Tangentially Elongated Gaussian Belief Propagation for Event-based Incremental Optical Flow Estimation

Supplemental material

A. Semi-dense estimation

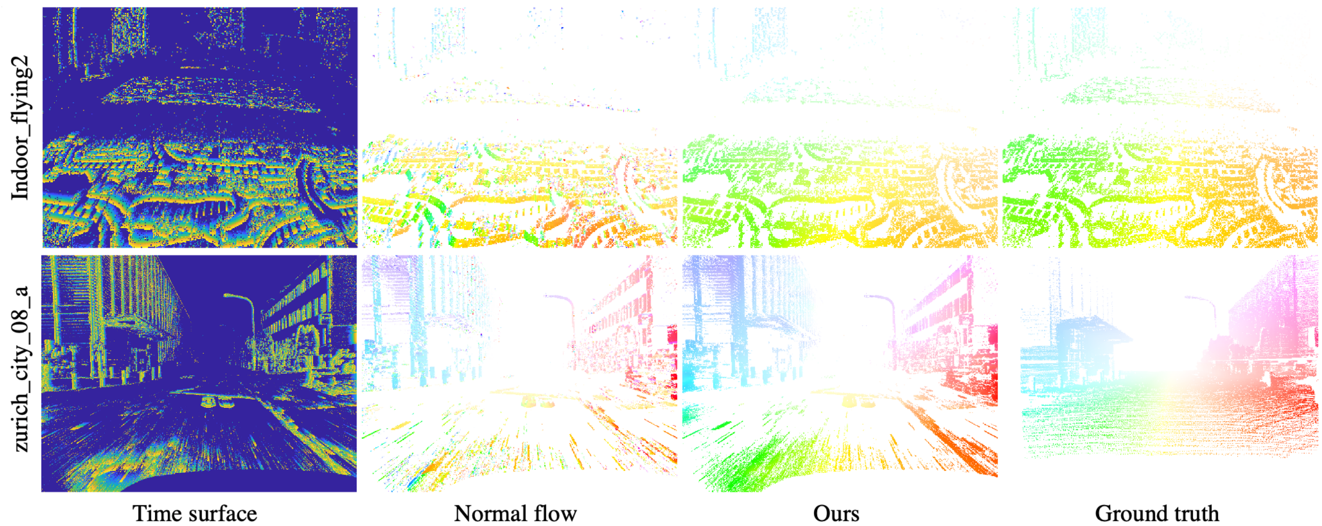

Figure A1. **Semi-dense estimation**. MVSEC (top) and DSEC (bottom). Refer to the `supp_MVSEC.mp4` for MVSEC and `supp_DSEC.mp4` for DSEC in the supplement.

In this supplemental section, we show the result of the TEGBP extension for *semi-dense* optical flow estimation. In the main paper, we described the method for estimating the flow at the sparse pixels, where the normal flow is observed (Sec. 3). We evaluate and report the results of the sparse estimation, mainly for the comparison with ARMS [2]. Normal flow estimation often fails, therefore, normal flow observation points are much more sparse than the raw event's points.

Our TEGBP can be easily extended to estimate the flow at any pixel point where we do not have an observation. In this section, we show the results on MVSEC and DSEC by estimating the full flow at events' points (semi-dense). By the semi-dense estimation, we can have an estimation on more pixels at the cost of a slight increase in the computational cost.

The extension to semi-dense (or dense) is simple. It is realized by sending the message to the pixel locations where we need an estimation. In the case of a semi-dense scenario, it is realized by setting the node having the observation of the raw event as *active* (irrespective of the plane fitting results).

Fig. A1 shows a snapshot from the semi-dense estimation on MVSEC and DSEC. Refer to the `supp_MVSEC.mp4` for MVSEC and `supp_DSEC.mp4` for DSEC in the supplement. We've selected, `indoor_flying2` from MVSEC and `zurich_city_08_a` from DSEC.

Note that, for this video demonstration, we estimate the semi-dense optical flow by exchanging messages at regular time intervals (e.g. a frame rate).

B. Ablation on the TEG parameters

In this section, we ablate the TEG parameter to demonstrate the key ingredient of the proposed TEGBP. As discussed in Sec. 3, our core idea for realizing the robust full-flow estimation is modeling the distribution of full-flow as TEG, Gaussian distribution having large variance along the tangential direction of the normal flow. We show the result by changing the variance along the tangential direction σ_t.

B.1. Toy example of Fig. 1

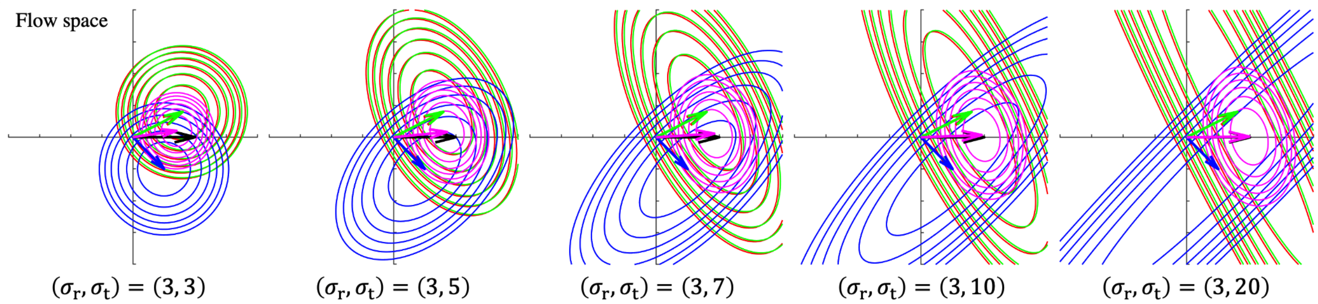

Figure A2. **TEG and its marginal for different** σ_t. Normal flow measurements modeled as TEG (RGB ellipse, having different σ_t) and its marginal with a uniform prior (magenta ellipse) are shown. The black arrow represents the true full flow.

Fig. A2 visualize the marginal distribution of the full flow in the case of toy example of Fig. 1 by changing σ_t. When σ_t is small and the TEG does not model the distribution of full flow from the normal flow measurement well, the full flow estimation, i.e, MAP solution of the marginal, is largely different from the true flow. As the σ_t becomes larger, and the TEG well represents the distribution of full flow, the MAP solution gets closer to the correct full flow. And the limit of $\sigma_t = \infty, \sigma_r = 0$, it equals to the true full flow as we showed in Sec. 3.3.2.

Our open-source repository includes the MATLAB script for this visualization to help understand our core algorithm. It does not utilize the multi-scale scheme (Sec. C.1) for simplicity. Refer to the comments in the code for the implementation detail of the TEG construction from normal flow measurements and the proposed message-passing scheme. (We've also added a pointer to the important equations in the comments).

B.2. ESIM_bricks

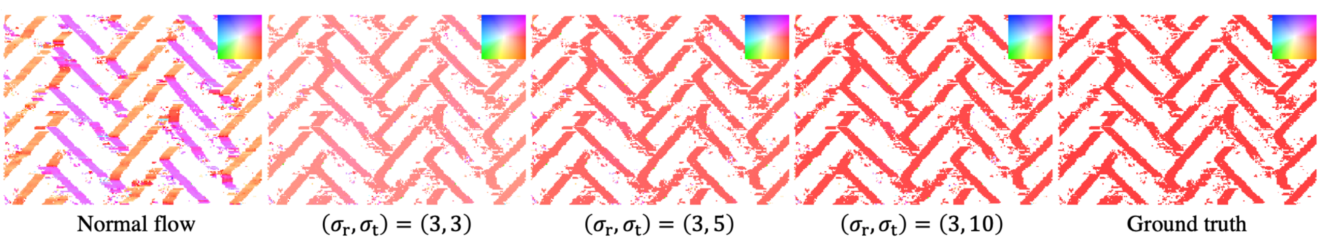

Figure A3. **Full flow estimation results using different** σ_t.

Fig. A3 shows the result by using different value of σ_t (refer to the detailed setup for Sec. 4.2). We'll get accurate flow as the TEG well represents the distribution of full flow by increasing σ_t.

C. Implementation Detail

C.1. Coarse-to-fine message-passing

We adopted the coarse-to-fine message-passing scheme of [10] to speed up the convergence. The measurement factor of the coarser layer l is computed as the sum of active measurement factor messages in a 2×2 block of finer layer $l-1$ as follows:

$$\boldsymbol{\eta}_i^l = \sum_{j \in \mathcal{B}_i \cap \mathcal{A}_t} \boldsymbol{\eta}_j^{l-1}, \quad \Lambda_i^l = \sum_{j \in \mathcal{B}_i \cap \mathcal{A}_t} \Lambda_j^{l-1}, \tag{A1}$$

where $\mathcal{B}_i$ is the block corresponding the node i.

Note that one can obtain similar results without C2F by naively increasing the number of iterations or adding edges between longer-distance nodes. We left the exploration of another C2F mechanism or more efficient message-passing scheme for future work.

C.2. Algorithm

The algorithm of the proposed TEGBP is listed in Algorithm 1. The message passing part which corresponds to Fig. 5 is listed separately in Algorithm 2.

Algorithm 1 TEGBP

for all incoming normal flow $e_i = \{\mathbf{x}_i, t_i, \mathbf{v}_i^{\perp}\}$ **do**
  Calculate the message from the data factor by Eq. 4 and Eq. 8 and set to node at pixel $\mathbf{x}_i$.
  Update hierarchical data factor by Eq. A1.
  for all l in $[L, \cdots, 1]$ **do**
    Update belief by Eq. 10
    Message passing (Algorithm 2)
    if $l > 1$ **then**
      Copy the messages from a coarser layer l to a finer layer $l-1$.
    end if
  end for
  Determine the estimated value $\mathbf{v}_i$ as the mean of Gaussian belief.
end for

Algorithm 2 Message passing at node i

Add node i to queue Q.
for all k in K hops **do**
  Pop all nodes in Q and add to the temporary queue Q'.
  for all j popped from Q' **do**
    Message passing by Eq. 11.
    for all $k \in \mathcal{A}_t \cap \mathcal{C}_j$ **do**
      Update belief by Eq. 10.
      Add node k to queue Q.
    end for
  end for
end for

D. Probability density of full flow along the tangential around the normal flow (σ_t)

In real-world data, the probability density of full flow is it is not uniform along the tangential direction of the normal flow but highly concentrated around the normal flow. Figure A4 shows the distribution in the MVSEC dataset.

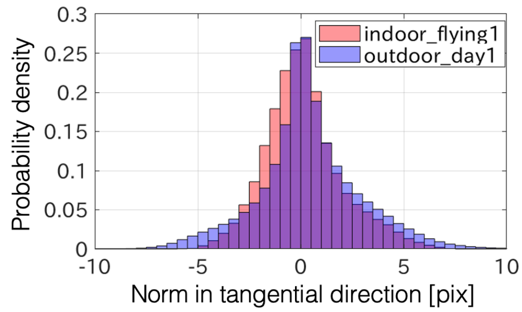

Figure A4. Full flow distribution around the normal flow on MVSEC dataset [33].