Normal-guided Garment UV Prediction for Human Re-texturing

Supplementary Material

1. Derivation of Equation (6)

Given Equation (2), $\widetilde{\mathbf{N}}_{\mathbf{x}} = f_u \times f_v$, the surface normal can be expressed as:

$$\widetilde{n}_x = \frac{\partial y}{\partial u}\frac{\partial z}{\partial v} - \frac{\partial y}{\partial v}\frac{\partial z}{\partial u}$$
$$\widetilde{n}_y = \frac{\partial z}{\partial u}\frac{\partial x}{\partial v} - \frac{\partial z}{\partial v}\frac{\partial x}{\partial u}$$
$$\widetilde{n}_z = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}, \tag{12}$$

where $f_u = \begin{bmatrix}\frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u}\end{bmatrix}^\mathsf{T}$, $f_v = \begin{bmatrix}\frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v}\end{bmatrix}^\mathsf{T}$, and $\widetilde{\mathbf{N}}_{\mathbf{x}} = \begin{bmatrix}\widetilde{n}_x & \widetilde{n}_y & \widetilde{n}_z\end{bmatrix}^\mathsf{T}$.

Given the expression of the surface normal, it is possible to express g_x and g_y in terms of f_u and f_v using the inverse function theorem (Equation (5)), $\mathbf{J}_g = \left(\mathbf{J}_{f^{1:2}}\right)^{-1}$:

$$\begin{bmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{bmatrix} = \begin{bmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{bmatrix}^{-1} \tag{13}$$

$$= \frac{1}{\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}}\begin{bmatrix}\frac{\partial y}{\partial v} & -\frac{\partial x}{\partial v}\\ -\frac{\partial y}{\partial u} & \frac{\partial x}{\partial u}\end{bmatrix} \tag{14}$$

$$= \frac{1}{\widetilde{n}_z}\begin{bmatrix}\frac{\partial y}{\partial v} & -\frac{\partial x}{\partial v}\\ -\frac{\partial y}{\partial u} & \frac{\partial x}{\partial u}.\end{bmatrix} \tag{15}$$

Therefore, the partial derivatives of g with respect to x and y can be written as:

$$g_x = \frac{1}{\widetilde{n}_z}\begin{bmatrix}\frac{\partial y}{\partial v}\\ -\frac{\partial y}{\partial u}\end{bmatrix}, \quad g_y = \frac{1}{\widetilde{n}_z}\begin{bmatrix}-\frac{\partial x}{\partial v}\\ \frac{\partial x}{\partial u}\end{bmatrix}. \tag{16}$$

To eliminate f_u and f_v from Equation (16), we derive a set of partial differential equations that are equivalent up to the choice of the coordinate system of the UV map:

$$\|g_x\|^2 = \frac{1}{\widetilde{n}_x^2}\left(\left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2\right)$$
$$= \frac{\widetilde{n}_x^2 + \widetilde{n}_z^2}{\widetilde{n}_z^2} \tag{17}$$

because from Equation (12), $\widetilde{n}_x^2 + \widetilde{n}_z^2$ can be expressed as:

$$\widetilde{n}_x^2 + \widetilde{n}_z^2 = \left(\frac{\partial y}{\partial u}\frac{\partial z}{\partial v} - \frac{\partial y}{\partial v}\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}\right)^2$$
$$= \left(\frac{\partial y}{\partial u}\right)^2\left(\frac{\partial z}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2\left(\frac{\partial z}{\partial u}\right)^2 - 2\frac{\partial y}{\partial u}\frac{\partial y}{\partial v}\frac{\partial z}{\partial u}\frac{\partial z}{\partial v}$$
$$+ \left(\frac{\partial x}{\partial u}\right)^2\left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial x}{\partial v}\right)^2\left(\frac{\partial y}{\partial u}\right)^2 - 2\frac{\partial x}{\partial u}\frac{\partial x}{\partial v}\frac{\partial y}{\partial u}\frac{\partial y}{\partial v}$$
$$= \left(\frac{\partial y}{\partial u}\right)^2\left(1 - \left(\frac{\partial y}{\partial v}\right)^2\right) + \left(\frac{\partial y}{\partial v}\right)^2\left(1 - \left(\frac{\partial y}{\partial u}\right)^2\right)$$
$$- 2\frac{\partial y}{\partial u}\frac{\partial y}{\partial v}\left(\frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial v}\right)$$
$$= \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 - 2\frac{\partial y}{\partial u}\frac{\partial y}{\partial v}\left(\frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial v}\right)$$
$$= \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2, \tag{18}$$

given Equation (1), or

$$\|f_u\|^2 = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2 = 1 \tag{19}$$
$$\|f_v\|^2 = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 = 1 \tag{20}$$
$$f_u^\mathsf{T} f_v = \frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial v} = 0. \tag{21}$$

Similarly, we have

$$\|g_y\|^2 = \frac{1}{\widetilde{n}_x^2}\left(\left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial x}{\partial v}\right)^2\right)$$
$$= \frac{\widetilde{n}_y^2 + \widetilde{n}_z^2}{\widetilde{n}_z^2}. \tag{22}$$

Further, the angle between the vector can be represented as:

$$g_x^\mathsf{T} g_y = -\frac{1}{\widetilde{n}_z^2}\left(\frac{\partial x}{\partial v}\frac{\partial y}{\partial v} + \frac{\partial x}{\partial u}\frac{\partial y}{\partial u}\right)$$
$$= \frac{\widetilde{n}_x\widetilde{n}_y}{\widetilde{n}_z^2} \tag{23}$$

because $\widetilde{n}_x\widetilde{n}_y$ can be written as:

$$\begin{aligned}
\widetilde{n}_x\widetilde{n}_y &= \left(\frac{\partial y}{\partial u}\frac{\partial z}{\partial v} - \frac{\partial y}{\partial v}\frac{\partial z}{\partial u}\right)\left(\frac{\partial z}{\partial u}\frac{\partial x}{\partial v} - \frac{\partial z}{\partial v}\frac{\partial x}{\partial u}\right) \\
&= -\frac{\partial y}{\partial u}\frac{\partial x}{\partial u}\left(\frac{\partial z}{\partial v}\right)^2 - \frac{\partial y}{\partial v}\frac{\partial x}{\partial v}\left(\frac{\partial z}{\partial u}\right)^2 \\
&+ \left(\frac{\partial z}{\partial u}\frac{\partial z}{\partial v}\right)\left(\frac{\partial y}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial v}\frac{\partial x}{\partial u}\right) \\
&= -\frac{\partial y}{\partial u}\frac{\partial x}{\partial u} + \frac{\partial y}{\partial u}\frac{\partial x}{\partial u}\left(\frac{\partial x}{\partial v}\right)^2 + \frac{\partial y}{\partial u}\frac{\partial x}{\partial u}\left(\frac{\partial y}{\partial v}\right)^2 \\
&- \frac{\partial y}{\partial v}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial v}\frac{\partial x}{\partial v}\left(\frac{\partial x}{\partial u}\right)^2 + \frac{\partial y}{\partial v}\frac{\partial x}{\partial v}\left(\frac{\partial y}{\partial u}\right)^2 \\
&- \left(\frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial v}\right)\left(\frac{\partial y}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial v}\frac{\partial x}{\partial u}\right) \\
&= -\left(\frac{\partial y}{\partial u}\frac{\partial x}{\partial u} + \frac{\partial y}{\partial v}\frac{\partial x}{\partial v}\right).
\end{aligned} \tag{24}$$

Equation (17), (22), (23) form a set of partial differential equations of g with respect to the surface normal where we solve these equations by minimizing the loss of $\mathcal{L}_{\mathrm{geo}}$.