# Revisiting the P3P Problem - Supplementary Material 

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## 1. Introduction

This supplementary material provides the following information: Section 2 provides the comparison of the different proposed methods for extracting the lines. Section 3 provides the the details for the danger cylinder and the failure cases.

## 2. Comparison of Our Methods

Since we have proposed several methods to extract the line, we show the comparison of our methods in Table 1 and Table 2. We can see that the direct method is the fastest, while the adjoint matrix method is more stable than the other two methods. Hence, we recommend to use the adjoint matrix based method.

| Method | Null space | Adjoint | Direct |
| :---: | :---: | :---: | :---: |
| Valid solutions | 16828556 | 16828556 | 16828546 |
| Unique | 16828556 | 16828556 | 16828546 |
| Duplicates | 0 | 0 | 0 |
| Good solution | 10000000 | 10000000 | 9999998 |
| No solution | 0 | 0 | 2 |
| Ground truth | 9999992 | 9999993 | 9999991 |
| Incorrect | 0 | 0 | 0 |

Table 1. Solution comparison with the current state-of-the-art solvers

| Timing (ns) | Null space | Adjoint | Direct |
| :---: | :---: | :---: | :---: |
| Mean | 237.7 | 224.2 | 211.7 |
| Median | 237.4 | 223.2 | 211.7 |
| Min | 236.7 | 222.8 | 211.2 |
| Max | 241.6 | 228.7 | 212.6 |

Table 2. Running times comparison averaged over $10^{7}$ trials with 10 times each.

## 3. Danger Cylinder and Failure Cases

As shown in Figure 1, $A, B, C$ defines a cylinder with the generatrix parallel to the normal of the plane $A B C$, and the optical center $O$ lies on the surface of this cylinder. Without loss of generality, we can assume $O$ is the origin, and points $A, B, C$ lie on the plane

$$
\begin{equation*}
(x-r)^{2}+y^{2}=r^{2}, \quad z=k \tag{1}
\end{equation*}
$$

where $r$ is the radius of the circle defined by $A B C$, and $k$ is the distance from $O$ to $A B C$. When generating data based on this cylinder, we find that the discriminant of the cubic equation is always zero. In general, case (d) and (f) are dominant, and case (e), (g) and (h) rarely happen. We have also found that it is possible for $\Delta=0$, and $\alpha=\beta=0$. However, a detailed analysis of how such cases occur in P3P problem is future work. The reason for most of the failure cases connect with danger cylinder is that the numerical instability may give incorrect results. For instance, in case (d) the two conics are tangent and have a double intersection, but due to the round-off error and numerical instability we may find zero intersection (failure case) or two intersections (duplicates).


Figure 1. The danger cylinder is defined as a circular cylinder circumscribing points $A, B, C$ with axis normal to the plane $A B C$ [1].

## References

[1] EH Thompson. Space resection: Failure cases. The Photogrammetric Record, 5(27):201-207, 1966. 1

