Learn from View Correlation: An Anchor Enhancement Strategy for Multi-view Clustering

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Abstract

In recent years, anchor-based methods have achieved promising progress in multi-view clustering. The performances of these methods are significantly affected by the quality of the anchors. However, the anchors generated by previous works solely rely on single-view information, ignoring the correlation among different views. In particular, we observe that similar patterns are more likely to exist between similar views so such correlation information can be leveraged to enhance the quality of the anchors, which is also omitted. To this end, we propose a novel plug-and-play anchor enhancement strategy through view correlation for multi-view clustering. Specifically, we construct a view graph based on aligned initial anchor graphs to explore inter-view correlations. By learning from view correlation, we enhance the anchors of the current view using the relationships between anchors and samples on neighboring views, thereby narrowing the spatial distribution of anchors on similar views. Experimental results on seven datasets demonstrate the superiority of our proposed method over other existing methods. Furthermore, extensive comparative experiments validate the effectiveness of the proposed anchor enhancement module when applied to various anchor-based methods.

1. Introduction

Large-scale scenarios have always been a hot concern with the data explosion in our real world. Among these scenarios, large-scale multi-view clustering (MVC) has also drawn increasing attention in recent years[2, 4, 8, 18, 35]. In general, large-scale MVC methods can be categorized into two types, *i.e.,* matrix factorization-based methods and anchor-based methods. The former methods [11, 24, 41, 50] aim to generate a low-dimensional representation for further clustering by decomposing the original data, thereby avoiding the computational costs of similarity matrix construction. However, their efficiency usually decreases along with increasing dimension, thus constraining their scalability[10, 36]. Inspired by prototype learning [16, 48], anchors are introduced into MVC to reduce the scope for clustering from n to m, which denotes the number of samples and anchors, respectively. Specifically, anchor-based MVC methods [13, 27, 45] construct an anchor graph ($n \times m$) rather than a similarity matrix ($n \times n$). It will alleviate the aforementioned computation and scalability problems.

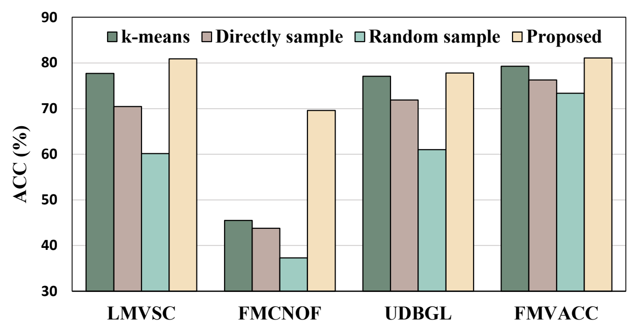


Figure 1. An example illustration of clustering results affected by initial anchors. The first two methods keep the anchors fixed, while the latter two methods update the anchors through iteration-optimization. Their clustering performances are influenced to varying degrees by different anchor generation approaches.

Anchor-based MVC methods typically consist of three procedures, including (1) anchor generation, (2) anchor graph construction, and (3) clustering. Early attempts usually generate anchors at first, which are fixed during constructing anchor graphs[31, 32, 47]. For example, SFMC [19] selects anchors by iterative sampling based on feature similarity. Besides, LMVSC [14] regards cluster centers generated by k-means of each view as the anchors. However, these methods are sensitive to the initial anchors (See Fig. 1), where the anchor selection is primitive and sometimes results in compromised performance.

To address the above issues, researchers make efforts on anchor optimization to guarantee better quality anchors[17, 26, 38]. For example, SMVSC [33] proposed a unified framework that combines anchor generation and anchor-graph construction, achieving iterative updates to optimize the anchors. Building upon this, UDBGL [7] introduced the Laplacian rank constraint to ensure that the anchor graph possesses a discrete cluster structure. The above iteration-optimization strategies actually aim to get a better combination of the anchor and its corresponding anchor graph instead of enhancing the quality of the anchors themselves. Meanwhile, to the best of our knowledge, almost all existing anchor optimization models belong to this type. Although they can get better and more stable clustering performance, they still suffer from the initial state for optimization[6, 15], which leaves us a great challenge as well as the opportunity, *i.e.,* **what is the new strategy to alleviate such problem?** In this paper, we provide a potential solution to it.

We observe that most anchor generation methods only rely on a single view[3, 12, 30]. Multiple works have proven that information of different views for the same sample will benefit the expressive ability of the sample [23, 29, 34, 49]. Similarly, different views for the same anchors should also be used to generate better anchors. Specifically, utilizing information across views and ensuring the consistency of anchor graphs on each view will benefit the anchor generation procedure. However, such a characteristic is not used for anchor enhancement in previous anchor-based MVC methods, and our strategy is developed based on it.

In this work, we propose an Anchor Enhancement strategy with View Correlation (AEVC) for multi-view clustering (MVC). The framework of AEVC is shown in Fig. 2. The proposed approach introduces a novel plug-and-play anchor enhancement strategy consisting of two modules: the view-graph learning module and the anchor enhancement module, which enhance the quality of anchors. Additionally, we improve the clustering performance by revising the existing anchor graph construction method based on view correlation. Specifically, we first align the initial anchors and construct a view-graph based on the aligned anchor graph to find the neighbors of each view. Then, new anchors for the current view are generated with the anchor graph of neighboring views, thereby enhancing the initial anchors. Finally, the inter-view correlations are combined to learn a consistent anchor graph between views with the enhanced anchors. AEVC optimizes anchors by absorbing information from similar views, improving initial anchor quality, and reducing distribution bias caused by the independent anchors generation in each view. The main contributions of our work can be summarized as follows:

- We propose AEVC, a plug-and-play anchor enhancement strategy for MVC based on view correlation, to address the issue of poor initial anchor quality affecting the performance of MVC.
- By learning from the view correlation, AEVC incorporates the anchor distribution information from neighboring views into the process of anchor enhancement. Theoretical and experimental validations demonstrate that the anchors enhanced by AEVC exhibit a tighter inter-view relationship.
- We compared our proposed approach to other anchor-based multi-view methods and demonstrated its superiority. Extensive experiments verify the effectiveness and generalization of the proposed AEVC strategy when integrated with typical anchor-based MVC methods.

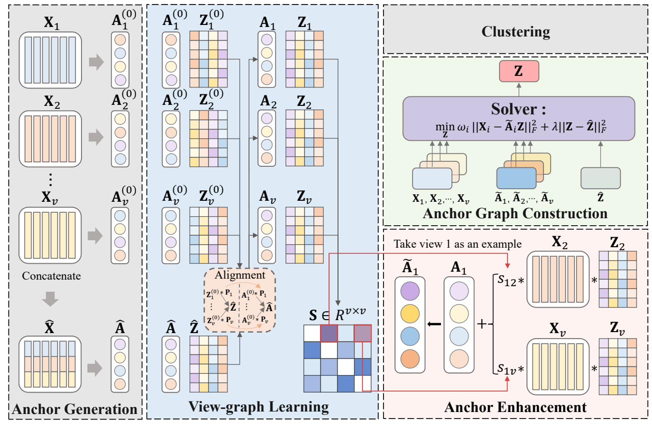


Figure 2. The framework of our proposed AEVC. Specifically, in the view-graph learning module, AEVC primarily performs anchor alignment and learns the view-graph. In the anchor enhancement module, AEVC leverages the neighboring relationships from the view-graph to enhance the anchors. Finally, AEVC executes anchor graph construction based on the enhanced anchors.

2. Related Work

2.1. Anchor-based Multi-view Clustering

To address the challenges posed by traditional multi-view clustering methods in handling large-scale datasets, Anchor-based MVC has been proposed in recent years[37, 45, 55]. The core idea of anchor-based MVC is to generate a set of representative anchors from the entire sample pool, where the number of anchors is typically much smaller than the total number of samples[25, 54]. In contrast to conventional methods that require establishing relationships among all samples, anchor-based approaches only involve computing similarities between anchors and samples. This significantly reduces the complexity associated with constructing the relationship graph. Additionally, adopting a reduced-dimensional graph further mitigates computational burdens, facilitating subsequent multi-view fusion and clustering with reduced computational overhead.

Traditional anchor-based MVC methods generate anchors by sampling[30, 31, 46]. Subsequent anchor graph construction and clustering are performed based on the initially determined anchors. For instance, Kang et al.[14]

and Yang et al. [46] advocate applying k-means to the original samples, selecting cluster centers as anchors to enhance their representativeness. Xia et al. [44] introduces a variance-based de-correlation anchor selection strategy, leveraging the internal data structure to augment the coverage capability of anchors. The performance of such methods is highly influenced by the quality of the initially generated anchors.

2.2. Anchor Optimzation Strategy

To tackle the above challenges, extensive research efforts have been devoted to anchor optimization in order to achieve superior anchor quality. Iteration-optimization methods propose to update anchors and anchor graphs, utilizing the anchor graph obtained from the previous iteration to update the anchors[51, 53, 56]. Sun et al. [33] initially propose learning anchors and constructing anchor graphs through iterative updates within the same framework. Liu et al. [26] further introduce Laplacian rank constraints, enabling the anchor graph to generate labels without directly needing post-processing. However, the results of iterative update algorithms are significantly influenced by the initial values of variables[6, 15]. Instead of directly enhancing the quality of the anchors themselves, the above methods primarily focus on obtaining an improved combination of the anchor and its corresponding anchor graph.

2.3. View Correlation Learning

Multi-view and multi-modal scenarios are widely seen nowadays [1, 20]. Among them, investigating the relationships between multiple objects is a crucial topic [5, 21, 22], especially for multi-view learning, which holds for anchor-based multi-view clustering[7, 44]. Some existing methods delve into the consistency of views by aligning anchors[42, 43, 52]. For instance, Li et al. [19] concatenates raw data from all views to select anchors, ensuring correspondence among anchors across views. Wang et al. [39] takes a step further by proposing an anchor alignment framework based on the features and structural information of anchors. Another category of methods employs early fusion to avoid conflicts between views[9, 28]. For instance, Wang et al. [40] use a projection matrix to learn shared anchors and anchor graphs in the latent space. However, the first category of methods solely utilizes alignment information between views, while the second category underutilizes complementary information among views. In the next section, we will introduce a novel approach based on a view graph, leveraging inter-view interactions to enhance anchors.

3. Method

In this section, we introduce our proposed Anchor Enhancement strategy with View Correlation (AEVC) for multi-view clustering. We begin by sequentially describing the three modules of AEVC. Subsequently, a complexity analysis of the proposed algorithm is provided. Lastly, a theoretical analysis is conducted to evaluate the effectiveness of AEVC.

3.1. Overview

Existing anchor-based MVC methods can be divided into three steps: (1) anchor generation, (2) anchor graph construction, (3) clustering. The clustering performance of existing anchor-based MVC methods heavily relies on the quality of initially generated anchors. Although iteration-optimization strategies alleviate this issue by exploring combinations of anchors and anchor graphs, they are still impacted by the optimization of the initial state. To address this problem, we propose an anchor enhancement strategy based on view correlation for multi-view clustering. Specifically, different from existing methods, our approach introduces a novel plug-and-play anchor enhancement strategy consisting of two modules: the view-graph learning module and the anchor enhancement module, which helps to enhance the quality of anchors. Furthermore, we revise the existing anchor graph construction method based on view correlation, further improving the clustering performance.

3.2. View-graph Learning

Given multi-view data $\{\mathbf{X}_i\}_{i=1}^{v}$, where $\mathbf{X}_i \in \mathbb{R}^{d_i \times n}$ is compose of n samples with d_i dimension. Denoting $\mathbf{A}_i^{(0)} \in \mathbb{R}^{d_i \times m}$ as the generated anchor matrix in the i-th view, and $\mathbf{Z}_i^{(0)} \in \mathbb{R}^{m \times n}$ represents the corresponding anchor graph, where m is the number of anchors. Due to the varying dimensions and inconsistent distributions of data across different views, it is not feasible to directly assess the distance between $\mathbf{X}_i$ and $\mathbf{X}_j$. However, it is easy to calculate the similarity between anchor graphs from different views due to the same size of anchor graphs.

Nevertheless, the independently generated anchors on each view introduce the Anchor-Unaligned Problem [39]. This means that directly computing the similarity between anchor graphs of different views would be inaccurate due to the misalignment of rows corresponding to anchors. Thus, an anchor alignment operation must be performed before view-graph learning.

Existing anchor alignment strategies typically directly align the anchors on the first view, which carries the risk of being influenced by noise view. Here, we first concatenate the multi-view data column-wise to obtain $\hat{\mathbf{X}} = [\mathbf{X}_1; \ldots; \mathbf{X}_v] \in \mathbb{R}^{\sum_{i=1}^{v} d_i \times n}$. Then, we generate the anchors $\hat{\mathbf{A}} \in \mathbb{R}^{\sum_{i=1}^{v} d_i \times m}$ and the corresponding anchor graph $\hat{\mathbf{Z}} \in \mathbb{R}^{m \times n}$. According to [39], we can obtain the permutation matrix $\mathbf{P}_i$ in the i-th view by minimizing the distance between $\hat{\mathbf{Z}}$ and the permuted $\mathbf{Z}_i^{(0)}$, the formula is as follows,

$$\min_{\mathbf{P}_i} \left\|\hat{\mathbf{Z}} - \mathbf{P}_i\mathbf{Z}_i^{(0)}\right\|_{\mathbf{F}}^2, \\ \text{s.t. } \mathbf{P}_i\mathbf{1} = \mathbf{1}, \mathbf{P}_i^\top\mathbf{1} = \mathbf{1}, \mathbf{P}_i \in \{0,1\}^{m\times m}. \tag{1}$$

Then the anchor matrix and anchor graph can be aligned with the permutation matrix as follows:

$$\begin{cases} \mathbf{A}_i = \mathbf{A}_i^{(0)}\mathbf{P}_i^\top, \\ \mathbf{Z}_i = \mathbf{P}_i\mathbf{Z}_i^{(0)}. \end{cases} \tag{2}$$

After performing the alignment operation, we calculate the view graph $\mathbf{S} \in \mathbb{R}^{v\times v}$ as follows,

$$\begin{cases} \mathbf{s}_{ij} = \frac{1}{\|\mathbf{Z}_i - \mathbf{Z}_j\|_{\mathbf{F}}^2}, & i \neq j, \\ \mathbf{s}_{ij} = 0, & i = j,. \end{cases} \tag{3}$$

where $\mathbf{s}_{ij}$ denotes the similarity between the i-th and the j-th view.

3.3. Anchor Enhancement Strategy

Most existing methods independently generate anchors on each view, ignoring the correlation between views. In fact, anchors on similar views should have similar distributions, which means that the corresponding anchor graphs should be close to each other. Based on this assumption, we propose to enhance the anchors of the current view with information on neighboring views. One intuitive idea is to directly update the anchors based on the anchors of neighboring views. However, the dimensions of anchors on different views are not the same, making direct transfer infeasible. Instead of utilizing feature consistency, we focus on structural consistency. Specifically, we reconstruct new anchors by leveraging the anchor graphs from neighboring views and the data from the current view, thereby enhancing the initial anchors. The formulation of the anchor enhancement on the i-th view is as follows:

$$\tilde{\mathbf{A}}_i = \frac{1}{\mu}(\mathbf{A}_i + \gamma \sum_{\mathbf{X}_j\in\Omega_\zeta(\mathbf{X}_i)} \hat{s}_{ij}\mathbf{X}_i\mathbf{Z}_j^\top). \tag{4}$$

where $\mu = 1+\gamma\sum_{\mathbf{X}_j\in\Omega_\zeta(\mathbf{X}_i)}\hat{s}_{ij}$ ensures that the enhanced anchors are a convex combination of the initial anchors and the reconstructed anchors. $\Omega_\zeta(\mathbf{X}_i)$ denotes the ζ-neighbors of the i-th view, which is defined as:

$$\Omega_\zeta(\mathbf{X}_i) = \arg\max_{\mathbf{X}_j} \sum_{j=1} |s_{ij}|. \tag{5}$$

γ is the enhancement rate, which is used to adjust the influence of the initial anchors on the enhanced version. $\hat{\mathbf{S}}$ represents the normalized view-graph based on the following settings, which controls the influence of different neighbors during anchor enhancement:

$$\hat{s}_{ij} \begin{cases} s_{ij}, & \mathbf{X}_j \in \Omega_\zeta(\mathbf{X}_i), \\ 0, & \mathbf{X}_j \notin \Omega_\zeta(\mathbf{X}_i). \end{cases} \tag{6}$$

By enhancing the anchors with Eq. (4), the quality of the initial anchors is improved. Moreover, the enhancement further achieves a better alignment between anchors on similar views. In the subsequent sections, we will analyze the aforementioned conclusions from both theoretical and experimental perspectives. Furthermore, our proposed anchor enhancement module is plug-and-play and we will showcase its performance improvement in the experiments by integrating different anchor-based multi-view clustering methods.

3.4. Anchor Graph Construction

After obtaining high-quality anchors through our proposed anchor enhancement module, it is crucial to construct a reliable anchor graph that ensures view consistency. Since the enhanced anchors are aligned, we can directly generate a common anchor graph. However, the contributions of different views in constructing the anchor graph are not equal. To mitigate the influence of view discrepancies on building a consistent representation, we assign higher weights to views that have stronger associations with other views. Specifically, the contribution weight $\omega_i = \sum_{j=1}^{v} s_{ij}$ for the i-th view is computed as the sum of its similarities with all other views. Furthermore, to avoid the impact of noise during anchor graph construction, we introduce the learned anchor graph $\hat{\mathbf{Z}}$ from the concatenated views as guidance. Ultimately, the anchor graph construction is formulated as follows:

$$\min_{\mathbf{Z}} \omega_i \left\|\mathbf{X}_i - \tilde{\mathbf{A}}_i\mathbf{Z}\right\|_{\mathbf{F}}^2 + \lambda\left\|\mathbf{Z} - \hat{\mathbf{Z}}\right\|_{\mathbf{F}}^2, \\ \text{s.t. } \mathbf{Z} \geq 0, \mathbf{Z}^\top\mathbf{1} = \mathbf{1}. \tag{7}$$

Due to the independence between each column of $\mathbf{Z}$, let z_j denote the j-th column of $\mathbf{Z}$. The aforementioned problem can be transformed into n Quadratic Programming(QP) problems, formulated as follows:

$$\min \tfrac{1}{2}\mathbf{z}_j^\top\mathbf{H}\mathbf{z}_j + f\mathbf{z}_j, \\ \text{s.t. } \mathbf{z}_j \geq 0, \mathbf{z}_j^\top\mathbf{1} = 1, \tag{8}$$

where the matrix $\mathbf{H} = 2\left(\sum_{i=1}^{v}\omega_i\tilde{\mathbf{A}}_i^\top\tilde{\mathbf{A}}_i + \lambda\mathbf{I}\right)$ and the vector $f = -2\left(\sum_{i=1}^{v}\omega_i\mathbf{X}_{i[:,j]}^\top\tilde{\mathbf{A}}_i + \lambda\hat{z}_j\right)$. The optimal $\mathbf{Z}$ can be obtained by solving the above QP problem. Finally, the results can be generated by performing k-means on the left singular vector of $\mathbf{Z}$ [14]. The entire procedure of AEVC is shown in Algorithm 1.

Algorithm 1: The proposed AEVC

Input: Multi-view datasets $\{\mathbf{X}_i\}_{i=1}^{v}$, the number of anchors m, the parameter γ, λ and the number of clusters k.
Output: Clustering results.

```
Initialize {A_i^(0)}_{i=1}^v, Â, {Z_i^(0)}_{i=1}^v and Ẑ;
// View-graph Learning
for i = 1 → v do
    Calculate permutation matrix P_i with Eq. (1);
    Perform alignment with Eq. (2);
Learn view-graph S as defined in Eq. (3);
// Anchor Enhancement
for i = 1 → v do
    Find ζ-neghbors Ω_ζ (X_i) as defined in Eq. (5);
Calculate normalized view-graph Ŝ with Eq. (6);
for i = 1 → v do
    Enhance A_i with Eq. (4);
// Anchor Graph Construction
Obtain Z by solving Eq. (7);
Perform k-means on the left singular vector of Z.
```

3.5. Discussion

Computational complexity. The computational complexity of the proposed AEVC mainly consists of three modules. The complexity of the view-graph learning module is $\mathcal{O}(m^3v + nmv)$. Specifically, the computation of v permutation matrices $\mathbf{P}_i$ has a complexity of $\mathcal{O}(m^3v)$, and the computation of the view graph has a complexity of $\mathcal{O}(nmv)$. The time complexity of the anchor enhancement module is $\mathcal{O}(nmdv)$, where $d = \sum_{i=1}^{v} d_v$, primarily arising from matrix multiplication during enhancement. The complexity of the anchor graph construction module is $\mathcal{O}(nm^3v + nmdv)$, where solving all QP problems requires a time complexity of $\mathcal{O}(nm^3v)$, and the involved matrix multiplication requires $\mathcal{O}(nm^3v + nmdv)$. Therefore, the overall computational complexity of AEVC is $\mathcal{O}(nm^3v + m^3v + nmdv + nmv)$, which exhibits linear complexity with respect to the number of samples.

Space complexity. During the execution of the AEVC algorithm, the main variables that need to be stored include matrices $\mathbf{X}_i$, $\mathbf{A}_i$, $\mathbf{Z}_i$, $\hat{\mathbf{A}}$, $\hat{\mathbf{Z}}$, $\mathbf{P}_i$, $\mathbf{S}$, and $\hat{\mathbf{S}}$. The overall space complexity of AEVC is $\mathcal{O}(nd + ndv + nmdv + v^2)$, which is also linear with respect to the number of samples and can be extended to large-scale scenarios.

Two Property of enhanced anchors. We have discovered that the anchors enhanced by the proposed AEVC exhibit stronger inter-view consistency. The theoretical analysis is provided below.

Assumption 1. *Given a two-view data $\{\mathbf{X}_1, \mathbf{X}_2\}$, denoting $\{\mathbf{A}_1, \mathbf{A}_2\}$ are the generated initial anchors, and $\{\mathbf{Z}_1, \mathbf{Z}_2\}$ are the corresponding anchor graphs under the assumption that $\mathbf{X}_1 = \mathbf{A}_1\mathbf{Z}_1$ and $\mathbf{X}_2 = \mathbf{A}_2\mathbf{Z}_2$.*

For the convenience of derivation, we simplify the anchor enhancement strategy in Eq. (4) with $\alpha \in (0, 1)$ as follows:

$$\begin{cases} \tilde{\mathbf{A}}_1 = \alpha\mathbf{A}_1 + (1-\alpha)\mathbf{X}_1\mathbf{Z}_2^\top, \\ \tilde{\mathbf{A}}_2 = \alpha\mathbf{A}_2 + (1-\alpha)\mathbf{X}_2\mathbf{Z}_1^\top. \end{cases} \tag{9}$$

The corresponding anchor graphs $\{\tilde{\mathbf{Z}}_1, \tilde{\mathbf{Z}}_2\}$ are constructed under the same assumption that $\mathbf{X}_1 = \tilde{\mathbf{A}}_1\tilde{\mathbf{Z}}_1$ and $\mathbf{X}_2 = \tilde{\mathbf{A}}_2\tilde{\mathbf{Z}}_2$. Then we can derive that

$$\begin{cases} \tilde{\mathbf{Z}}_1 = \alpha\mathbf{Z}_1 + (1-\alpha)\mathbf{Z}_2, \\ \tilde{\mathbf{Z}}_2 = \alpha\mathbf{Z}_2 + (1-\alpha)\mathbf{Z}_1. \end{cases} \tag{10}$$

We can deduce the following properties:

Property 1. *The disparity among the anchor graphs on each view is reduced after anchor enhancement. Mathematically,*

$$\left\|\tilde{\mathbf{Z}}_1 - \tilde{\mathbf{Z}}_2\right\|_{\mathbf{F}} < \left\|\mathbf{Z}_1 - \mathbf{Z}_2\right\|_{\mathbf{F}}. \tag{11}$$

Proof. According to Eq. (10), we can derive that

$$\begin{aligned} \left\|\tilde{\mathbf{Z}}_1 - \tilde{\mathbf{Z}}_2\right\|_{\mathbf{F}} &= \left\|(2\alpha-1)(\mathbf{Z}_1 - \mathbf{Z}_2)\right\|_{\mathbf{F}} \\ &= |2\alpha-1| \cdot \left\|\mathbf{Z}_1 - \mathbf{Z}_2\right\|_{\mathbf{F}} \\ &< \left\|\mathbf{Z}_1 - \mathbf{Z}_2\right\|_{\mathbf{F}}. \end{aligned} \tag{12}$$

This completes the proof. □

Property 1 implies that anchor enhancement leads to a more consistent distribution of anchors in similar views, thereby establishing a tighter alignment.

Property 2. *Assuming there exists a value $\beta \in (0, 1)$ such that $\mathbf{Z} = \beta\mathbf{Z}_1 + (1-\beta)\mathbf{Z}_2$ is the optimal consistent anchor graph among all views, the enhanced anchor graph is closer to the optimal version after enhancement than before for all $\alpha \in (2\beta - 1, 1) \cap (0, 1)$. Mathematically,*

$$\left\|\tilde{\mathbf{Z}}_1 - \mathbf{Z}\right\|_{\mathbf{F}} < \left\|\mathbf{Z}_1 - \mathbf{Z}\right\|_{\mathbf{F}}. \tag{13}$$

Proof. According to Eq. (10), we can derive that

$$\begin{aligned} \left\|\tilde{\mathbf{Z}}_1 - \mathbf{Z}\right\|_{\mathbf{F}} &= \left\|(\alpha-\beta)(\mathbf{Z}_1 - \mathbf{Z}_2)\right\|_{\mathbf{F}} \\ &= |\alpha-\beta| \cdot \left\|\mathbf{Z}_1 - \mathbf{Z}_2\right\|_{\mathbf{F}} \\ &< |1-\beta| \cdot \left\|\mathbf{Z}_1 - \mathbf{Z}_2\right\|_{\mathbf{F}} \\ &= \left\|(1-\beta)(\mathbf{Z}_1 - \mathbf{Z}_2)\right\|_{\mathbf{F}} \\ &= \left\|\mathbf{Z}_1 - \mathbf{Z}\right\|_{\mathbf{F}}. \end{aligned} \tag{14}$$

This completes the proof. □

Property 2 implies that anchor enhancement helps reduce the gap between the anchor graph on each view and the optimal consistent anchor graph. It is easy to derive the multi-view extended versions of the above properties.

4. Experiment

4.1. Experiment Setup

Datasets. Seven multi-view datasets are employed in our experiments: Dermatology[1] contains diagnostic information for six types of skin diseases. ForestTypes[2] consists of satellite images of different forest types. BDGP[3] comprises both images and textual descriptions of Drosophila embryos. Reuters[4] is a news text dataset. Hdigit[5] consists of a set of handwritten digital images. VGGFace2[6] is a large-scale face recognition dataset. CIFAR100[7] is a subset of a large-scale tiny image dataset. Detail description is shown in Table 1.

Compared methods. We compare the proposed AEVC with existing state-of-the-art anchor-based MVC methods: Large-scale multi-view subspace clustering in linear time (LMVSC)[14]; Scalable Multi-view Subspace Clustering with Unified Anchors (SMVSC)[33]; Fast multi-view clustering via nonnegative and orthogonal factorization (FMCNOF)[46]; Fast Parameter-Free Multi-View Subspace Clustering With Consensus Anchor Guidance (FPMVS-CAG)[40]; Efficient Multi-View Clustering via Unified and Discrete Bipartite Graph Learning (UDBGL)[7]; Fast multi-view clustering via ensembles: Towards scalability, superiority, and simplicity (FastMICE)[10]; Flexible and Diverse Anchor Graph Fusion for Scalable Multi-View Clustering (FDAGF)[55]; Align then Fusion: Generalized Large-scale Multi-view Clustering with Anchor Matching Correspondences (FMVACC)[39].

Implementation details. For the above-compared algorithm, we have tuned their parameters to align with the relevant literature. We conducted a search for three hyperparameters in the proposed AEVC method using predefined lists. Specifically, the anchor number m is chosen from the set $[k, 2k, 5k]$, where k represents the cluster number. The enhancement rate γ is selected from $[0.1, 1, 10, 100]$, and the balancing hyperparameter λ is chosen from the range $[0.0001, 0.01, 1, 100]$. To ensure a fair comparison, algorithms relying on k-means for result acquisition underwent 50 independent runs, and the results were averaged for accuracy assessment. Our experimental evaluation incorporated four distinct metrics: clustering Accuracy (ACC), Normalized Mutual Information (NMI), Purity, and F-score.

[1]http://archive.ics.uci.edu/ml/datasets/Dermatology
[2]http://archive.ics.uci.edu/dataset/333/
[3]https://www.fruitfly.org/
[4]http://www.research.att.com/%7Elewis/reuters21578.html
[5]https://cs.nyu.edu/%7Eroweis/data.html
[6]https://www.robots.ox.ac.uk/vgg/data/vgg_face2/
[7]https://www.cs.toronto.edu/%7Ekriz/cifar.html

Table 1. Benchmark datasets description.

Dataset	n	v	k	$d_p(p = 1, \ldots, v)$
Dermatology	358	2	6	12/22
ForestTypes	523	3	4	9/9/9
BDGP	2500	3	5	1000/500/250
Reuters	7200	5	6	4819/4810/4892/4858/4777
Hdigit	10000	2	10	784/256
VGGFace2	36287	4	100	3944/576/512/640
CIFAR100	50000	4	100	944/576/512/640

4.2. Superiority of AEVC

We compare our proposed AEVC with eight state-of-the-art anchor-based multi-view clustering methods on seven datasets, as illustrated in Table 2. From the results, we have the following observations:

- The experimental results reveal that our proposed algorithm consistently outperforms other anchor-based MVC methods across all datasets, confirming the superiority of AEVC. In terms of ACC, AEVC surpasses the second-best algorithm by 6.9%, 4.1%, 1.9%, 9.9%, 1.3%, 15.8%, and 9.5% on different datasets.
- Compared to previous methods based on iteratively optimized anchors (SMVSC, FPMVS-CAG, UDBGL, FDAGF, FMVACC), AEVC exhibits superior clustering performance, emphasizing the effectiveness of our anchor enhancement approach over iteration-optimization methods.

4.3. Effectiveness of Enhanced Anchors

We carried out an ablation study to demonstrate the effectiveness of the proposed anchor enhancement strategy. In our approach, the anchor enhancement strategy typically comprises two steps: anchor alignment in the view-graph learning module and anchor enhancement module. We denote the method of removing the entire anchor enhancement strategy as a baseline and compare it with those where anchor alignment and enhancement are sequentially incorporated. From the experimental results shown in Table 3, it is observed that anchor alignment leads to an improvement in clustering performance, as misalignment of anchors can result in incorrect multi-view fusion[39]. Furthermore, with the application of our anchor enhancement strategy, there is a further improvement in clustering performance, providing strong evidence for the effectiveness of the anchor enhancement strategy.

To further demonstrate the effectiveness of the enhanced anchors, we incorporate randomly generated anchors from the original data at different proportions and showcase the corresponding clustering performance in Fig. 3(a). For instance, when the proportion is 10%, it means that 90%

Table 2. Clustering performance comparison of eight anchor-based MVC methods on seven datasets. The best is marked in bold and underlined, the second best is marked in bold.

Datasets	LMVSC	SMVSC	FMCNOF	FPMVS-CAG	UDBGL	FastMICE	FDAGF	FMVACC	Proposed
					ACC				
Dermatology	79.02±0.07	78.64±0.05	62.01±0.00	82.96±0.07	84.08±0.00	87.15±0.00	84.41±7.90	84.39±4.34	**93.20±4.50**
ForestTypes	77.68±0.04	72.82±0.01	45.51±0.00	72.22±0.05	77.06±0.00	77.06±0.00	76.40±7.13	79.29±0.23	**82.53±0.09**
BDGP	49.52±0.02	37.22±0.02	31.08±0.00	32.62±0.01	39.28±0.00	49.00±0.00	47.30±3.16	59.51±4.00	**60.65±0.62**
Reuters	27.19±0.01	24.41±0.01	19.25±0.00	17.33±0.00	22.32±0.00	26.00±0.00	25.04±0.50	27.22±0.24	**29.91±0.02**
Hdigit	86.87±0.08	65.99±0.03	32.84±0.00	64.22±0.05	31.39±0.00	85.13±0.00	77.69±7.83	86.55±4.08	**88.01±2.97**
VGGFace	6.09±0.00	7.21±0.00	3.47±0.00	6.02±0.00	5.39±0.00	5.26±0.00	6.50±0.33	6.68±0.16	**8.35±0.07**
CIFAR100	8.92±0.00	8.48±0.00	4.40±0.00	7.46±0.00	7.83±0.00	9.81±0.00	7.47±0.13	7.04±0.12	**10.74±0.08**
					NMI				
Dermatology	70.17±0.04	66.62±0.03	54.24±0.00	71.90±0.05	87.41±0.00	82.73±0.00	79.70±4.34	76.27±2.19	**88.73±3.01**
ForestTypes	53.56±0.02	46.56±0.01	16.99±0.00	48.20±0.04	52.04±0.00	56.53±0.00	54.80±5.35	55.33±0.15	**59.63±0.09**
BDGP	25.85±0.02	9.85±0.01	10.29±0.00	10.02±0.01	14.90±0.00	26.03±0.00	21.90±1.95	**35.55±4.53**	30.92±0.65
Reuters	5.67±0.01	5.45±0.01	0.64±0.00	0.13±0.00	1.79±0.00	**7.23±0.00**	3.36±0.48	5.05±0.08	7.07±0.01
Hdigit	**89.33±0.03**	56.16±0.02	28.57±0.00	55.80±0.03	19.04±0.00	88.67±0.00	70.00±2.60	76.96±3.09	78.09±1.59
VGGFace	11.92±0.00	14.13±0.00	5.81±0.00	12.33±0.00	11.00±0.00	10.79±0.00	12.32±0.33	12.97±0.15	**14.33±0.11**
CIFAR100	15.49±0.00	14.83±0.00	8.49±0.00	13.87±0.00	14.83±0.00	16.82±0.00	13.93±0.15	11.45±0.16	**16.94±0.14**
					Purity				
Dermatology	80.97±0.04	80.35±0.04	62.85±0.00	83.55±0.07	84.64±0.00	87.15±0.00	87.86±4.51	85.38±2.04	**94.03±2.35**
ForestTypes	77.99±0.02	72.82±0.01	45.51±0.00	72.73±0.04	77.06±0.00	78.93±0.00	77.61±4.74	79.29±0.23	**82.53±0.09**
BDGP	49.64±0.02	37.80±0.01	33.32±0.00	34.82±0.01	40.16±0.00	49.44±0.00	47.36±3.08	59.73±3.67	**60.65±0.62**
Reuters	28.22±0.00	25.12±0.01	19.78±0.00	17.89±0.00	23.31±0.00	28.07±0.00	25.45±0.59	27.50±0.17	**30.84±0.02**
Hdigit	**88.93±0.06**	66.28±0.03	32.85±0.00	64.34±0.05	31.94±0.00	88.43±0.00	79.82±5.66	86.59±4.01	88.22±2.25
VGGFace	7.02±0.00	7.73±0.00	3.50±0.00	6.33±0.00	5.77±0.00	6.11±0.00	7.17±0.33	7.47±0.17	**9.43±0.08**
CIFAR100	10.41±0.00	9.09±0.00	4.49±0.00	7.84±0.00	8.69±0.00	11.11±0.00	8.28±0.13	7.89±0.14	**12.29±0.13**

Table 3. Ablation study on anchor enhancement module.

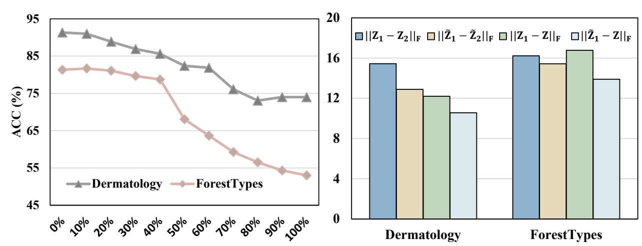
Methods	Dermatology	ForestTypes	BDGP
Baseline	90.87±3.60	70.76±0.05	54.38±0.10
+Align	91.82±3.78	75.51±0.17	57.97±0.09
+Align+Enh.	**93.20±4.50**	**82.53±0.09**	**60.65±0.62**

of the reconstructed anchors and 10% of random anchors are included during the enhancement process. From the results, as the proportion of randomly generated anchors increases, the clustering performance on both datasets continuously deteriorates, thus validating the effectiveness of our enhanced anchors.

To verify the two properties of enhanced anchors, we calculate the values on both sides of the equations in the properties on two datasets. As shown in Fig. 3(b), the difference between the anchor graphs in the two views before and after anchor enhancement decreases ($\left\|\tilde{\mathbf{Z}}_1 - \tilde{\mathbf{Z}}_2\right\|_{\mathbf{F}} < \|\mathbf{Z}_1 - \mathbf{Z}_2\|_{\mathbf{F}}$), validating Property 1. Additionally, the difference between the anchor graph and the optimal consistent anchor graph reduces before and after anchor enhancement ($\left\|\tilde{\mathbf{Z}}_1 - \mathbf{Z}\right\|_{\mathbf{F}} < \|\mathbf{Z}_1 - \mathbf{Z}\|_{\mathbf{F}}$), validating Property 2.

4.4. Effectiveness of Anchor Graph Construction

To further validate the effectiveness of the revised anchor graph construction module, we contrast our proposed approach with two versions without the view weight and regularized term respectively, as illustrated in Table 4. After incorporating weights learned from the views during fusion, there is a noticeable improvement in the clustering performance of the method. This highlights the advantage of assessing the importance of views in multi-view clustering



(a) Validation of enhancement strategy (b) Validation of Property 1 & 2

Figure 3. Validation of the proposed AEVC's effectiveness: (a) As the proportion of randomly generated anchors increases, the corresponding clustering performance decreases monotonically. (b) After anchor enhancement, the differences in anchor graphs between different views are reduced, and the enhanced anchor graph is closer to the optimal consensus anchor graph.

Table 4. Ablation study on revised anchor graph construction module. Other results are provided in supplementary materials.

Methods	Dermatology	ForestTypes	BDGP
w/o. view weight	91.60±6.26	81.45±0.00	60.50±0.46
w/o. regularized term	89.98±3.40	79.75±0.10	60.64±0.62
Proposed	**93.20±4.50**	**82.53±0.09**	**60.65±0.62**

Table 5. Performance comparison between various anchor-based MVC w./w.o. our AEVC strategy.

Methods	Dermatology	ForestTypes	BDGP	Reuters	Hdigit	VGGFace	CIFAR100
Fixed Anchors Methods							
FMCNOF	62.01±0.00	45.51±0.00	31.08±0.00	19.25±0.00	32.84±0.00	3.47±0.00	4.40±0.00
FMCNOF-AE	**73.18±0.00**	**69.60±0.00**	**37.64±0.00**	**22.79±0.00**	**34.31±0.00**	3.21±0.00	**5.05±0.00**
LMVSC	79.02±0.07	77.68±0.04	49.52±0.02	27.19±0.01	86.87±0.08	6.09±0.00	8.92±0.00
LMVSC-AE	**88.23±7.80**	**80.90±5.49**	**52.13±0.34**	**28.84±0.67**	85.74±5.72	**6.83±0.16**	8.81±0.16
Updated Anchors Methods							
UDBGL	84.08±0.00	77.06±0.00	39.28±0.00	22.32±0.00	31.39±0.00	5.39±0.00	7.83±0.00
UDBGL-AE	81.01±0.00	**77.82±0.00**	**46.92±0.00**	**28.53±0.00**	**44.60±0.00**	**5.65±0.00**	**8.57±0.00**
FMVACC	84.39±4.34	79.29±0.23	59.51±4.00	27.22±0.24	86.55±4.08	6.68±0.16	7.04±0.12
FMVACC-AE	**88.41±3.13**	**81.08±0.23**	59.36±3.03	**33.12±2.49**	**88.42±2.93**	**7.13±0.17**	**7.28±0.15**

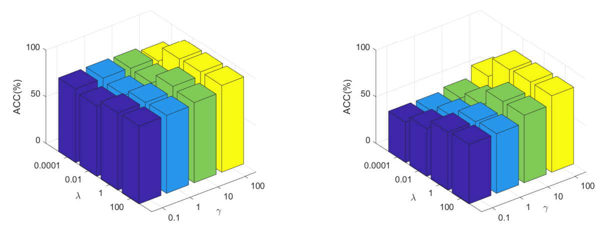
through the proposed strategy. After removing the regularization term, there was a decline in performance across different datasets, which further confirms the effectiveness of constructing anchor graphs guided by $\hat{\mathbf{Z}}$.

4.5. Transferability Analysis

To validate the transferability of AEVC, Table 5 presents a comparison of clustering accuracy between various types of anchor-based multi-view clustering methods with and without our anchor enhancement module. In this experiment, we selected two methods from each of the two categories employing different anchor strategies. Specifically, FMCNOF and LMVSC represent methods with fixed anchors, while UDBGL and FMVACC belong to methods with iteratively optimized anchors. As observed in the table, the clustering performance of the four anchor-based multi-view clustering methods experiences significant improvement across the majority of datasets after incorporating our proposed module. For instance, on the Reuters dataset, the clustering performance is notably enhanced by 18.4%, 6.1%, 27.8%, and 21.7% for FMCNOF, LMVSC, UDBGL, and FMVACC, respectively. This substantial improvement provides strong evidence for the effectiveness of AEVC when transferred to other anchor-based methods.

4.6. Sensitivity Analysis

Our proposed method involves three hyperparameters: the number of anchors m, the anchor enhancement rate γ, and the hyperparameter for balancing the regularization term λ. Due to space constraints, we primarily analyze the sensitivity of the latter two parameters here, while the analysis regarding m will be presented in the supplementary materials. To demonstrate the impact of γ and λ, we fix m and conduct grid searches within specified ranges. Results presented in Fig. 4 indicate that our method is significantly influenced by the anchor enhancement rate, as the extreme rate can disrupt the balance between newly generated and original anchors. Specifically, on the Dermatology dataset, setting γ to 100 yields the highest clustering accuracy.



(a) Dermatology (b) ForestTypes

Figure 4. Sensitivity analysis of λ and γ on two benchmark datasets. Others are displayed in supplementary materials.

5. Conclusion

In this paper, we propose an Anchor Enhancement strategy with View Correlation for multi-view clustering termed AEVC. Different from existing anchor-based MVC methods, AEVC introduces a novel plug-and-play anchor enhancement strategy consisting of two modules: the view-graph learning module and the anchor enhancement module. It can effectively enhance the quality of anchors, which is proven both experimentally and theoretically. Besides, we propose a revised anchor graph construction module based on the anchor enhancement strategy, which further improves the clustering performances. Experimental results show the promising capacity of AEVC, especially when integrating with other anchor-based methods. In the future, we will explore adaptive learning of enhancement rate.

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