# Efficient Solution of Point-Line Absolute Pose

Supplementary Material

## A. P2P1L in the coplanar case

Here, we describe the modified version of the P2P1L solver (Sec. 2.1) capable of handling the coplanar case. The input and output to this solver are the same as those of the original P2P1L solver.

The solver is identical to the original P2P1L solver until Equation (12). Then, we express values $R_{11}, R_{31}, R_{22}$ in terms of $R_{21}, R_{23}$ as

$$\begin{aligned} R_{11} &= \frac{b_2a_1 - b_1a_2}{b_1b_2X_2(Y_3 - Y_4)} \\ &\cdot\left(\left(Y_3X_4 - Y_4X_3 + \frac{b_1a_2(X_2Y_3 - X_1)}{b_2a_1 - b_1a_2}\right)R_{21} \right. \\ &\left. + Z_4Y_3R_{23}\right) \\ R_{31} &= \left(\frac{b_2 - b_1}{b_1b_2X_2(Y_3 - Y_4)}\left(Y_3X_4 - Y_4X_3 + \right.\right. \\ &\left.\left.\frac{b_1a_2(X_2Y_3 - X_1)}{b_2a_1 - b_1a_2}\right) + \frac{a_1 - a_2}{b_2a_1 - b_1a_2}\right)R_{21} \\ &+ \frac{(b_2 - b_1)Z_4Y_3}{b_1b_2X_2(Y_3 - Y_4)}R_{23}, \\ R_{22} &= \left(\frac{1}{Y_3(Y_3 - Y_4)}\left(Y_3X_4 - Y_4X_3 + \right.\right. \\ &\left.\left.\frac{b_1a_2(X_2Y_3 - X_1)}{b_2a_1 - b_1a_2}\right) - \frac{X_3}{Y_3} - \frac{b_1a_2X_2}{Y_3(b_2a_1 - b_1a_2)}\right)R_{21} \\ &+ \frac{Z_4}{Y_3 - Y_4}R_{23} \end{aligned} \tag{42}$$

Then, we substitute (42) into (9), to obtain two bivariate quadratic constraints in $R_{21}$ and $R_{23}$. We can write them in matrix form as

$$\begin{pmatrix} c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{pmatrix} \cdot \begin{pmatrix} R_{21}^2 \\ R_{21}R_{23} \\ R_{23}^2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{43}$$

where the coefficients $c_1, c_2, c_3, d_1, d_2, d_3$ are rational functions of the problem data. Similarly to the original solver, we apply the change of variables

$$u = R_{21}^2, \quad v = R_{23}/R_{21} \tag{44}$$

Subtracting the two equations in (43), we obtain

$$(c_1 - d_1)u + (c_2 - d_2)uv + (c_3 - d_3)uv^2 = 0.$$

Assuming $u \neq 0$, we therefore have the univariate quadratic equation in $v$

$$(c_1 - d_1) + (c_2 - d_2)v + (c_3 - d_3)v^2 = 0. \tag{45}$$

We recover value $v$ as one of the roots of (45) and $u$ as

$$u = (c_1 + c_2v + c_3v^2)^{-1}, \quad \text{or} \quad u = (d_1 + d_2v + d_3v^2)^{-1}. \tag{46}$$

After the values $u$, $v$ are recovered, we obtain $R_{21}, R_{23}$ according to (43) and $R_{11}, R_{31}, R_{22}$ according to (42). The rest of the solver is identical to the original solver.

## B. Evaluation of the pose error.

In this section, we show the pose estimation errors obtained in the RANSAC experiment, following the same experimental setup as in Sec. 3.2. The results are presented in Table 6. As expected, the errors are the same for all solvers. This demonstrates that our solvers can achieve improved runtimes without sacrificing any accuracy.

| | P2P1L | | | | P1P2L | |
|---|---|---|---|---|---|---|
| Dataset | OUR | 3Q3 | *Ram.* SVD | *Ram.* LU | OUR | 3Q3 |
| Model House | 0.251, 0.429 | 0.251, 0.429 | 0.251, 0.429 | 0.251, 0.429 | 0.251, 0.429 | 0.251, 0.429 |
| Corridor | 0.573, 0.580 | 0.573, 0.580 | 0.573, 0.580 | 0.573, 0.580 | 0.573, 0.580 | 0.573, 0.580 |
| Merton I | 0.005, 3.2e-4 | 0.005, 3.2e-4 | 0.005, 3.2e-4 | 0.005, 3.2e-4 | 0.005, 3.2e-4 | 0.005, 3.2e-4 |
| Merton II | 0.003, 4.6e-4 | 0.003, 4.6e-4 | 0.003, 4.6e-4 | 0.003, 4.6e-4 | 0.003, 4.6e-4 | 0.003, 4.6e-4 |
| Merton III | 0.007, 0.001 | 0.007, 0.001 | 0.007, 0.001 | 0.007, 0.001 | 0.007, 0.001 | 0.007, 0.001 |
| Library | 0.011, 0.002 | 0.011, 0.002 | 0.011, 0.002 | 0.011, 0.002 | 0.011, 0.002 | 0.011, 0.002 |
| Wadham | 0.007, 0.001 | 0.007, 0.001 | 0.007, 0.001 | 0.007, 0.001 | 0.007, 0.001 | 0.007, 0.001 |

Table 6. **RANSAC error**, on Oxford Multi-view dataset [1], in degrees. Every cell shows rotation and translation error.