Power Bundle Adjustment for Large-Scale 3D Reconstruction Supplementary

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In this supplementary material we provide additional details to augment the content of the main paper. Section 1 contains a proof of Proposition 1 in the main paper. In Section 2 we evaluate different levels of noises to highlight the consistence of our solver. In Section 3 we tabulate the percentage of solved problems of the performance profiles (Sec. 5.2.) for each tolerance $\tau \in \{0.1, 0.01, 0.03, 0.001\}$ and for each solver. In Section 4 we list the evaluated problems from the BAL dataset.

1. Proof of Proposition 1

Firstly, simple product expansion gives

$$(I - M)(I + ... + M^i) = I - M^{i+1} \,. \tag{A}$$

Since the spectral norm is sub-multiplicative and

$$\|M\| < 1 \,, \tag{B}$$

it is straightforward that

$$\|M^i\| \leq \|M\|^i \underset{i \to \infty}{\longrightarrow} 0 \,. \tag{C}$$

Thus,

$$M^i \underset{i \to \infty}{\longrightarrow} \mathbf{0} \,. \tag{D}$$

Taking the limit of both sides in (A) gives (1).

Secondly,

$$R = \sum_{i=m+1}^{\infty} M^i = M^{m+1} \sum_{i=0}^{\infty} M^i = M^{m+1}(I - M)^{-1} \,. \tag{E}$$

It follows that

$$\|R\| = \|M^{m+1} \sum_{i=0}^{\infty} M^i\| \leq \|M\|^{m+1} \sum_{i=0}^{\infty} \|M\|^i \,. \tag{F}$$

Since $\|M\| < 1$ we have

$$\sum_{i=0}^{\infty} \|M\|^i = \frac{1}{1 - \|M\|} \,, \tag{G}$$

which directly leads to the inequality

$$\|R\| \leq \frac{\|M\|^{m+1}}{1 - \|M\|} \,. \tag{H}$$

2. Consistence

In Sec. 5.2. initial landmark and camera positions are perturbated with a small Gaussian noise $(m, \sigma) = (0, 0.01)$. We observe that the relative performance of solvers is similar for different noise levels. Fig. 1 and 2 illustrate the consistence of our results with different initial noises $\sigma = 0.05$ and $\sigma = 0.1$.

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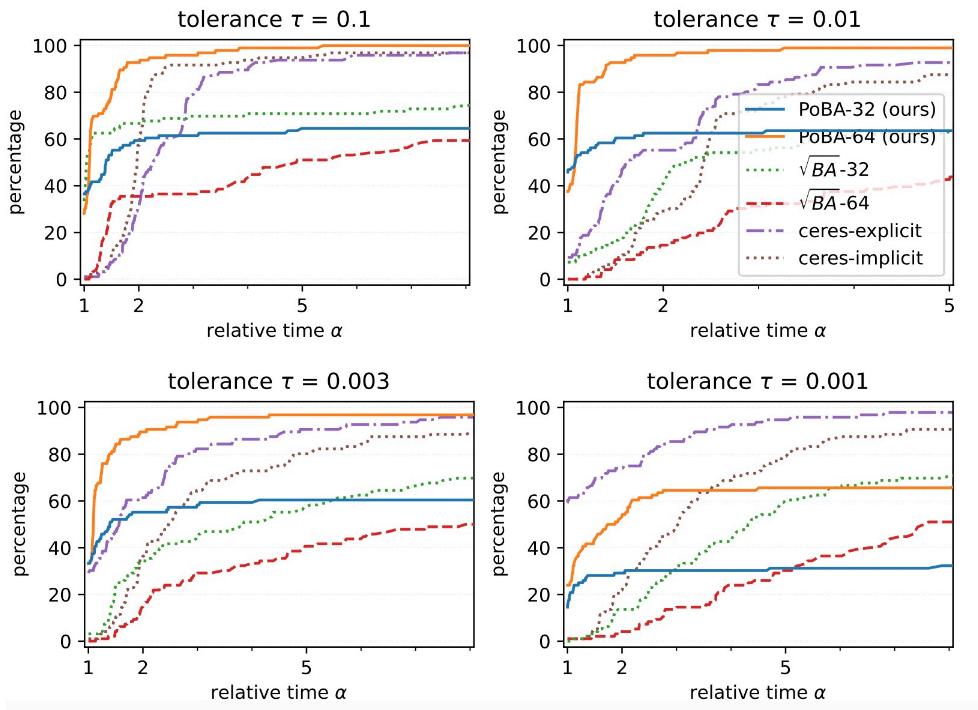

Figure 1. Performance profiles for all BAL problems show the percentage of problems solved to a given accuracy tolerance $\tau \in \{0.1, 0.01, 0.003, 0.001\}$ with relative runtime α. Initial landmark and camera positions are disturbed with a Gaussian noise $(0, 0.05)$.

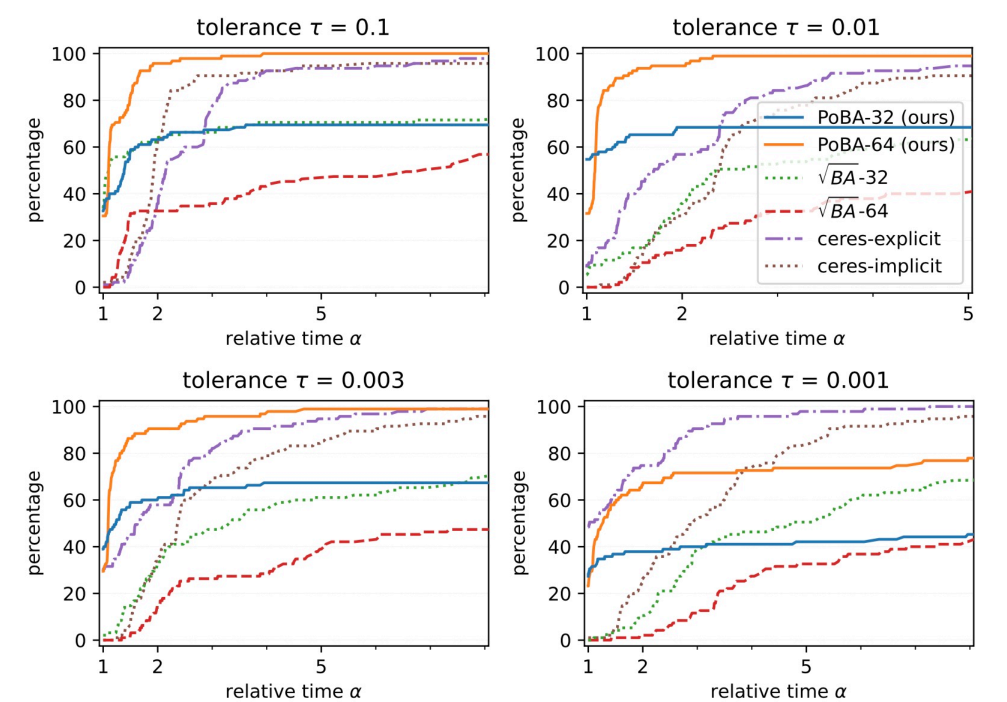

Figure 2. Performance profiles for all BAL problems show the percentage of problems solved to a given accuracy tolerance $\tau \in \{0.1, 0.01, 0.003, 0.001\}$ with relative runtime α. Initial landmark and camera positions are disturbed with a Gaussian noise $(0, 0.1)$.

3. Tables of solved problems associated to the performance profiles

Solver	$\alpha = 1$	$\alpha = 3$	$\alpha = \infty$
PoBA-64 (ours)	7%	<u>100%</u>	<u>100%</u>
PoBA-32 (ours)	<u>62%</u>	88%	88%
$\sqrt{BA}$-64	0%	60%	81%
$\sqrt{BA}$-32	31%	90%	98%
ceres-explicit	0%	49%	95%
ceres-implicit	0%	81%	95%

Solver	$\alpha = 1$	$\alpha = 3$	$\alpha = \infty$
PoBA-64 (ours)	18%	<u>98%</u>	<u>98%</u>
PoBA-32 (ours)	<u>60%</u>	84%	84%
$\sqrt{BA}$-64	0%	62%	79%
$\sqrt{BA}$-32	22%	83%	97%
ceres-explicit	2%	66%	90%
ceres-implicit	0%	80%	90%

Solver	$\alpha = 1$	$\alpha = 3$	$\alpha = \infty$
PoBA-64 (ours)	20%	<u>90%</u>	93%
PoBA-32 (ours)	<u>44%</u>	75%	79%
$\sqrt{BA}$-64	0%	58%	84%
$\sqrt{BA}$-32	22%	<u>90%</u>	<u>98%</u>
ceres-explicit	14%	71%	90%
ceres-implicit	0%	66%	91%

Solver	$\alpha = 1$	$\alpha = 3$	$\alpha = \infty$
PoBA-64 (ours)	13%	52%	58%
PoBA-32 (ours)	8%	26%	27%
$\sqrt{BA}$-64	0%	63%	85%
$\sqrt{BA}$-32	21%	<u>88%</u>	<u>98%</u>
ceres-explicit	<u>54%</u>	83%	90%
ceres-implicit	4%	75%	90%

Table 1. Percentage of solved problems of the performance profiles (Sec. 5.2.) for each solver and for tolerance $\tau = 0.1$ (upper left), $\tau = 0.01$ (upper right), $\tau = 0.003$ (lower left) and $\tau = 0.001$ (lower right). We conclude that PoBA is particularly well suited for very fast/low-accurate ($\tau = 0.1$), fast/medium-accurate ($\tau = 0.01$) and slow/high-accurate ($\tau = 0.003$) applications.

4. Problems Table

	cameras	landmarks	observations
ladybug-49	49	7,766	31,812
ladybug-73	73	11,022	46,091
ladybug-138	138	19,867	85,184
ladybug-318	318	41,616	179,883
ladybug-372	372	47,410	204,434
ladybug-412	412	52,202	224,205
ladybug-460	460	56,799	241,842
ladybug-539	539	65,208	277,238
ladybug-598	598	69,193	304,108
ladybug-646	646	73,541	327,199
ladybug-707	707	78,410	349,753
ladybug-783	783	84,384	376,835
ladybug-810	810	88,754	393,557
ladybug-856	856	93,284	415,551
ladybug-885	885	97,410	434,681
ladybug-931	931	102,633	457,231
ladybug-969	969	105,759	474,396
ladybug-1064	1,064	113,589	509,982
ladybug-1118	1,118	118,316	528,693
ladybug-1152	1,152	122,200	545,584
ladybug-1197	1,197	126,257	563,496
ladybug-1235	1,235	129,562	576,045
ladybug-1266	1,266	132,521	587,701
ladybug-1340	1,340	137,003	612,344

ladybug-1469	1,469	145,116	641,383
ladybug-1514	1,514	147,235	651,217
ladybug-1587	1,587	150,760	663,019
ladybug-1642	1,642	153,735	670,999
ladybug-1695	1,695	155,621	676,317
ladybug-1723	1,723	156,410	678,421
	cameras	landmarks	observations
trafalgar-21	21	11,315	36,455
trafalgar-39	39	18,060	63,551
trafalgar-50	50	20,431	73,967
trafalgar-126	126	40,037	148,117
trafalgar-138	138	44,033	165,688
trafalgar-161	161	48,126	181,861
trafalgar-170	170	49,267	185,604
trafalgar-174	174	50,489	188,598
trafalgar-193	193	53,101	196,315
trafalgar-201	201	54,427	199,727
trafalgar-206	206	54,562	200,504
trafalgar-215	215	55,910	203,991
trafalgar-225	225	57,665	208,411
trafalgar-257	257	65,131	225,698
	cameras	landmarks	observations
dubrovnik-16	16	22,106	83,718
dubrovnik-88	88	64,298	383,937
dubrovnik-135	135	90,642	552,949
dubrovnik-142	142	93,602	565,223
dubrovnik-150	150	95,821	567,738
dubrovnik-161	161	103,832	591,343
dubrovnik-173	173	111,908	633,894
dubrovnik-182	182	116,770	668,030
dubrovnik-202	202	132,796	750,977
dubrovnik-237	237	154,414	857,656
dubrovnik-253	253	163,691	898,485
dubrovnik-262	262	169,354	919,020
dubrovnik-273	273	176,305	942,302
dubrovnik-287	287	182,023	970,624
dubrovnik-308	308	195,089	1,044,529
dubrovnik-356	356	226,729	1,254,598
	cameras	landmarks	observations
venice-52	52	64,053	347,173
venice-89	89	110,973	562,976
venice-245	245	197,919	1,087,436
venice-427	427	309,567	1,695,237
venice-744	744	542,742	3,054,949
venice-951	951	707,453	3,744,975
venice-1102	1,102	779,640	4,048,424
venice-1158	1,158	802,093	4,126,104
venice-1184	1,184	815,761	4,174,654
venice-1238	1,238	842,712	4,286,111
venice-1288	1,288	865,630	4,378,614

venice-1350	1,350	893,894	4,512,735
venice-1408	1,408	911,407	4,630,139
venice-1425	1,425	916,072	4,652,920
venice-1473	1,473	929,522	4,701,478
venice-1490	1,490	934,449	4,717,420
venice-1521	1,521	938,727	4,734,634
venice-1544	1,544	941,585	4,745,797
venice-1638	1,638	975,980	4,952,422
venice-1666	1,666	983,088	4,982,752
venice-1672	1,672	986,140	4,995,719
venice-1681	1,681	982,593	4,962,448
venice-1682	1,682	982,446	4,960,627
venice-1684	1,684	982,447	4,961,337
venice-1695	1,695	983,867	4,966,552
venice-1696	1,696	983,994	4,966,505
venice-1706	1,706	984,707	4,970,241
venice-1776	1,776	993,087	4,997,468
venice-1778	1,778	993,101	4,997,555

	cameras	landmarks	observations
final-93	93	61,203	287,451
final-394	394	100,368	534,408
final-871	871	527,480	2,785,016
final-961	961	187,103	1,692,975
final-1936	1,936	649,672	5,213,731
final-3068	3,068	310,846	1,653,045
final-4585	4,585	1,324,548	9,124,880
final-13682	13,682	4,455,575	28,973,703

Table 2. List of all 97 BAL problems including number of cameras, landmarks and observations. These are the problems evaluated for performance profiles in the main paper.