Topology Preserving Local Road Network Estimation from Single Onboard Camera Image - Supplementary Material

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1. Code

The github page: https://github.com/ybarancan/TopologicalLaneGraph

2. Theory

2.1. Assumptions

In this subsection, we provide the additional assumptions we make on top of the ones listed in the main paper.

A curve cannot appear more than once in a minimal cycle. This holds because the divergence and convergence of the lanes result in centerlines with different identities.

Direction assumption. Let the direction of a minimal cycle MC_i be defined as $D(MC_i) \in \{0, 1\}$, i.e. 0=clockwise and 1= counter-clockwise. It induces an ordering of the segments in the minimal cycle relative to an intersection point P in the cycle such that $u_i > u_j$ for any two segment u_i and u_j in the minimal cycle if u_i appears later than u_j when traversing the cycle starting from P in the direction $D(MC_i)$. Similarly an intersection point $p_i > p_j$ if p_i appears later than p_j.

Consider two minimal cycles MC_i and MC_j where at least one curve U_n appears in both cycles. The segment of U_n in MC_i is u_n and the segment in MC_j is u'_n. Since a curve is continuous there is a path connecting u_n to u'_n such that it only consists of U_n. Let this path be $VG(u_n) = VG(u'_n)$ and the intersection point that this path intersects MC_i be $PP(u_n)$ and the point it intersects in MC_j be $PP(u'_n)$. The other intersection points that define their respective segments is $K(u_n)$ and $K(u'_n)$ respectively.

We define that two minimal cycles are neighbors if they share at least one intersection point. Based on this, we have the following property (direction property): Let $P = PP(u_n)$ and $D(MC_i)$ be the direction from $PP(u_n)$ to $K(u_n)$ through u_n. Then $D(MC_j) \neq D(MC_i)$ and there exists a path from $K(u_n)$ to $PP(u'_n)$ entirely on two minimal cycles and following the direction of the minimal cycle it is in such that it does not intersect U_n except at $K(u_n)$ and $PP(u'_n)$. If MC_i and MC_j are not neighbors, then the path can teleport to MC_j at any intersection point other than $PP(u'_n)$ or $K(u'_n)$.

For minimal cycles with shared segments or no neighborhood at all, this means: $\exists \quad (D(MC_i), D(MC_j)) \quad | \quad D(MC_i) \neq D(MC_j) \;\&\; \forall \; PP(u_n) \in \{U_n \cap MC_i\}, \forall K(u_n) \in \{U_n \cap MC_i\} \; ; \; PP(u_n) \lesseqgtr K(u_n) \implies PP(u'_n) \lesseqgtr K('_n)$. This means there exists a tuple of opposing directions such that for all the shared curves in both minimal cycles, if the port point of the curve in a cycle appears later/earlier than its corresponding non-port point, the same order has to be preserved in the other cycle. Note that, this holds for all reference points that the directions are defined on. Moreover, negation of both directions does not affect the condition. Therefore, for any arbitrarily assigned $D(MC_i)$, $D(MC_j)$ such that $D(MC_i) \neq D(MC_j)$ has to support this condition. In Fig 1 subfigure i-a, we focus on shared curve C_1 in minimal cycles A and C. These minimal cycles are not neighbors. The port point of C_1 in A, $PP(c_1)$, is the blue dot and the port point in C, $PP(c'_1)$, is the red dot. Thus, the path should be from green dot , $K(c_1)$, to red dot, $PP(c'_1)$. The path is shown with green arrows while the directions of the minimal cycles are shown by the black arrows surrounding the letters.

If two minimal cycles share intersection points but no segments, the condition is trivially satisfied for the curves at these intersection points since $PP(u'_n) = PP(u_n)$ and the path is simply either of the minimal cycles. In Fig 1 subfigure i-b, this case is shown. The port point of C_3 in D, $PP(c_3)$, is the same point as $PP(c'_3)$ and represented by the red dot. While the green dots represent the $K(c'_3)$ in E and $K(c_3)$ in D. The green arrows in D show the path where the beginning point is $K(c_3)$. The arrow in E shows the path when the beginning point is $K(c'_3)$. In both cases, the entire paths are in either of the minimal cycles, hence obeying the assumption.

This assumption is suitable for our case since the lanes tend to have limited curvature. Therefore, for a minimal cycle that violates this assumption, the curves need to have significantly varying curvatures. An example with two minimal cycles with same minimal covers violating direction assumption is given in Fig. 2.

Accessibility assumption. This assumption states that for all pairs of minimal cycles, it is possible to draw two artificial curves such that the curves start at different intersection points on one cycle and end on different intersection points on the other cycle. The curves cannot intersect and the resulting area bounded by the two curves and two minimal cycles is not intersected by any other curves that are in both of the minimal cycles except on the cycles. In other words, pick two minimal cycles and remove any curve that does not appear in any of those two cycles. Then, if a curve appears in only one of the cycles, only keep the segment of the curve that is in one of the considered minimal cycles. If a curve appears in both cycles, keep its segments in both cycles as well as its VG. In the remaining structure it is possible to draw two paths from two distinct intersection points on one cycle to two distinct intersection points on the other cycle such that these two paths do not intersect and no curve intersects the area bounded by the these two artificial curves and the minimal cycles. This property is trivially satisfied by the minimal cycles that share a segment since the two endpoints of a shared segment serve as two distinct points on both cycles.

We show some example cases for this assumption in Fig 1 subfigure ii. In a) we see the complete lane graph. In b) we show the mentioned area with shaded region when the minmal cycles A and D are selected. Since these cycles share an intersection point, it can act as one of the two paths needed to enclose the area. The other path is from green curve to blue curve. In c), we see the case if we focus on minimal cycles A and C. Since the gray curve is not in any of the minimal cycles in focus, we simply ignore it. Moreover, since red and blue curves only appear in one of the cycles, we ignore their segments that are not in the minimal cycles A and C. However, black and green curves appear in both cycles, thus all of their segments are included. The proposed area is the shaded region that satisfies the assumption.

This assumption is suitable for our case since merging or divergence of lanes are represented with different road segments that correspond to different curves in our formulation. This assumption's underlying intuition is that curves are short and limited in their change in curvature. .

Assumption 1. *Any two curves can intersect at most once.*

Assumption 2. *No curve can intersect with itself.*

Assumption 3. *No curve is floating, i.e. every curve is **connected** (hence intersecting) with another curve in its start and endpoints.*

Assumption 4. *Curves can appear in one minimal cycle at most once.*

Assumption 5. *Direction assumption*

Assumption 6. *Accessibility assumption*

Lemma 2.1. *A minimal closed polycurve (minimal cycle) MC is uniquely identified by its minimal cover B*

Proof. Assume there is more than one minimal cycles defined by the same minimal cover and let the first minimal cycle be MC_1 with $D(MC_1)$ and the second MC_2 with $D(MC_2)$. There are two scenarios: the cycles share at least one segment or not. Let's start with the former case.

For proving the shared segment case of the lemma, let $D(MC_1) \neq D(MC_2)$. Pick the intersection point where the last shared segment ends (end defined by $D(MC_1)$) as the reference point. Segments $u_0, ..., u_N$ refer to the segments other than the shared ones in MC_1 with the order induced by the reference point and $D(MC_1)$. The curves these segments belong to are $U_0, ..., U_N$. Similarly, $w_0, ..., w_M$ are the segments in MC_2 with the same reference point but $D(MC_2)$ with the curves $W_0, ..., W_M$. Note that this means u_0 and w_0 are neighbors as well as u_N and w_M. Assume $W_0 \neq U_0$ and $W_j = U_0$ where $j > 1$ since W_0 and U_0 already intersect. We begin by deciding whether $PP(u_0) > K(u_0)$ or not. Let $PP(u_0) > K(u_0)$. From the **Assumption 5**, if $VG(u_0)$ follows $D(MC_1)$, $VG(u_0)$ covers the space around $u_0, ...u_N, w_M, ...u'_0$ and otherwise $u_0, w_0, ...u'_0$. That is, since $PP(u_0) > K(u_0)$, $PP(u'_0) > K(u'_0)$. In both cases, $VG(w_0)$ has to intersect with $VG(u_0)$ to connect W_0 with MC_1. This is true because MC_1 and MC_2 are minimal cycles and thus, no path can pass through these cycles. See Fig 3 for visualization. Since W_0 and U_0 are neighbors, intersection of $VG(w_0)$ and $VG(u_0)$ would mean W_0 and U_0 intersecting twice, thus violating **Assumption 1**. Therefore, $PP(u_0) < K(u_0)$. In this case, $PP(u_0) = PP(w_0)$ to avoid intersection twice. This means they share the port points. Moreover, since a curve cannot appear more than once in a minimal cycle (**Assumption 4**), we know that $N = M$.

Statement 1: Consider two segments u_i and u_{i+1} in MC_1. If they do not share a port point, $u'_{i+1} > u'_i$. The reason is $VG(u_i)$ either covers $[u_i, ..., u_0, w_0, ..., u'_i]$ or $[u_i, ..., u_{|U|-1}, w_{|W|-1}, ..., u'_i]$ based on **Assumption 5**. In either case, $VG(u_{i+1})$ cannot reach $[u_i, ..., w_0, ..., u'_i]$, without intersecting $VG(u_i)$ since the area in MC_1 and MC_2 cannot be intersected because they are minimal cycles. Note that this holds if MC_1 and MC_2 share at least one segment.

Now consider u_1; We know that u_0 and u_1 do not a share port point because u_0 already has a port point with its inter-

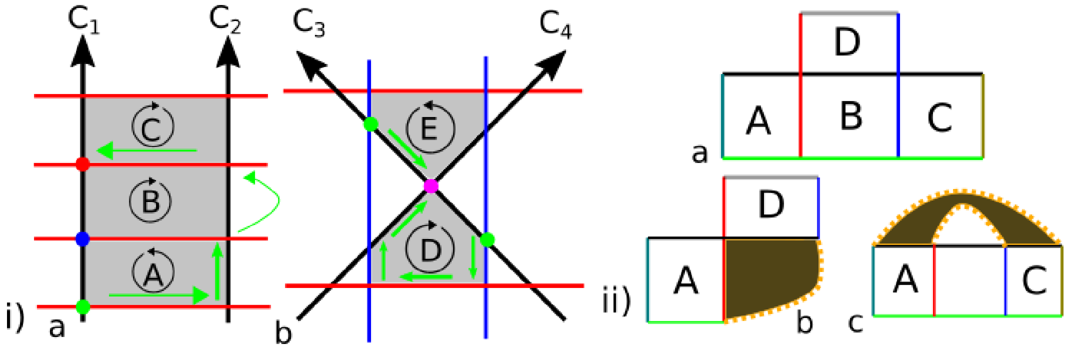

Figure 1. i) The direction property for 2 most common types of minimal cycles. a) We consider cycles A and C with focus on curve C_1. Green dot represents $K(C_1)$, red dot represents $PP(C_1')$ and blue dot is $PP(C_1)$. The green arrows show the path from green dot to red dot, teleporting from A to C over B. In b), we show how the shared intersection point trivially satisfies the property. We focus on C_3 where green dots show $K(C_3)$ and $K(C_3')$. In this example, $PP(C_3')$ = $PP(C_3)$, thus red and blue dots coincide and shown in purple. ii) Accessibility property and the resulting areas (shaded) for 2 pairs of minimal cycles (A-D) and (A-C). For A-D, the shared intersection point acts as the first artificial path.

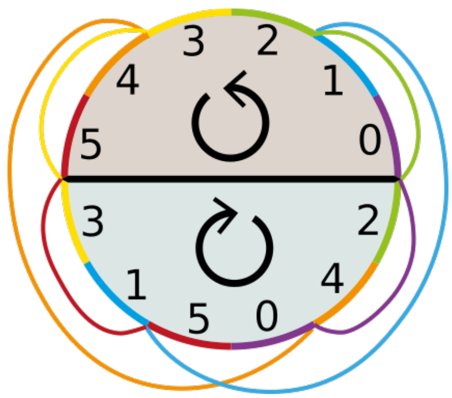

Figure 2. The **simplest** counter example with shared segments if direction assumption is violated. Obviously, it is very unlikely that such a structure would occur in a lane graph. The orange curve (curve 4) and blue curve (curve 1) violate the assumption.

section with w_0. Having two port points would mean U_0 itself is a cycle which is not allowed by **Assumption 2**. From Statement 1, we know that $u_1' > u_0'$. If $PP(u_1) > K(u_1)$, $PP(u_1') > K(u_1')$. Therefore, $u_1' - 1$ (the neighbor of u_1' that appears just before u_1') cannot share a port point with u_1'. This means from Statement 1, $(u_1' - 1)'$ has to appear before u_1. However, only u_0 appears before u_1 and U_0 and U_1 cannot be neighbors in MC_2 since they are already neighbors in MC_1. Therefore, we reach the conclusion that $PP(u_1) < K(u_1)$.

Since $PP(u_1) < K(u_1)$, U_2 has to appear after U_1. With a similar argument as U_1, $PP(u_2) > K(u_2)$ is impossible because it would require its neighbor with a lower order in MC_2 to either be U_0 or U_1 to avoid intersecting $VG(u_2)$ twice. This means $PP(u_2) < K(u_2)$. Iteratively applying this argument, we reach that $u_N' > u_{N-1}' > ... > u_1' > u_0'$. This means the same order has to be preserved which indicates neighbors in MC_1 are also neighbors in MC_2 violating the rule of at most one intersection between any two curves.

Now let $W_0 = U_0$. The same argument carries over. Simply remove U_0 from U and W_0 from W and apply the same procedure as the case $W_0 \neq U_0$.

No shared segment. We know that there exists an area between two cycles such that no shared curves intersect from **Assumption 6**. Since MC_1 and MC_2 have the same list of curves, all curves are shared. Therefore, no curve in either MC_1 or MC_2 intersects this area. This implies, MC_1 and MC_2 can be essentially considered as neighbors. We can apply the same arguments as used in case with shared segment. For this, consider the area that no curve can intersect **(Assumption 6)** and the part of MC_1 and MC_2 it intersects. Let the curve it intersects on MC_1 be U_0 and correspondingly W_0 in MC_2. Moreover, let the intersection region of this artificial curve and MC_1 be an artificial segment a and correspondingly a' in MC_2 with this artificial curve being A. We know $PP(u_0) \not> a$ from the shared segment case. The reason being $u_0' - 1$ would be completely cut off from MC_1 by $VG(U_0)$ and A. Thus, $PP(u_0) < a$. Similar to the shared segment case, this means u_1' appears after u_0' in MC_2. Moreover, $PP(u_1) > K(u_1)$ would imply that $u_1' - 1$ would be completely cut-off from MC_1. Therefore $PP(u_1) < K(u_1)$, which implies u_2' appears after u_1'. The same iterative argument shows that $u_N' > u_{N-1}' > ... > u_1' > u_0'$.

□

Lemma 2.2. *Let a set of curves C_1 and the induced intersection orders I_1 form the structure $T_1 = (C_2, I_2)$. Apply-*

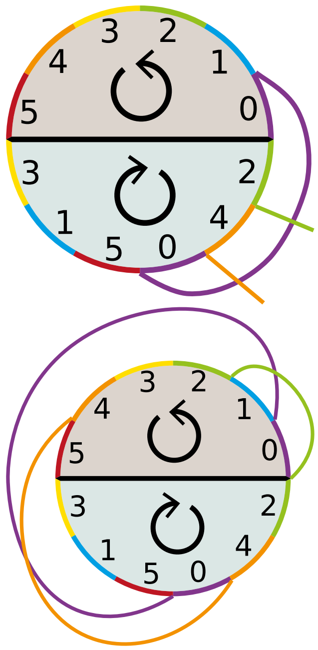

Figure 3. The illustration of the proof for Lemma 1.1. Due to the direction assumption, curve 0 blocks curve 4 and curve 2 accessing the upper minimal cycle (MC_1 in the proof). This situation happens regardless of the route $VG(U_0)$ takes (here it is the purple path connecting the curve 0 between two minimal cycles).

ing any deformations on the curves in C_1 (but no removal or addition of curves) and the resulting induced intersection orders create $T_2 = (C_2, I_2)$. $I_1 = I_2 \iff MC_1 == MC_2$. In other words, the global intersection orders of the two structures are same if and only if the set of minimal cycles are same.

Proof. Let's begin by proving the forward statement. Consider a MC in G_1 and the set of intersection points and the curves creating this MC. Since G_2 has the same I as G_1, a closed polycurve CC in G_2 can be formed by the same set of intersection points and curves. Assume some curve C_p intersects the area of closed polycurve CC and hence CC is not a minimal cycle. We know that no curve is floating from **Assumption 3** so C_p has to intersect some other curve either in CC or on CC. Either way, this means alteration in I of at least 2 curves.

For the opposite direction, consider a pair of identical MCs (one in G_1 and one in G_2). The cycles are formed by the same curve segments by definition. The curve segments are, in turn, defined by the intersection points on their respective curves. Since we know no curve is removed or added, the identical minimal cycles in the structures are formed by the identical intersection points. Since minimal cycles create a partition of the space, each curve segment has to appear in at least one minimal cycle. Thus, the set of intersection points $I_1 = I_2$. □

3. Architectures

Here, we provide some details regarding the architectures.

3.1. MC-Polygon-RNN (Ours/PRNN)

Polygon-RNN produces $2N$ feature maps where N is the number of initial points (which is the same as the number of centerlines). Since we use three control points, Polygon-RNN does two iterations to produce the rest of the control points. The N final feature maps are then passed through an MLP to produce N feature vectors. In the transformer NLP setting, these vectors correspond to the transformer encoder output which is the processed input sequence. Thus, in the transformer decoder, cycle queries attend to all the centerlines to produce the output.

3.2. TR-RNN

In order to investigate the feasibility of estimating the order of intersections directly, we opted for an architecture that combines transformers and RNNs. Specifically, on top of the base transformer model, we use an RNN. The input to the RNN is V_c, i.e. the processed query vectors. Each processed query vector is processed by the RNN independently. Let the RNN input vector be $V_c(i)$, i.e. ith processed query vector. We refer to it as the reference curve. At each time step t of the RNN, the output is a probability distribution over the estimated curves and the boundary curves, as well as an end token. Therefore, at each time step t, the output is $N+K+1$ dimensional, with N estimated curves, K boundary curves and one end token. Whenever the RNN outputs the end token, we gather the estimates at the previous steps. These outputs form the ordered sequence of intersections for the reference curve. Since the query vectors are processed jointly by the transformer, V_c carries information about the whereabouts of the other curves as well. We measure the performance of this RNN by using the same order metric 'I-Order' on the RNN estimates.

4. Metrics

We introduced 2 new metrics that measure topological structure accuracy. Here we present additional explanations on the calculation and the justifications for these metrics.

4.1. Minimal Cycle Minimal Cover

In order measure the performance of the methods on estimating the minimal cycles, we introduce a precision-recall based metric. The metric operates on min-matched set of curves. Similar to the connectivity metric, let $R(i)$ be the index of the target that the ith estimation is matched to and $S(n)$ be the set of indices of estimations that are

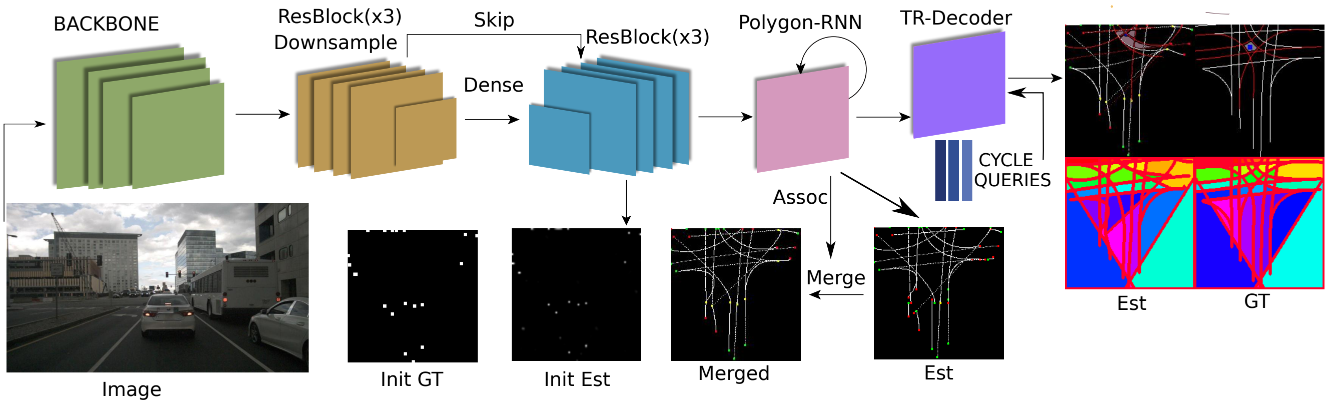

Figure 4. MC-Poly-RNN architecture is formed by addition of a transformer decoder on top of Polygon-RNN's last feature map outputs.

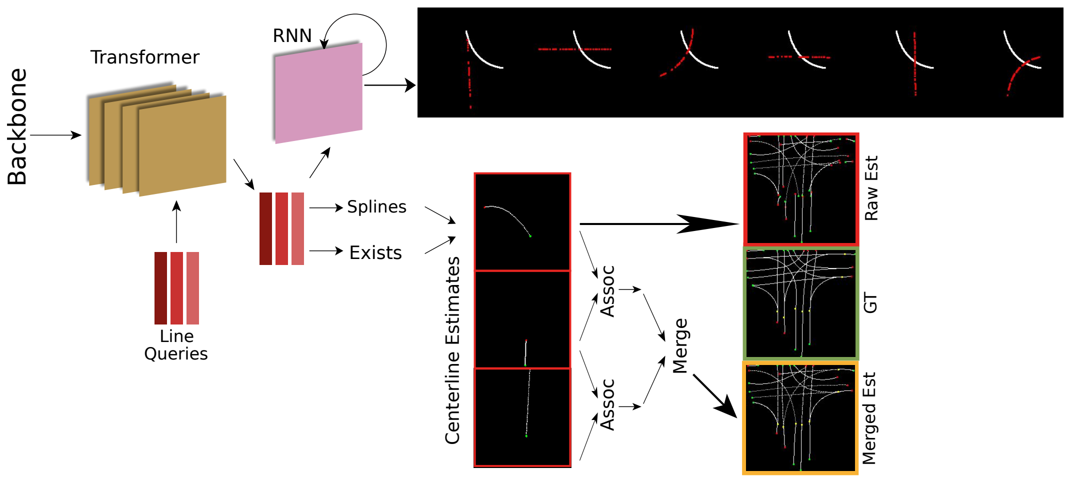

Figure 5. TR-RNN has an RNN operating on the transformer outputs. For each transformer processed curve query vector, it outputs a sequence of fixed sized ($N + K$ dimensional) vectors. Each such vector indicate the probability distribution over the estimated curves that the current reference query intersects in that order. In the figure, we show the reference query with white curve and the selected intersecting curve at each time step with red. This sequence is a direct estimation of the intersection order.

matched to target n. Now let the GT minimal cycles be $MC_{GT} \in [0,1]^{M' \times N' + K}$ while $MC_{Est} \in [0,1]^{M \times N + K}$. We will form true positive (TP), false positive (FP) and false negative (FN) matrices of size $M' \times M \times N' + K$.

TP(i, j, k) = 1 if $MC_{GT}(i,k) = 1$ & $\exists\ n \mid ((R(n) = k)$ & $(MC_{Est}(j,n) = 1))$. In words, TP(i, j, k) is 1 if ith true minimal cycle includes kth true curve and jth estimated cycle has a curve in it that is matched to kth true curve in matching.

FN(i, j, k) = 1 if $MC_{GT}(i,k) = 1$ & $\nexists\ n \mid ((R(n) = k)$ & $(MC_{Est}(j,n) = 1))$.

FP(i, j, k) = 1 if $MC_{GT}(i,k) = 0$ & $\exists\ n \mid ((R(n) = k)$ & $(MC_{Est}(j,n) = 1))$.

We sum up the TP, FP and FN matrices along the last dimension to obtain TP', FP' and FN'. Then $H \in \mathbb{R}^{M' \times M} = FN' + FP'$. We run Hungarian matching on this matrix and we simply select the resulting indices from TP', FP' and FN' and take the sum to get the statistics. If $M' > M$, we consider all the positive entries in unmatched true minimal cycles as false negative.

H-GT-F and H-EST-F . MC-F measures the minimal cycle accuracy of the resulting lane graph. However, we also want to measure the performance of the minimal cycle detection by the sub-network. Therefore, we apply the same procedure described above on the estimates of the MC detection sub-network. **H-GT-F** measures the F-score of the detection subnetwork's estimates compared to the true minimal cycles. This uses the same procedure as MC-F. Therefore, this measures how good the detection head is in detecting the minimal cycles that **should be** created in the estimated lane graph. **H-EST-F** measures the minimal cycle detection network's performance in detecting the minimal cycles that are induced by the estimated lane graph. Thus, it shows how good the detection network is in detecting the minimal cycles **that are created** in the estimated lane graph. In summary, **MC-F** is a similarity measure between induced lane graph and true lane graph, **H-GT-F** is between detection estimates and true lane graph and **H-EST-F** is between detection estimates and induced lane graph. Together, these metrics give a full picture of the performance of the detection network as well as the whole network in estimating the lane graph.

4.2. Intersection Order

In order to measure the intersection order, we get the best match for each true curve. Let $M(i)$ be the index of the target that the ith estimation is matched to and $S(n)$ be the set of indices of estimations that are matched to target n. Then the best match curve for a true curve n is $R_n = \arg\min_s \mathrm{L1}(C_n, s), s \in S(n)$. For the set of true curves C_n with $|S(n)| > 0$, given the matched pairs, we extract the intersection orders. We similarly extract the intersection orders for the estimated curves that are the best match for some true curve. Given the set of matched intersection orders we run Levenshtein edit distance and normalize it by the length of intersection order sequence of the true curve. For the unnmatched true curves, we consider the distance to be 2 (the normalized distance that would result from removing all the elements in the estimated sequence and adding the elements in the true sequence if the estimated sequence is the same length as true sequence). The I-Order of a frame is then the mean over the normalized edit distances.

RNN-Order. The procedure explained above in Section 4.2 is applied to the outputs of TR-RNN to measure how accurate the RNN is in estimating the intersection orders. Instead of the orders extracted from the estimated lane graph, we simply use the RNN outputs.

5. Results

We provided detailed statistical results in the main paper. Here, we provide more visual results. Visuals confirm the statistical findings that the proposed formulation provides improvement in preservation of the topological structure of the true road network.

References

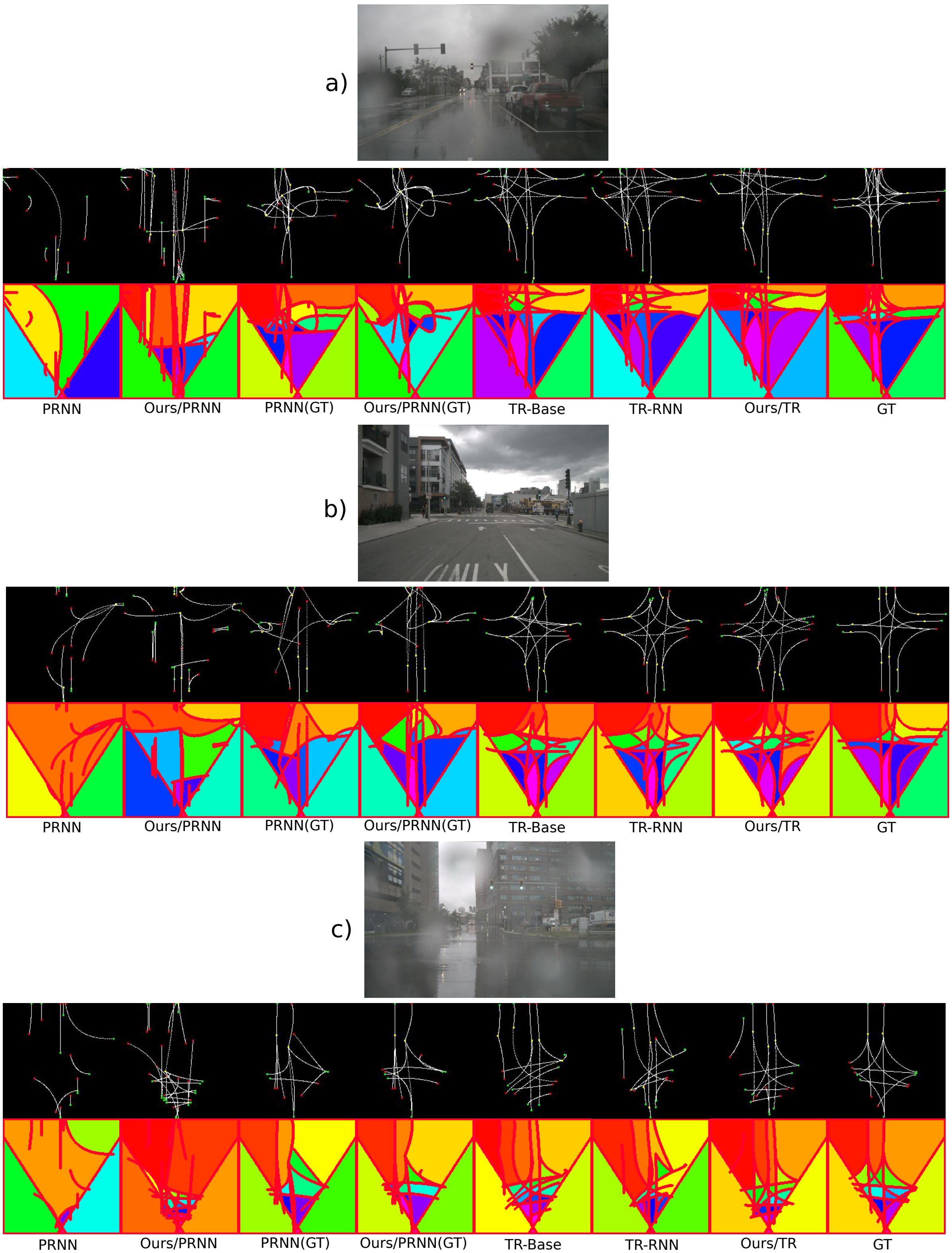

Figure 6. For 3 samples image (above) and the lane graphs and induced minimal cycles (below) in Nuscenes dataset.

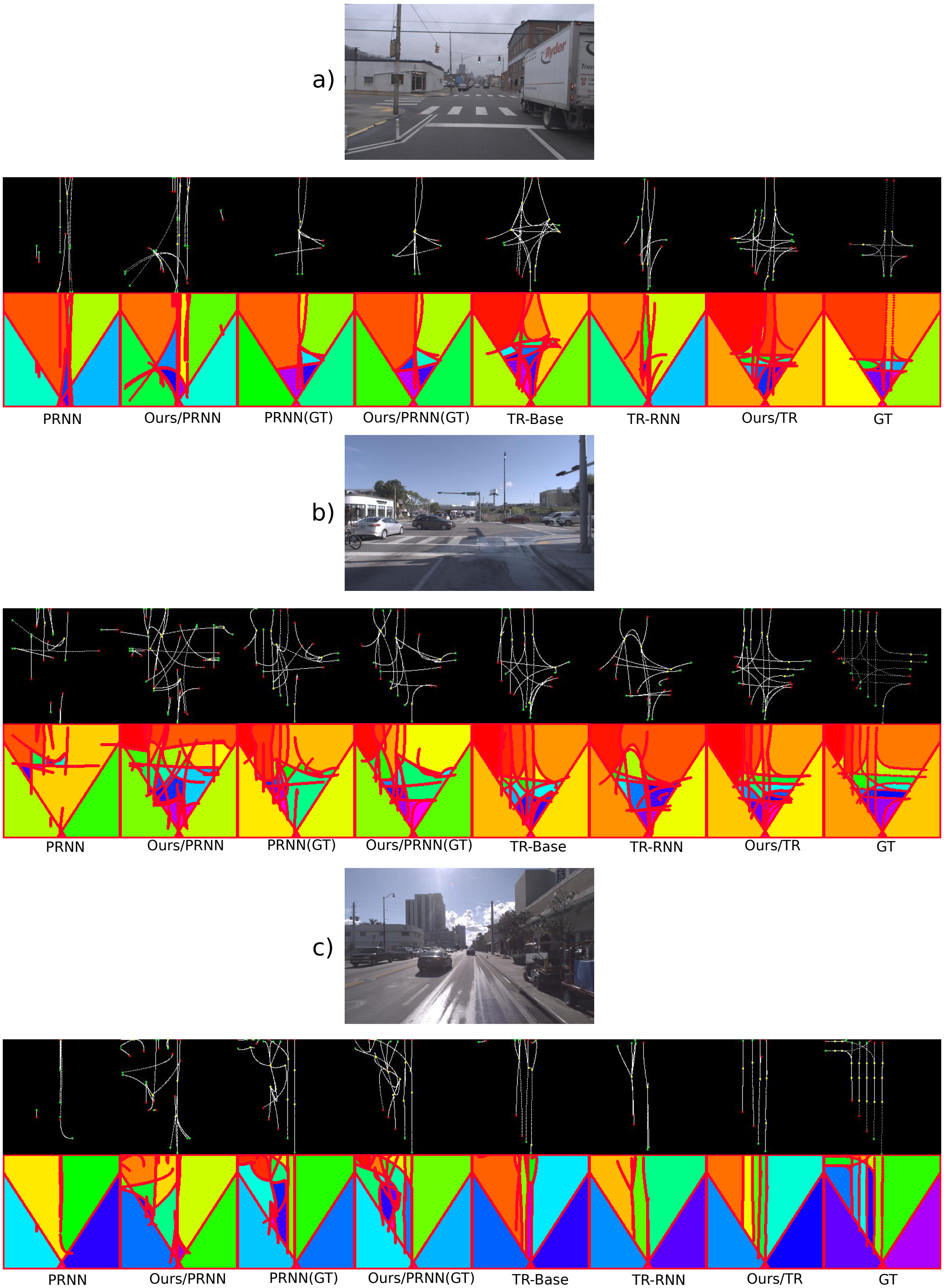

Figure 7. For 3 samples image (above) and the lane graphs and induced minimal cycles (below) in Argoverse dataset.