Learning Federated Visual Prompt in Null Space for MRI Reconstruction (Supplementary Material)

Contents

The following items are included in our supplementary material:

A. Rationality Analysis of the Approximation

To verify the rationality of the approximate null space of the global prompts [2], we denote the proportion of the sum of singular values of $\mathbf{\Lambda}_2^{z+1}$ in the sum of singular values of $\mathbf{\Lambda}^{z+1}$ as

$$R = \frac{\sum \operatorname{diag}\left\{\mathbf{\Lambda}_2^{z+1}\right\}}{\sum \operatorname{diag}\left\{\mathbf{\Lambda}^{z+1}\right\}}, \tag{1}$$

where "diag" indicates the diagonal elements. If the value of R is very small, the sum of the smallest singular values $\boldsymbol{U}_2^{z+1}$ can be ignored, allowing the null space to be approximated through the spatial range of $\boldsymbol{U}_2^{z+1}$. We record the R values of different layers under the `In-Federation` and `Out-of-Federation` scenarios in Fig. A1. As can be seen from this figure, the proportion R in each layer is smaller than 10^{-7}, indicating that the selected $\boldsymbol{U}_2^{z+1}$, which is last $\gamma\%$ of singular values in $\mathbf{\Lambda}_2^{z+1}$, can be ignored. Additionally, we observe that the value of R in the `Out-of-Federation` scenario is slightly greater than the value of R in the `In-Federation` scenario. We infer that this is primarily due to the fact that the distribution of the test set in the `Out-of-Federation` scenario is never seen by the training set, resulting in more severe catastrophic forgetting and thus a slightly higher R value. However, the values of R are still quite close to 0 in both two scenarios. As a result, our selected $\boldsymbol{U}_2^{z+1}$ is a reasonable approximation of the null space of the global prompt.

B. Communication Efficiency Analysis over SOTAs

To further evaluate our federated visual prompt mechanism regards to tackle the issue ❸ *catastrophic forgetting*, we record the reconstruction accuracy over state-of-the-arts under different communication rounds in Fig. A2. The number of the local epochs of each method is fixed at 10. As can be seen from this figure, all the baseline algorithms reached stability after 30 rounds while our method converges within 10 rounds. Although these methods also adopt the same pre-trained model as ours, the catastrophic forgetting due to the data heterogeneity mechanism still leads to slower convergence. For example, FedBN [1] (see the line in Fig. A2) applies the batch normalization on each local client to alleviate the client-shift. However, when there are only a few local training data, batch normalization is performed at the feature level, still resulting in deviations and affecting convergence. Especially in the `Out-of-Federation` scenario, where testing data is unseen by the local models during training, the performance degrades significantly. However, the convergence speed for the FedReg [3] (see the line in Fig. A2), which tries to minimize catastrophic forgetting, is still subpar because the full fine-tuning mechanism still introduces some deviation. On the other hand, FedReg [3] adds the regularization loss on the local side increasing the complexity of the algorithm. In contrast, our method updates the local prompts in the approximate null space of global prompts, which can avoid forgetting previously acquired knowledge and accelerating convergence (see the line in Fig. A2).

C. Additional Qualitative Results

We provide additional qualitative improvements with regard to the reconstructed images with their PSNR and SSIM values and corresponding error maps in Fig. A3 and Fig. A4. The results are consistent with our previous results, *i.e.*, our method provides the best-quality reconstructed images, and error maps with the least texture, indicating that our FedPR is effective in relieving the issues ❶-❸.

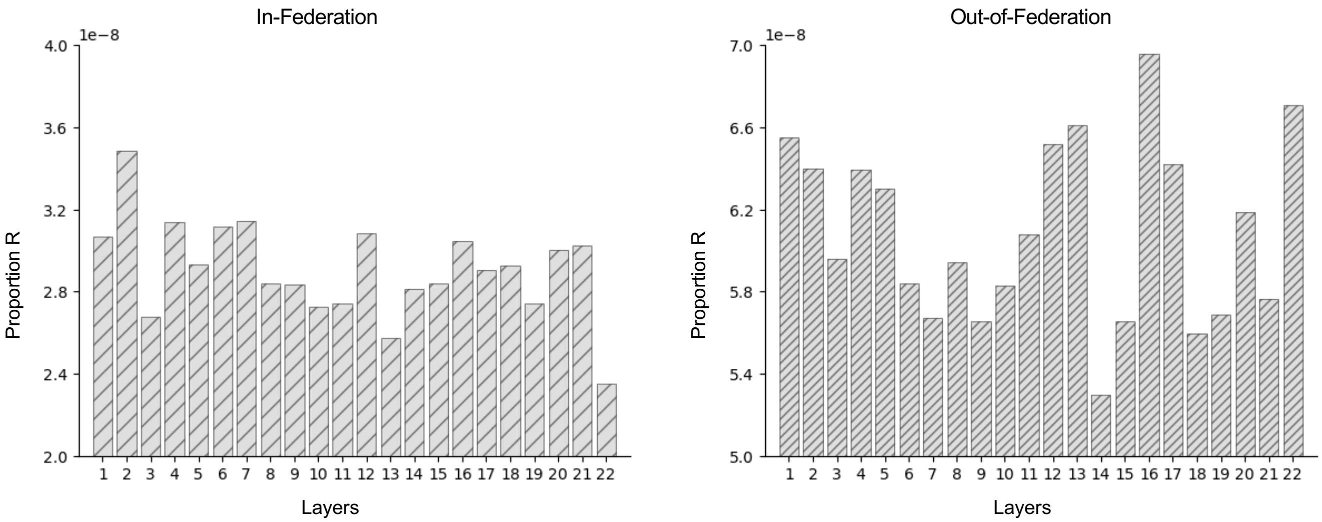

Figure A1. **Rationality analysis** of the approximate null space under `In-Federation` and `Out-of-Federation` scenarios.

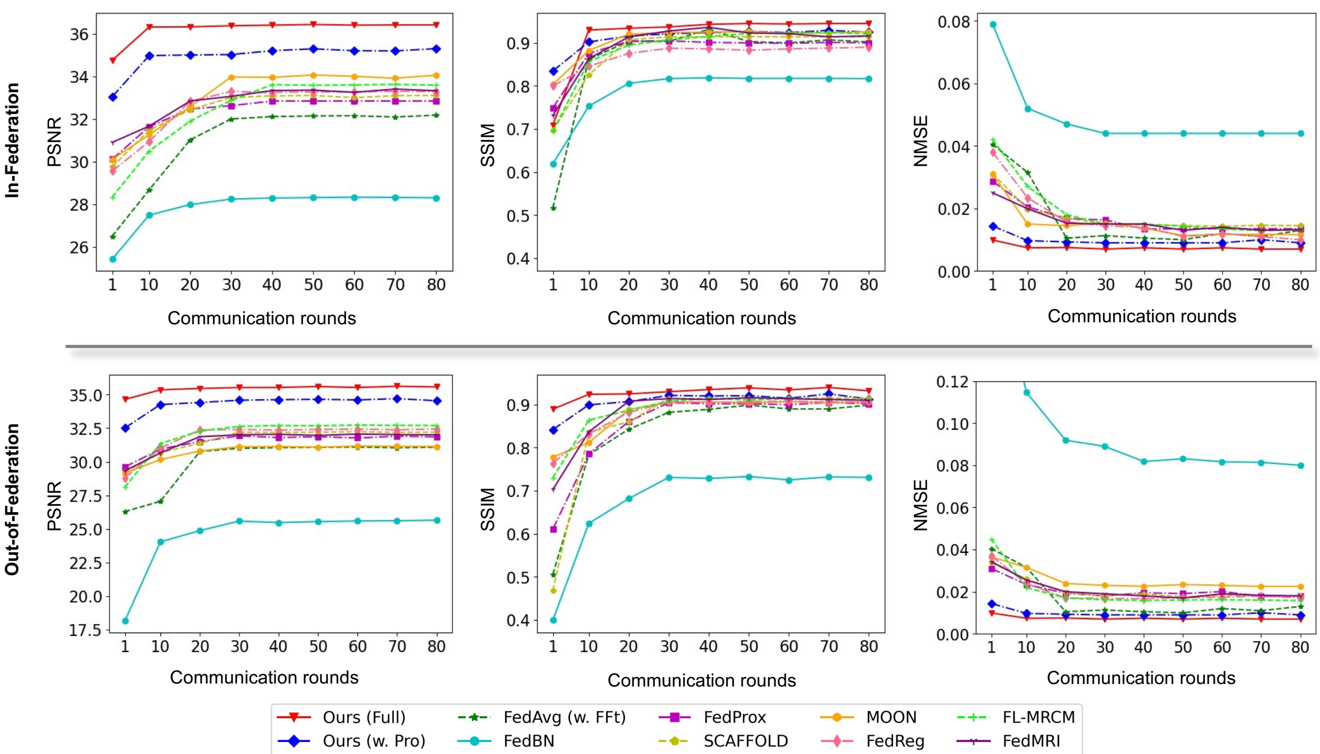

Figure A2. **Communication Efficiency Analysis** over state-of-the-arts versus PSNR, SSIM, and NMSE under `In-Federation` and `Out-of-Federation` scenarios.

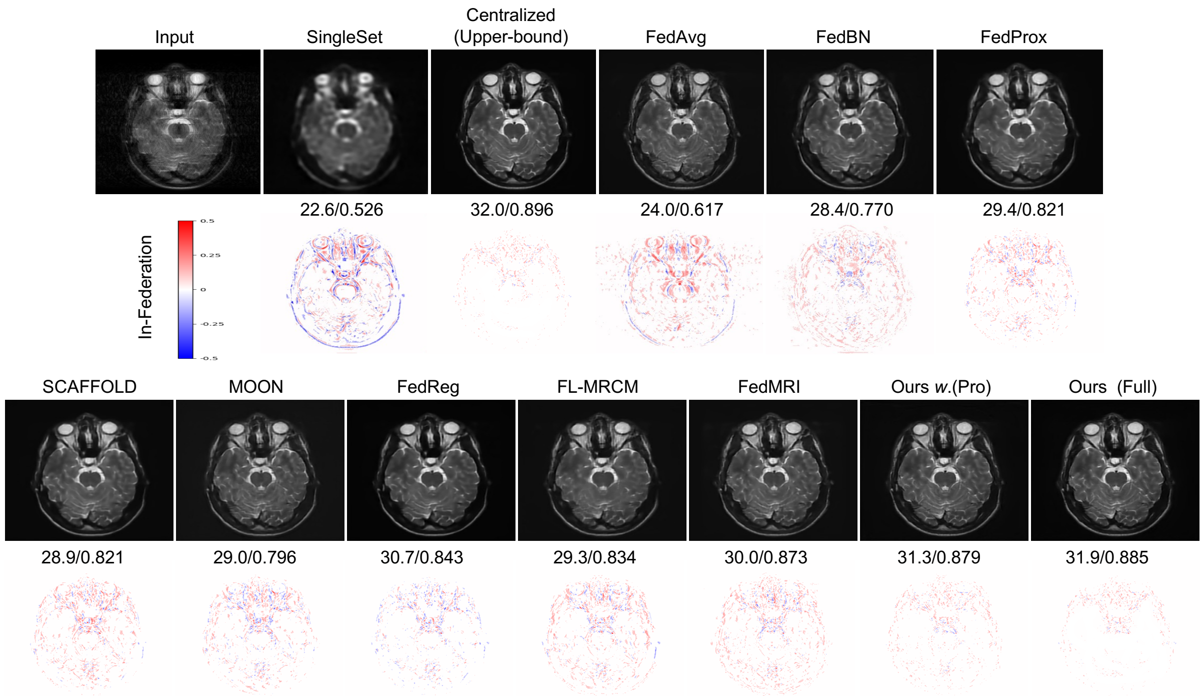

Figure A3. **Qualitative comparison** of different algorithms in terms of reconstruction images and error maps with corresponding quantitative measurements in PSNR/SSIM under `In-Federation` scenario. The less texture in the error map, the better reconstruction quality.

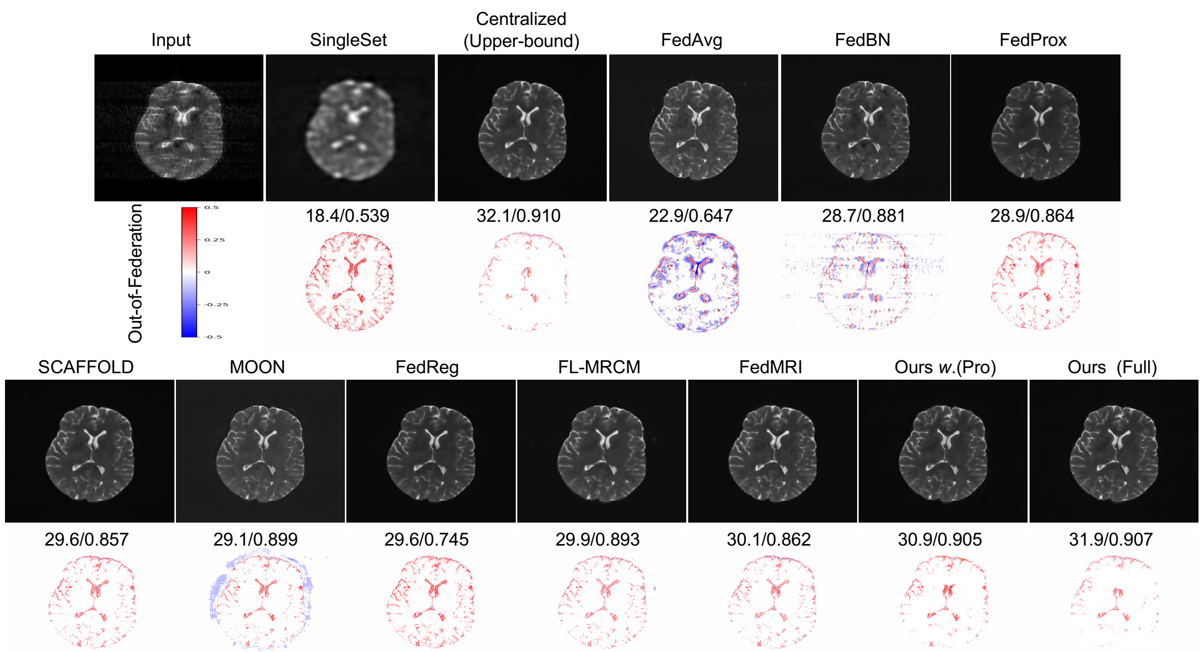

Figure A4. **Qualitative comparison** of different algorithms in terms of reconstruction images and error maps with corresponding quantitative measurements in PSNR/SSIM under `Out-of-Federation` scenario. The less texture in the error map, the better reconstruction quality.

References

[1] Xiaoxiao Li, Meirui Jiang, Xiaofei Zhang, Michael Kamp, and Qi Dou. Fedbn: Federated learning on non-iid features via local batch normalization. *arXiv preprint arXiv:2102.07623*, 2021. 1

[2] Shipeng Wang, Xiaorong Li, Jian Sun, and Zongben Xu. Training networks in null space of feature covariance for continual learning. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 184–193, 2021. 1

[3] Chencheng Xu, Zhiwei Hong, Minlie Huang, and Tao Jiang. Acceleration of federated learning with alleviated forgetting in local training. *arXiv preprint arXiv:2203.02645*, 2022. 1