	3516.5	297916	10509.61