

SeaThru-NeRF: Neural Radiance Fields in Scattering Media

Appendix

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[sea-thru-nerf.github.io](https://github.com/seathru-nerf/seathru-nerf)

Derivation of Eqs. (10-11)

We suggested the formulation

$$C(\mathbf{r}) = \int_{t_n}^{t_f} T(t) \left(\sigma^{\text{obj}}(t) \mathbf{c}^{\text{obj}}(t) + \sigma^{\text{med}}(t) \mathbf{c}^{\text{med}}(t) \right) dt$$

with transmittance:

$$T(t) = \exp \left(- \int_{t_n}^t \left(\sigma^{\text{obj}}(s) + \sigma^{\text{med}}(s) \right) ds \right),$$

using separate color and density parameters for the *object* and *medium*.

In the discretized version, given a sampling $\{s_i\}_{i=0}^N$ ($s_0 = t_n, s_N = t_f$) with intervals of length $\delta_i = s_{i+1} - s_i$, and assuming the parameters $\sigma^{\text{obj}}, \mathbf{c}^{\text{obj}}, \sigma^{\text{med}}, \mathbf{c}^{\text{med}}$ to be constant per interval, we can write the transmittance at a sampling point s_i , with as a piece-wise constant sum instead of the integral:

$$\begin{aligned} T(s_i) &= \exp \left(- \int_{t_n}^{s_i} \left(\sigma^{\text{obj}}(s) + \sigma^{\text{med}}(s) \right) ds \right) \\ &= \exp \left(- \sum_{j=0}^{i-1} (\sigma_j^{\text{obj}} + \sigma_j^{\text{med}}) \delta_j \right). \end{aligned}$$

In the discretized rendering equation

$$\hat{C}(\mathbf{r}) = \sum_i C_i(\mathbf{r})$$

the contribution $C_i(\mathbf{r})$ of the i th interval is given by:

$$\begin{aligned} C_i(\mathbf{r}) &= \int_{s_i}^{s_{i+1}} T(t) (\sigma_i^{\text{obj}} \mathbf{c}_i^{\text{obj}} + \sigma_i^{\text{med}} \mathbf{c}_i^{\text{med}}) dt \\ &= (\sigma_i^{\text{obj}} \mathbf{c}_i^{\text{obj}} + \sigma_i^{\text{med}} \mathbf{c}_i^{\text{med}}) \cdot \int_{s_i}^{s_{i+1}} T(t) dt \end{aligned}$$

where we can compute:

$$\begin{aligned} \int_{s_i}^{s_{i+1}} T(t) dt &= \int_{s_i}^{s_{i+1}} \exp \left(- \int_{t_n}^t (\sigma^{\text{obj}}(s) + \sigma^{\text{med}}(s)) ds \right) dt \\ &= \int_{s_i}^{s_{i+1}} \exp \left(- \int_{t_n}^{s_i} (\sigma^{\text{obj}}(s) + \sigma^{\text{med}}(s)) ds \right) \cdot \\ &\quad \exp \left(- \int_{s_i}^t (\sigma_i^{\text{obj}} + \sigma_i^{\text{med}}) ds \right) dt \\ &= \int_{s_i}^{s_{i+1}} T(s_i) \cdot \exp \left(-(t - s_i) \cdot (\sigma_i^{\text{obj}} + \sigma_i^{\text{med}}) \right) dt \\ &= T(s_i) \cdot \left[- \frac{\exp \left(-(t - s_i) \cdot (\sigma_i^{\text{obj}} + \sigma_i^{\text{med}}) \right)}{\sigma_i^{\text{obj}} + \sigma_i^{\text{med}}} \right]_{s_i}^{s_{i+1}} \\ &= T(s_i) \cdot \frac{1 - e^{-(\sigma_i^{\text{obj}} + \sigma_i^{\text{med}}) \delta_i}}{\sigma_i^{\text{obj}} + \sigma_i^{\text{med}}} \end{aligned}$$

which finally results in

$$C_i(\mathbf{r}) = T(s_i) \left(1 - e^{-(\sigma_i^{\text{obj}} + \sigma_i^{\text{med}}) \delta_i} \right) \frac{\sigma_i^{\text{obj}} \mathbf{c}_i^{\text{obj}} + \sigma_i^{\text{med}} \mathbf{c}_i^{\text{med}}}{\sigma_i^{\text{obj}} + \sigma_i^{\text{med}}}$$

as required.