

# IDGI: A Framework to Eliminate Explanation Noise from Gradients Integration (Appendix)

## 1. Theorem

**Theorem 1** Given a function  $f_c(x) : R^n \rightarrow R$ , points  $x_j, x_{j+1}, x_{j_p} \in R^n$ , then the gradient of the function with respect to each point in the space  $R^n$  forms the conservative vector fields  $\vec{F}$  and further define the hyperplane  $h_j = \{x : f_c(x) = f_c(x_j)\}$  in  $\vec{F}$ . Assume the Riemann Integration accurately estimates the line integral of the vector field  $\vec{F}$  from points  $x_j$  to  $x_{j+1}$  and  $x_{j_p}$  e.g.  $\int_{x_j}^{x_{j_p}} \frac{\partial f_c(x)}{\partial x} dx \approx \frac{\partial f_c(x_j)}{\partial x_j} (x_{j_p} - x_j)$ , and  $x_j \in h_j, x_{j_p}, x_{j+1} \in h_{j+1}$ . Then:

$$\int_{x_j}^{x_{j+1}} \frac{\partial f_c(x)}{\partial x} dx \approx \int_{x_j}^{x_{j_p}} \frac{\partial f_c(x)}{\partial x} dx.$$

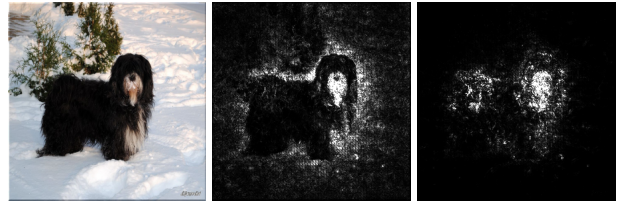
*Proof.*

$$\begin{aligned} \int_{x_j}^{x_{j+1}} \frac{\partial f_c(x)}{\partial x} dx &= f_c(x_{j+1}) - f_c(x_j) \\ &\approx \frac{\partial f_c(x_j)}{\partial x_j} (x_{j+1} - x_j) \\ &= f_c(x_{j_p}) - f_c(x_j) \\ &= \frac{\partial f_c(x_j)}{\partial x_j} (x_{j_p} - x_j) \\ &= \int_{x_j}^{x_{j_p}} \frac{\partial f_c(x)}{\partial x} dx \end{aligned} \quad (1)$$

## 2. IG with IDGI

The Integrated Gradients algorithm requires a specified reference image to compute the attribution. One often selects a black or white picture as a reference point, resulting in the zero attribution value to pixels with the same value as the reference. This is due to the fact that these pixel values do not change while traveling from the reference image to the original image. However, these pixels may still be crucial for the classifier to make the decision, and merit attribution differs from zero. For example, as shown in Figure 1, the *Xception* model makes the prediction correctly on the given image has a black dog. When utilizing IG for providing an explanation, the body of the dog will be assigned zero attributions since the reference (black) image has the same

pixel value as the dog. Intuitively, the explanation method should give non-zero values to these black pixels, since they represent the dog’s body and are assumed to be significant characteristics. Alternatively, if the attribution value is zero, it is likely because the feature is insignificant and not because of the explanation method’s design. In contrast to the original IG, IG with IDGI might potentially assign non-zero values to pixels with the same value as the reference picture, a desirable trait for a superior explanation technique.



(a) Original Image (b) IG (c) IG+IDGI

Figure 1. Original image is predicted *Tibetan terrier* from *Xception* classifier. Both **1b** and **1c** are attributions from IG and IG+IDGI with the black image as reference. Since the pixels are also black for the original image on the dog region, by design, IG is not able to assign important values to those pixels, however, ID+IDGI overcomes the issue.

## 3. Visual Examples

We present more visual examples in Figs. 2 to 6.

## 4. Distribution by Normalized Entropy and MS-SSIM

We present more distribution that compares Normalized Entropy and MS-SSIM in Figs. 7 to 17.

## 5. AIC and SIC with XRAI

Tab. 1 and Tab. 2 show the result of AIC and SIC for all methods and its version with XRAI. Similarly, Tab. 3 and Tab. 4 show the result of AIC and SIC with MS-SSIM for all methods and its version with XRAI.

AUC of AIC							
Models	IG-based Methods						Other
	IG	+Ours	GIG	+Ours	BlurIG	+Ours	
<i>DenseNet121</i>	.161	<b>.300</b>	.141	<b>.252</b>	.192	<b>.230</b>	.087
<i>DenseNet169</i>	.160	<b>.288</b>	.154	<b>.254</b>	.181	<b>.216</b>	.089
<i>DenseNet201</i>	.185	<b>.307</b>	.182	<b>.269</b>	.213	<b>.246</b>	.110
<i>InceptionV3</i>	.203	<b>.343</b>	.189	<b>.338</b>	.266	<b>.301</b>	.127
<i>MobileNetV2</i>	.098	<b>.233</b>	.114	<b>.204</b>	.145	<b>.197</b>	.068
<i>ResNet50V2</i>	.162	<b>.253</b>	.162	<b>.248</b>	.189	<b>.210</b>	.108
<i>ResNet101V2</i>	.177	<b>.268</b>	.163	<b>.253</b>	.198	<b>.215</b>	.116
<i>ResNet151V2</i>	.186	<b>.281</b>	.165	<b>.258</b>	.205	<b>.229</b>	.112
<i>VGG16</i>	.145	<b>.244</b>	.141	<b>.199</b>	.181	<b>.222</b>	.108
<i>VGG19</i>	.153	<b>.263</b>	.150	<b>.219</b>	.204	<b>.240</b>	.117
<i>Xception</i>	.238	<b>.404</b>	.239	<b>.381</b>	.309	<b>.355</b>	.174
With XRAI							
<i>DenseNet121</i>	.438	<b>.479</b>	.460	<b>.460</b>	.437	<b>.452</b>	.434
<i>DenseNet169</i>	.468	<b>.508</b>	.483	<b>.492</b>	.466	<b>.480</b>	.462
<i>DenseNet201</i>	.439	<b>.476</b>	.460	<b>.468</b>	.442	<b>.461</b>	.449
<i>InceptionV3</i>	.477	<b>.506</b>	.472	<b>.513</b>	.479	<b>.503</b>	.496
<i>MobileNetV2</i>	.407	<b>.442</b>	.437	<b>.435</b>	.410	<b>.436</b>	.424
<i>ResNet50V2</i>	.402	<b>.433</b>	.428	<b>.438</b>	.409	<b>.410</b>	.417
<i>ResNet101V2</i>	.415	<b>.447</b>	.433	<b>.445</b>	.416	<b>.422</b>	.424
<i>ResNet151V2</i>	.410	<b>.443</b>	.421	<b>.435</b>	.406	<b>.416</b>	.412
<i>VGG16</i>	.393	<b>.423</b>	.422	<b>.418</b>	.402	<b>.413</b>	.396
<i>VGG19</i>	.386	<b>.416</b>	.417	<b>.414</b>	.396	<b>.408</b>	.393
<i>Xception</i>	.486	<b>.521</b>	.507	<b>.525</b>	.492	<b>.520</b>	.511

Table 1. AUC of AIC

AUC of SIC							
Models	IG-based Methods						Other
	IG	+Ours	GIG	+Ours	BlurIG	+Ours	
<i>DenseNet121</i>	.054	<b>.228</b>	.036	<b>.157</b>	.085	<b>.134</b>	.015
<i>DenseNet169</i>	.052	<b>.230</b>	.045	<b>.170</b>	.083	<b>.130</b>	.016
<i>DenseNet201</i>	.068	<b>.241</b>	.058	<b>.183</b>	.109	<b>.155</b>	.019
<i>InceptionV3</i>	.087	<b>.294</b>	.061	<b>.286</b>	.171	<b>.232</b>	.029
<i>MobileNetV2</i>	.020	<b>.145</b>	.023	<b>.111</b>	.043	<b>.103</b>	.011
<i>ResNet50V2</i>	.077	<b>.210</b>	.067	<b>.201</b>	.099	<b>.158</b>	.025
<i>ResNet101V2</i>	.095	<b>.231</b>	.070	<b>.201</b>	.117	<b>.165</b>	.026
<i>ResNet151V2</i>	.101	<b>.249</b>	.065	<b>.212</b>	.122	<b>.177</b>	.025
<i>VGG16</i>	.046	<b>.166</b>	.039	<b>.104</b>	.082	<b>.141</b>	.021
<i>VGG19</i>	.046	<b>.177</b>	.041	<b>.115</b>	.098	<b>.151</b>	.023
<i>Xception</i>	.119	<b>.363</b>	.107	<b>.336</b>	.218	<b>.296</b>	.054
With XRAI							
<i>DenseNet121</i>	.407	<b>.464</b>	.435	<b>.445</b>	.403	<b>.428</b>	.404
<i>DenseNet169</i>	.450	<b>.496</b>	.465	<b>.475</b>	.439	<b>.458</b>	.435
<i>DenseNet201</i>	.427	<b>.473</b>	.449	<b>.462</b>	.419	<b>.449</b>	.432
<i>InceptionV3</i>	.450	<b>.493</b>	.449	<b>.499</b>	.441	<b>.481</b>	.477
<i>MobileNetV2</i>	.351	<b>.398</b>	.391	<b>.394</b>	.353	<b>.393</b>	.374
<i>ResNet50V2</i>	.401	<b>.439</b>	.430	<b>.445</b>	.404	<b>.412</b>	.418
<i>ResNet101V2</i>	.424	<b>.463</b>	.445	<b>.464</b>	.419	<b>.428</b>	.433
<i>ResNet151V2</i>	.413	<b>.453</b>	.423	<b>.445</b>	.401	<b>.424</b>	.414
<i>VGG16</i>	.343	<b>.382</b>	.381	<b>.376</b>	.352	<b>.368</b>	.347
<i>VGG19</i>	.337	<b>.376</b>	.374	<b>.373</b>	.347	<b>.362</b>	.344
<i>Xception</i>	.458	<b>.502</b>	.486	<b>.508</b>	.465	<b>.503</b>	.488

Table 2. AUC of SIC

## References

- [1] Andrei Kapishnikov, Tolga Bolukbasi, Fernanda Viégas, and Michael Terry. Xrai: Better attributions through regions. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pages 4948–4957, 2019. [8](#), [9](#), [10](#), [11](#)

AUC of AIC with MS-SSIM							
Models	IG-based Methods						Other
	IG	+Ours	GIG	+Ours	BlurIG	+Ours	
<i>DenseNet121</i>	.229	<b>.305</b>	.231	<b>.280</b>	.216	<b>.277</b>	.186
<i>DenseNet169</i>	.241	<b>.314</b>	.249	<b>.297</b>	.218	<b>.289</b>	.205
<i>DenseNet201</i>	.254	<b>.323</b>	.262	<b>.303</b>	.237	<b>.303</b>	.216
<i>InceptionV3</i>	.264	<b>.333</b>	.268	<b>.333</b>	.264	<b>.323</b>	.228
<i>MobileNetV2</i>	.179	<b>.259</b>	.197	<b>.238</b>	.186	<b>.241</b>	.150
<i>ResNet50V2</i>	.225	<b>.277</b>	.239	<b>.274</b>	.209	<b>.260</b>	.198
<i>ResNet101V2</i>	.235	<b>.284</b>	.243	<b>.277</b>	.215	<b>.265</b>	.206
<i>ResNet151V2</i>	.247	<b>.302</b>	.250	<b>.292</b>	.227	<b>.284</b>	.212
<i>VGG16</i>	.205	<b>.271</b>	.212	<b>.245</b>	.204	<b>.259</b>	.179
<i>VGG19</i>	.211	<b>.275</b>	.220	<b>.252</b>	.214	<b>.266</b>	.188
<i>Xception</i>	.281	<b>.362</b>	.293	<b>.356</b>	.284	<b>.345</b>	.254
With XRAI							
<i>DenseNet121</i>	.342	<b>.376</b>	.360	<b>.367</b>	.336	<b>.369</b>	.351
<i>DenseNet169</i>	.375	<b>.407</b>	.386	<b>.397</b>	.368	<b>.398</b>	.382
<i>DenseNet201</i>	.354	<b>.388</b>	.370	<b>.380</b>	.355	<b>.387</b>	.370
<i>InceptionV3</i>	.357	<b>.384</b>	.355	<b>.386</b>	.348	<b>.390</b>	.373
<i>MobileNetV2</i>	.310	<b>.339</b>	.333	<b>.334</b>	.310	<b>.339</b>	.329
<i>ResNet50V2</i>	.302	<b>.326</b>	.320	<b>.330</b>	.302	<b>.322</b>	.317
<i>ResNet101V2</i>	.316	<b>.342</b>	.329	<b>.342</b>	.312	<b>.334</b>	.327
<i>ResNet151V2</i>	.314	<b>.341</b>	.321	<b>.334</b>	.308	<b>.335</b>	.321
<i>VGG16</i>	.314	<b>.339</b>	.334	<b>.334</b>	.319	<b>.336</b>	.319
<i>VGG19</i>	.309	<b>.333</b>	.330	<b>.329</b>	.315	<b>.332</b>	.315
<i>Xception</i>	.370	<b>.402</b>	.391	<b>.406</b>	.372	<b>.408</b>	.396

Table 3. AUC of SIC with MS-SSIM

AUC of SIC with MS-SSIM							
Models	IG-based Methods						Other
	IG	+Ours	GIG	+Ours	BlurIG	+Ours	
<i>DenseNet121</i>	.184	<b>.263</b>	.188	<b>.239</b>	.172	<b>.236</b>	.139
<i>DenseNet169</i>	.205	<b>.282</b>	.214	<b>.263</b>	.182	<b>.256</b>	.166
<i>DenseNet201</i>	.212	<b>.286</b>	.221	<b>.265</b>	.194	<b>.266</b>	.170
<i>InceptionV3</i>	.211	<b>.287</b>	.215	<b>.285</b>	.214	<b>.276</b>	.179
<i>MobileNetV2</i>	.126	<b>.204</b>	.144	<b>.187</b>	.130	<b>.188</b>	.096
<i>ResNet50V2</i>	.196	<b>.254</b>	.213	<b>.250</b>	.177	<b>.236</b>	.167
<i>ResNet101V2</i>	.210	<b>.265</b>	.221	<b>.256</b>	.188	<b>.244</b>	.180
<i>ResNet151V2</i>	.221	<b>.282</b>	.227	<b>.270</b>	.197	<b>.261</b>	.186
<i>VGG16</i>	.163	<b>.234</b>	.174	<b>.210</b>	.166	<b>.224</b>	.137
<i>VGG19</i>	.173	<b>.240</b>	.186	<b>.219</b>	.177	<b>.233</b>	.149
<i>Xception</i>	.223	<b>.312</b>	.233	<b>.304</b>	.229	<b>.293</b>	.194
With XRAI							
<i>DenseNet121</i>	.290	<b>.332</b>	.309	<b>.324</b>	.282	<b>.324</b>	.306
<i>DenseNet169</i>	.327	<b>.364</b>	.338	<b>.356</b>	.314	<b>.353</b>	.338
<i>DenseNet201</i>	.311	<b>.349</b>	.326	<b>.345</b>	.301	<b>.350</b>	.333
<i>InceptionV3</i>	.300	<b>.334</b>	.295	<b>.342</b>	.291	<b>.343</b>	.323
<i>MobileNetV2</i>	.238	<b>.270</b>	.264	<b>.270</b>	.239	<b>.273</b>	.262
<i>ResNet50V2</i>	.273	<b>.305</b>	.294	<b>.308</b>	.270	<b>.299</b>	.295
<i>ResNet101V2</i>	.291	<b>.323</b>	.305	<b>.322</b>	.283	<b>.312</b>	.306
<i>ResNet151V2</i>	.286	<b>.322</b>	.294	<b>.313</b>	.277	<b>.314</b>	.302
<i>VGG16</i>	.256	<b>.285</b>	.280	<b>.280</b>	.260	<b>.284</b>	.264
<i>VGG19</i>	.252	<b>.278</b>	.276	<b>.277</b>	.258	<b>.279</b>	.262
<i>Xception</i>	.311	<b>.345</b>	.331	<b>.352</b>	.311	<b>.353</b>	.341

Table 4. AUC of SIC with MS-SSIM

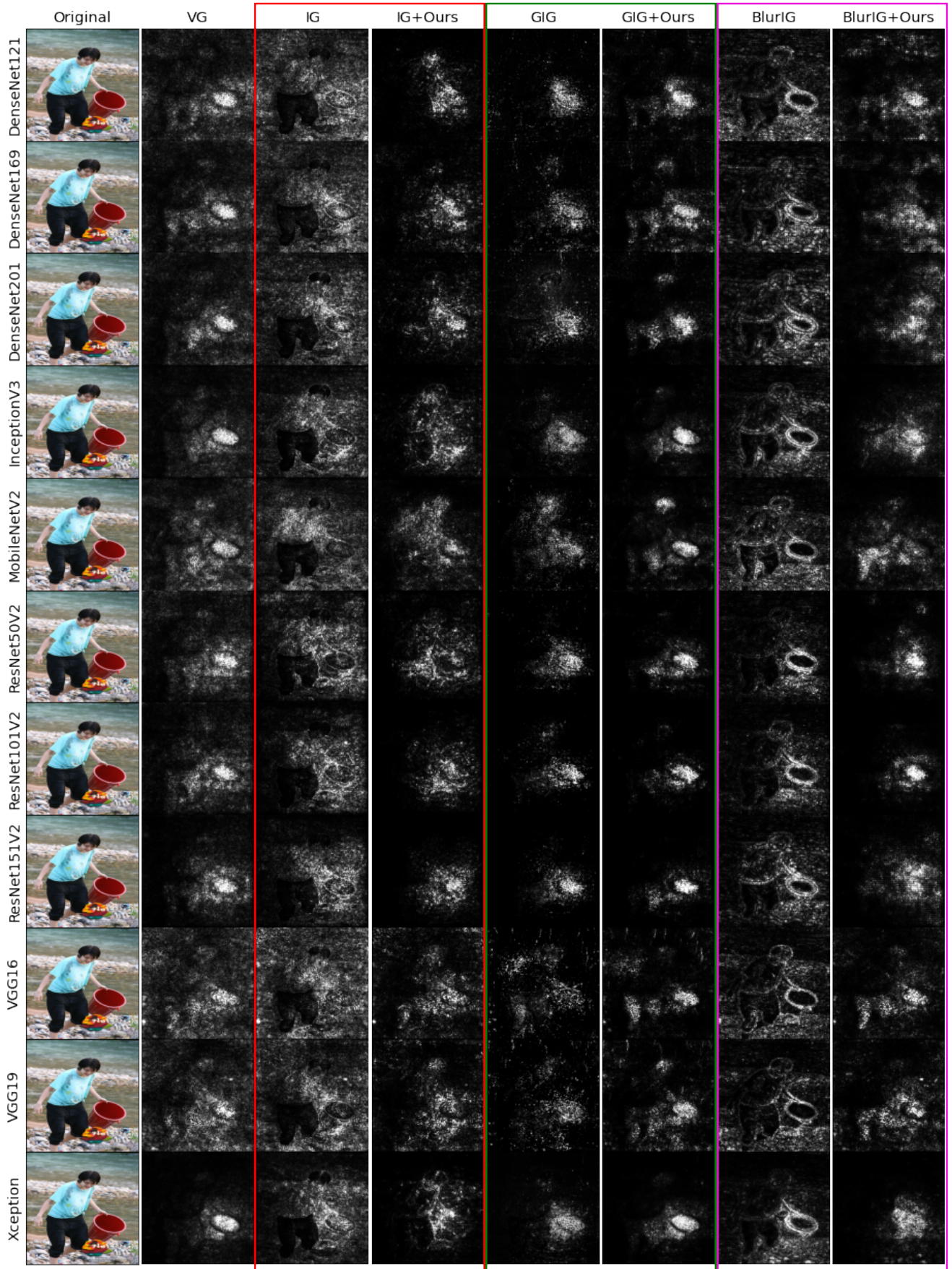


Figure 2. Predicted Label for all models: *bucket*

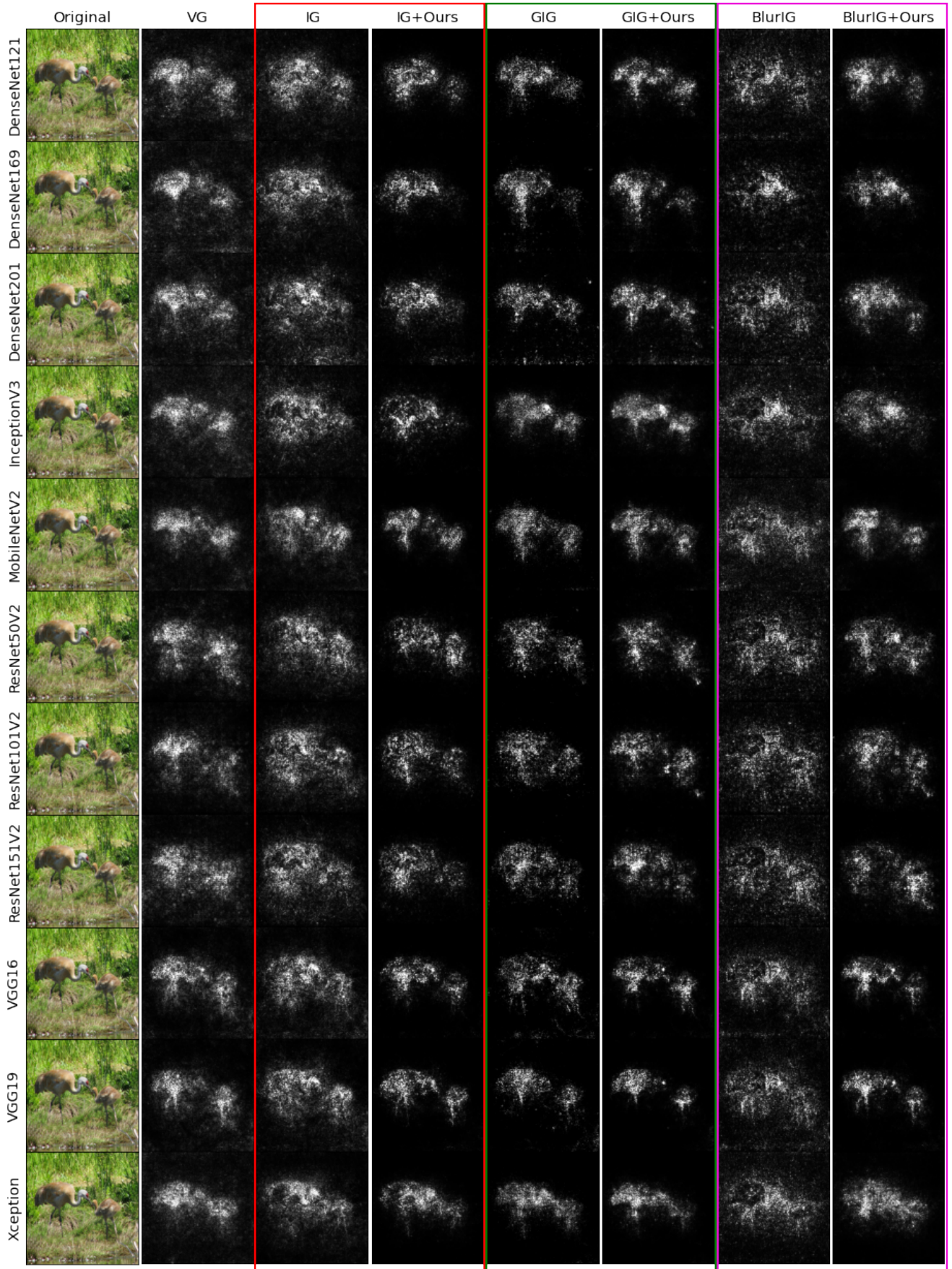


Figure 3. Predicted Label for all models: *crane*

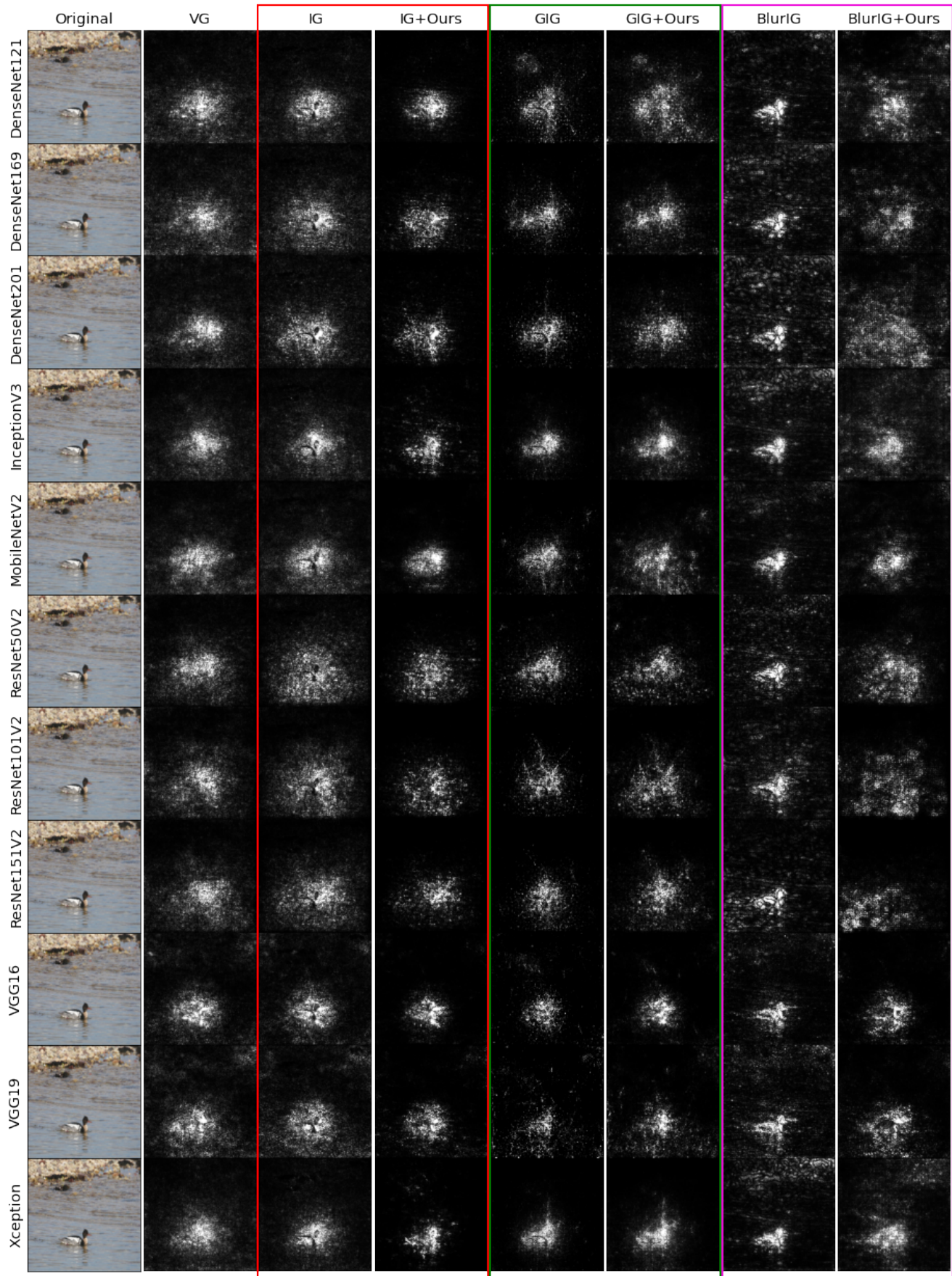


Figure 4. Predicted Label for all models: *mergus serrator*

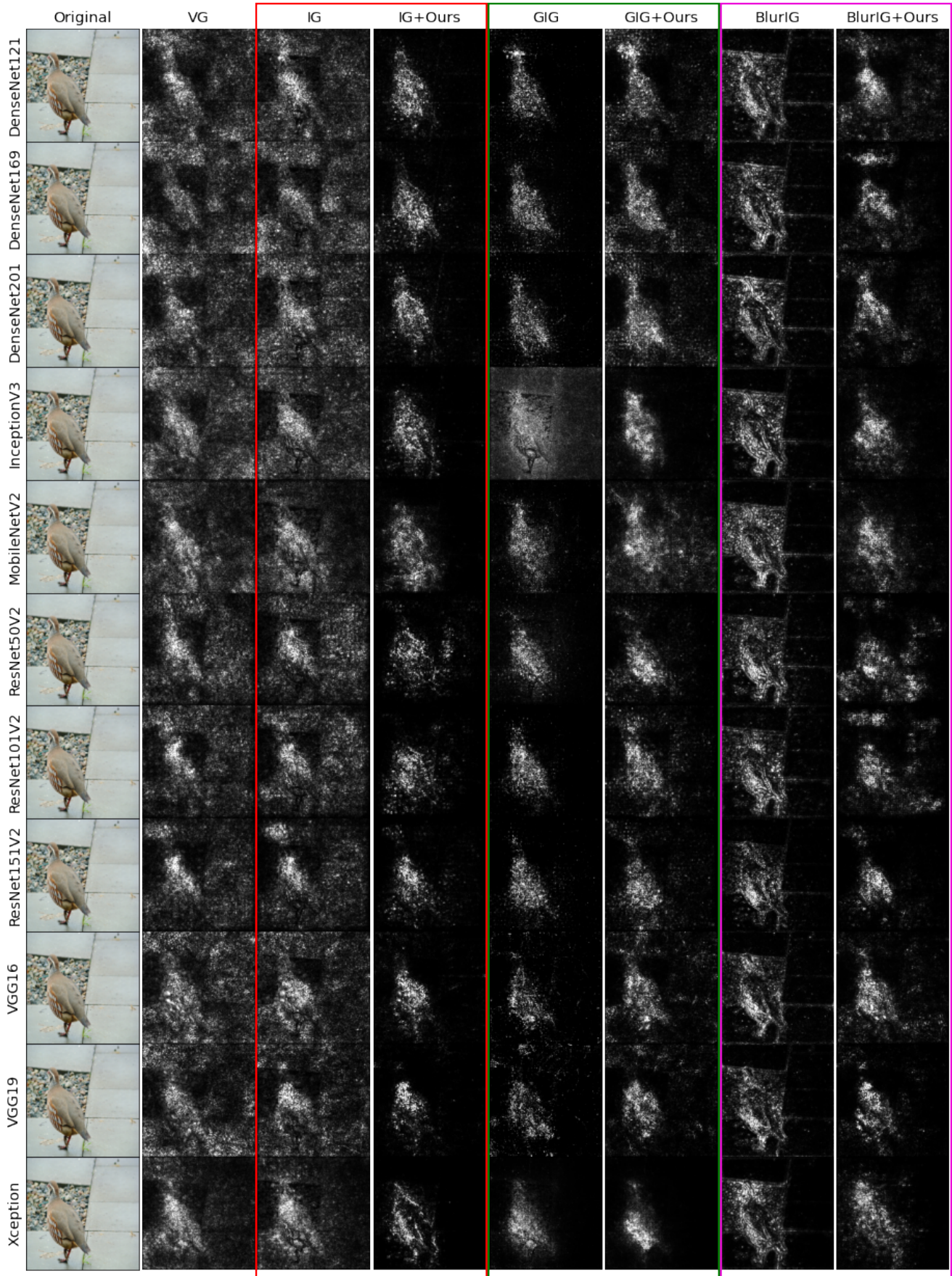


Figure 5. Predicted Label for all models: *partridge*

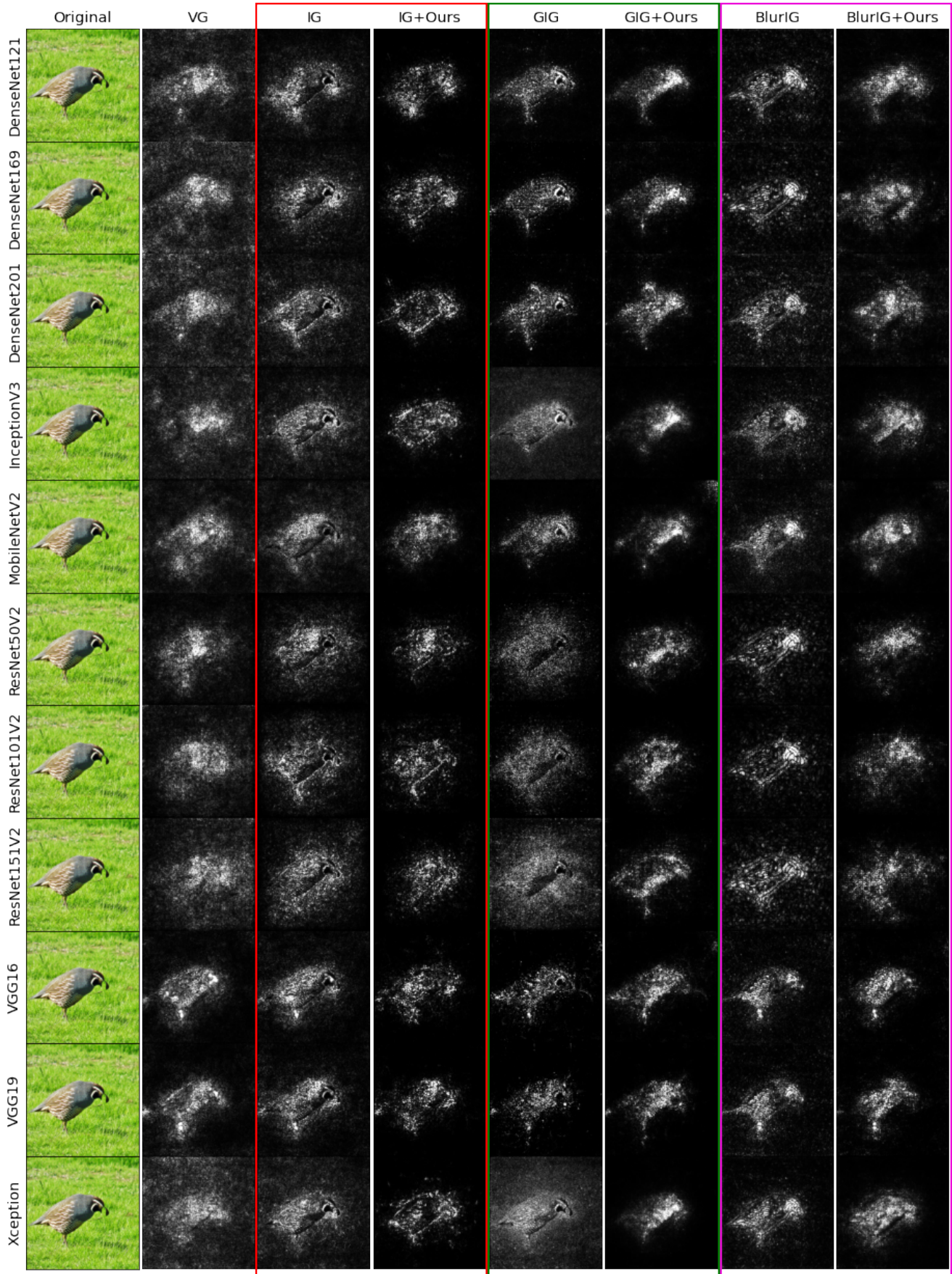


Figure 6. Predicted Label for all models: *quail*

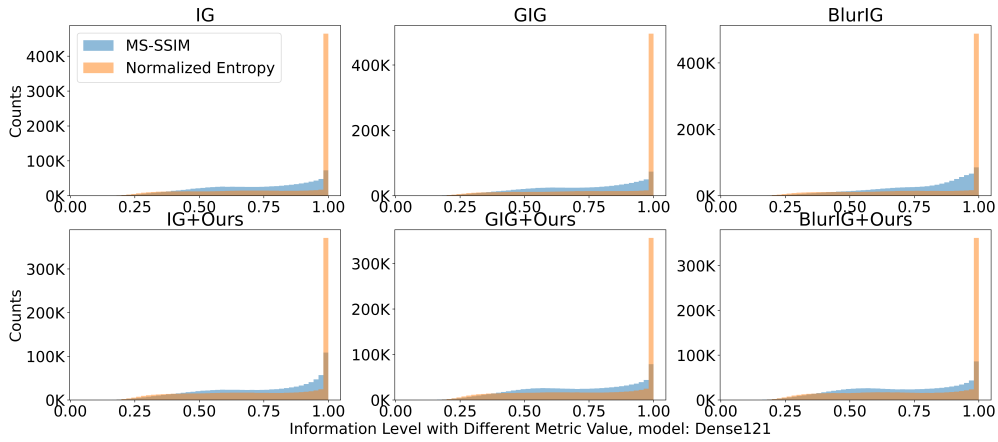


Figure 7. Modified distribution of bokeh images over MS-SSIM and Normalized Entropy [1]. Model: *DenseNet121*

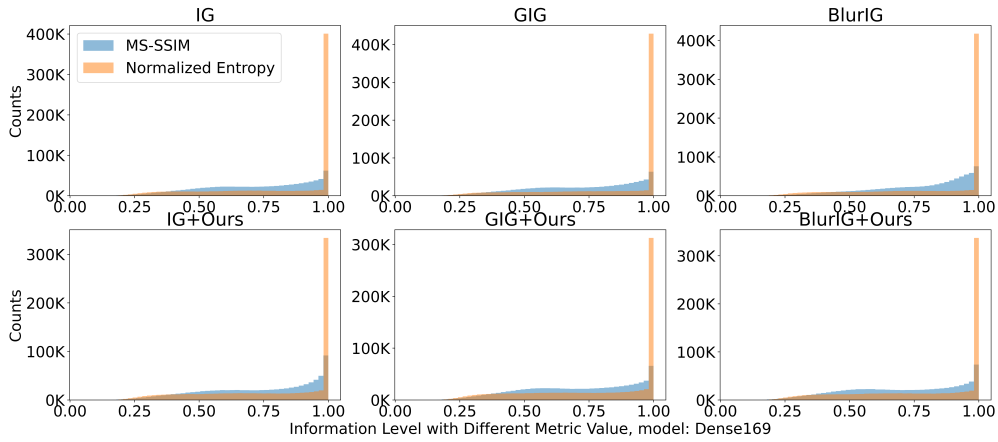


Figure 8. Modified distribution of bokeh images over MS-SSIM and Normalized Entropy [1]. Model: *DenseNet169*

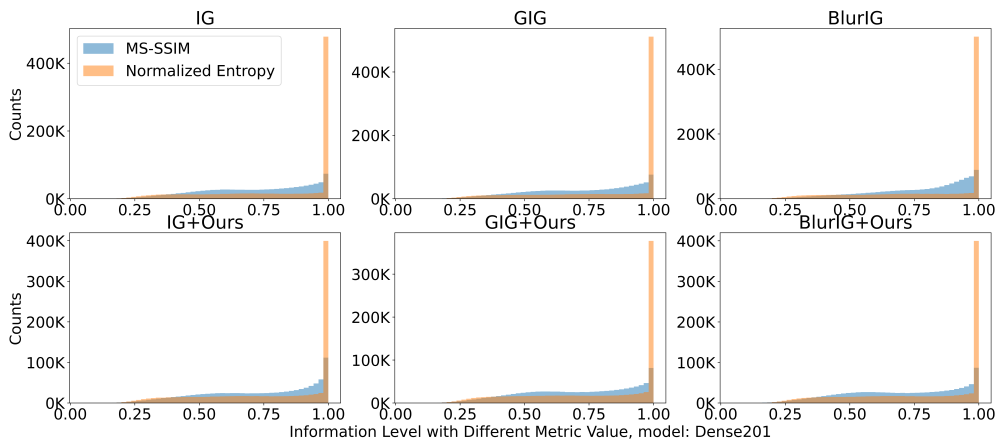


Figure 9. Modified distribution of bokeh images over MS-SSIM and Normalized Entropy [1]. Model: *DenseNet201*



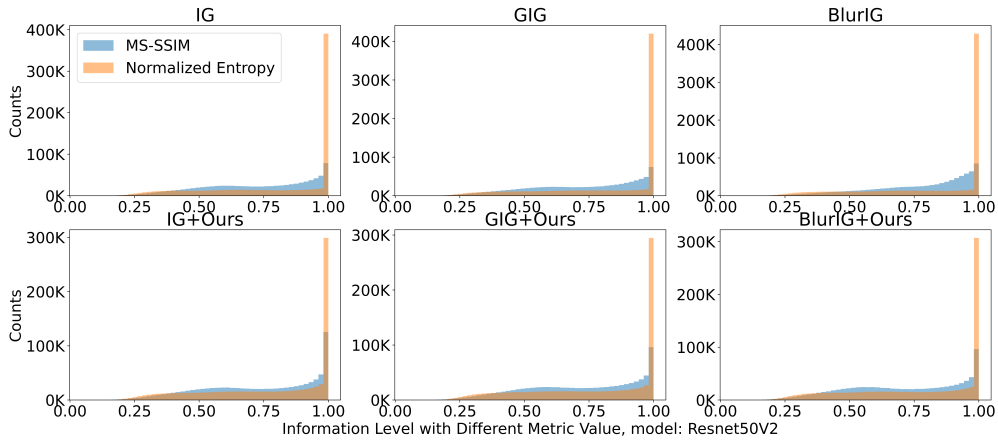


Figure 10. Modified distribution of bokeh images over MS-SSIM and Normalized Entropy [1]. Model: *Resnet50V2*

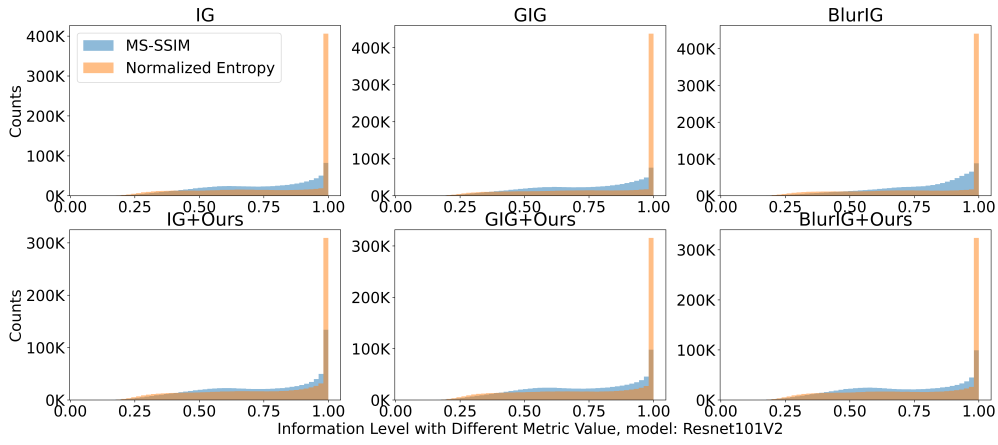


Figure 11. Modified distribution of bokeh images over MS-SSIM and Normalized Entropy [1]. Model: *Resnet101V2*

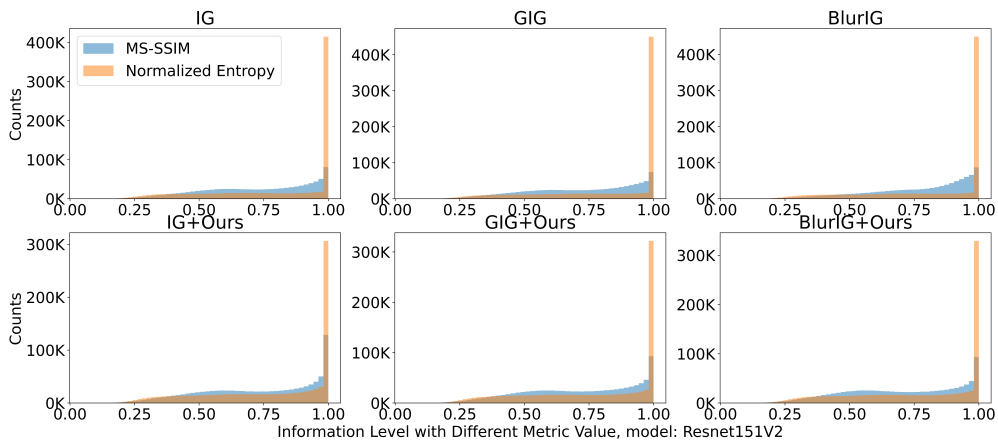


Figure 12. Modified distribution of bokeh images over MS-SSIM and Normalized Entropy [1]. Model: *Resnet151V2*

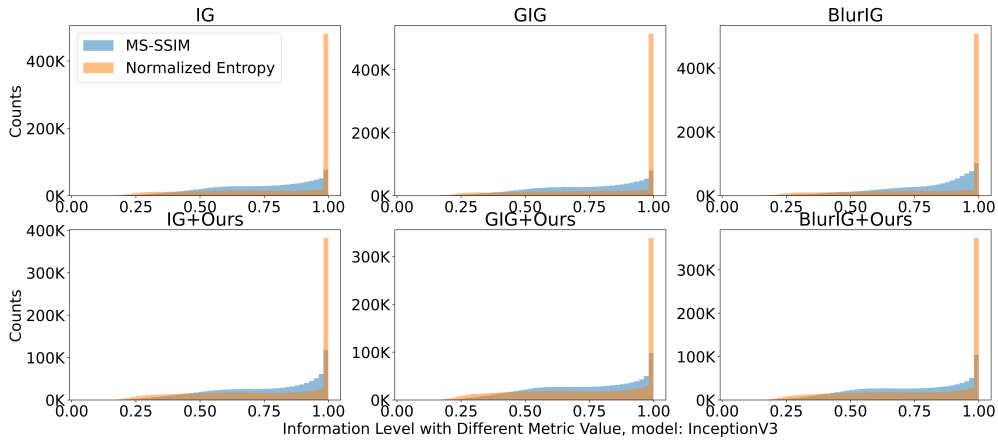


Figure 13. Modified distribution of bokeh images over MS-SSIM and Normalized Entropy [1]. Model: *InceptionV3*

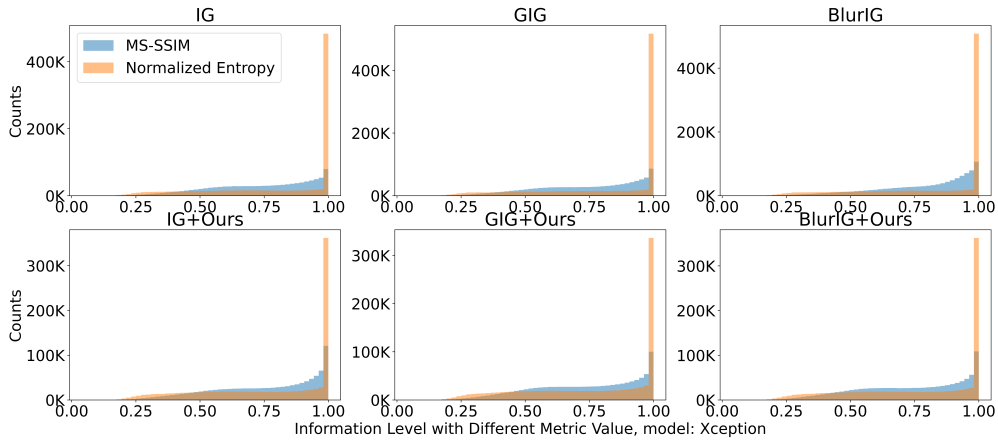


Figure 14. Modified distribution of bokeh images over MS-SSIM and Normalized Entropy [1]. Model: *Xception*

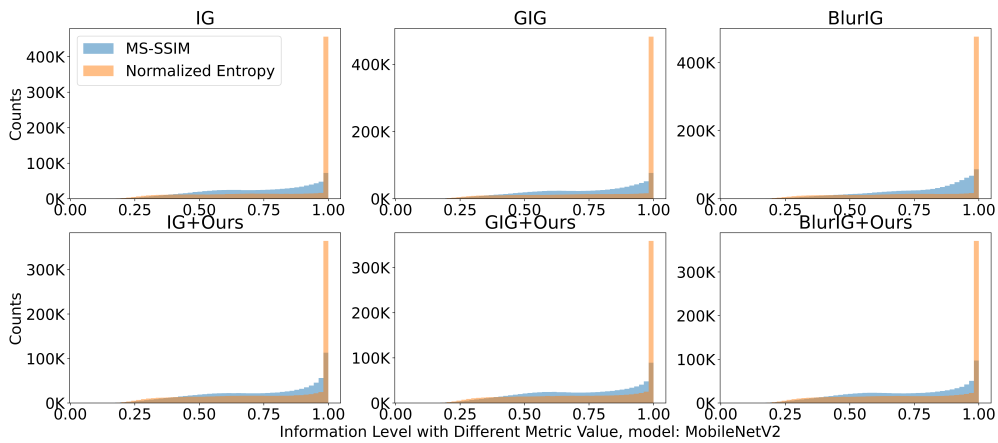


Figure 15. Modified distribution of bokeh images over MS-SSIM and Normalized Entropy [1]. Model: *MobileNetV2*

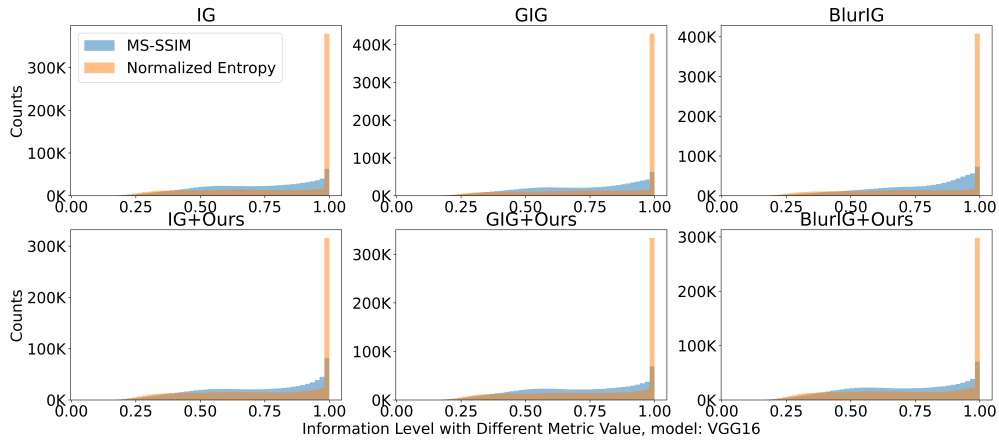


Figure 16. Modified distribution of bokeh images over MS-SSIM and Normalized Entropy [1]. Model: *VGG16*

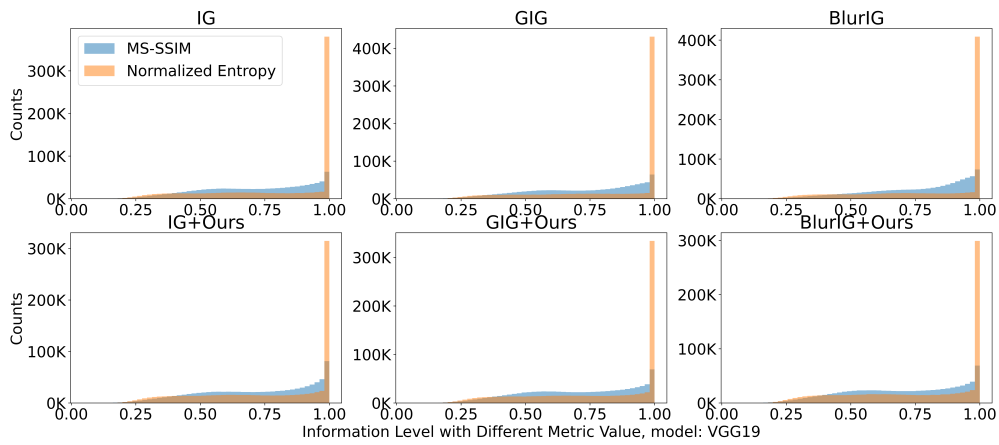


Figure 17. Modified distribution of bokeh images over MS-SSIM and Normalized Entropy [1]. Model: *VGG19*