Supplementary Materials of Deep Hashing with Minimal-Distance-Separated Hash Centers

Liangdao Wang[1], Yan Pan[1]*, Cong Liu[1], Hanjiang Lai[1], Jian Yin[1], Ye Liu[1,2]
[1]School of Computer Science and Engineering, Sun Yat-Sen University. [2]Big Data Department, Lizhi Inc.
wangld5@mail2.sysu.edu.cn, panyan5@mail.sysu.edu.cn

We analyze the convergence of the the following optimization problem (Eq.(5) in the main paper).

$$\begin{aligned} &\min_{h_i, v_1, v_2, v_3} \sum_{j:j\neq i} h_i^T h_j \\ &s.t.\ \ h_i^T H_{\sim i} + v_3 = (q-2d)1_{c-1}, v_3 \in R_+^{c-1}, \\ &\quad\ \ h_i = v_1,\ h_i = v_2, v_1 \in \mathcal{V}_{box},\ v_2 \in \mathcal{V}_{sph}. \end{aligned} \tag{1}$$

The augmented Lagrange function w.r.t. Eq.(1) is:

$$\begin{aligned} &L(h_i, v_1, v_2, v_3, k_1, k_2, k_3) = \sum_{j\neq i} h_i^T h_j + k_1^T(h_i - v_1) + \\ &\frac{\mu}{2}||h_i - v_1||_2^2 + k_2^T(h_i - v_2) + \frac{\mu}{2}||h_i - v_2||_2^2 \\ &+ k_3^T(h_i^T H_{\sim i} + v_3 - e) + \frac{\mu}{2}||h_i^T H_{\sim i} + v_3 - e||_2^2 \\ &s.t.\ v_1 \in V_{box}, v_2 \in V_{sph}, v_3 \in R_+^{c-1}, \end{aligned} \tag{2}$$

where $e = (q-2d)1_{c-1}$, and k_1, k_2, k_3 are Lagrange multipliers.

We adopt the ℓ_p-box ADMM method to solve Eq.(1), which is shown in the red lines in Algorithm 1. Next we will show that, under mild assumptions, by using this ℓ_p-box ADMM scheme, the optimization problem Eq.(1) will converge.

We first present two assumptions that are adapted from [1], by using our notations.

Assumption 1 Let μ^t be the value of the parameter μ at t-th iteration. Then the parameter μ is a finite value, i.e., $\lim_{t\to\infty} \mu^t \in (0, \infty)$.

Assumption 2 Let $k^t = (k_1^t, k_2^t, k_3^t)$ be the values of the Lagrange multipliers k_1, k_2 and k_3 at t-th iteration. Then the parameter sequence $\{k^t\}$ satisfies a) $\sum_{t=0}^{\infty} ||k^{t+1} - k^t||_2^2 < \infty$, which also hints that $k^{t+1} - k^t \to 0$ and b) k^t is bounded for all t.

Note that, if we replace the range constraint $S_b(z) = \{0 \le z \le 1\}$ by a similar range constraint $\{-1 \le z \le 1\}$, we can verify that the optimization problem Eq.(1) is a special case of the problem (8) in [1]. By the Proposition 2 in [1], the problem (8) in [1] will converge under some assumptions. Similarly, we have the following theorem.

*Corresponding Author

Algorithm 1 Optimization Procedure to Generate Hash Centers

Initialize: initialize $h_1, ..., h_c$ by Hadamard matrix and Bernoulli sampling. $\rho = 1.02, \max_\mu = 10^{10}, \epsilon = 10^{-6}, T = 50$.
For t = 1,2,...,T
  For i=1,2, . . . , c
    Set $v_1, v_2, v_3, k_1, k_2, k_3$ to be zero vectors. Set $\mu = 10^{-6}$.
    Repeat
      Update h_i via Eq.(7) in the main paper.
      Update v_1, v_2, v_3 via Eq.(10) in the main paper.
      Update k_1, k_2, k_3 via Eq.(11) in the main paper.
      Update μ by $\mu \leftarrow \min(\rho\mu, max_\mu)$.
    Until $\max(||h_i - v_1||_\infty, ||h_i - v_2||_\infty, ||h_i^T H_{\sim i} + v_3 - e||_\infty) \le \epsilon$.
  End For
  $T \leftarrow T + 1$.
End For
Output: h_i $(i = 1, 2, ...c)$

Theorem 1 (adapted from Proposition 2 in [1]) Given Assumptions 1 and 2, then any cluster point of the whole variable sequence $\{(h_i^t, v_1^t, v_2^t, v_3^t, k_1^t, k_2^t, k_3^t)\}_0^\infty$ generated by the ADMM method will satisfy the KKT conditions of the problem Eq.(1). Moreover, $\{(h_i^t, v_1^t, v_2^t)\}_0^\infty$ will converge to the binary solutions.

The proof of Theorem 1 is nearly identical to the proof of Proposition 2 in [1] because the optimization problem Eq.(1) can be regarded as a special case of the problem (8) in [1]. The difference in range constraints $S_b(z)$ has no impact on the proof.

Then, we illustrate that, by using the ADMM method in Algorithm 1, Assumption 1 and 2 hold and the optimization problem Eq.(1) will converge.

With the update rules $\mu \leftarrow \min(\rho\mu, max_\mu)$ for the parameter μ, we can see that μ is always bounded by a constant $\max_\mu = 10^{10}$. Hence Assumption 1 holds.

With the update rules for k_1, k_2 and k_3 (Eq.(11) in the main paper) and the stopping criteria $\max(||h_i - v_1||_\infty, ||h_i - v_2||_\infty, ||h_i^T H_{\sim i} + v_3 - e||_\infty) \le \epsilon$, we can verify that Assumption 2 holds.

Hence, by applying Theorem 1, the optimization problem Eq.(1) will converge.

References

[1] Baoyuan Wu and Bernard Ghanem. ℓ_p-Box ADMM: A Versatile Framework for Integer Programming. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 41(7):1695–1708, 2019. 1