Supplementary Material
DiffAvatar: Simulation-Ready Garment Optimization with Differentiable Simulation

Figure 1. After recovering simulation-ready assets, we can easily generate novel simulation results.

1. DiffXPBD: Differentiable Simulation

We provide details on the implementation of our differentiable simulator which builds upon DiffXPBD [1]. The simulation moves the states forward in time using $\mathbf{q}_{n+1} = \mathbf{F}_n(\mathbf{q}_{n+1}, \mathbf{q}_n, \mathbf{u})$, see Eq. (2). The adjoint states $\hat{\mathbf{Q}}$ are computed in a backward pass using

$$\hat{\mathbf{q}}_{n-1} = \left(\frac{\partial \mathbf{F}_{n-1}}{\partial \mathbf{q}_n}\right)^\top \hat{\mathbf{q}}_{n-1} + \left(\frac{\partial \mathbf{F}_n}{\partial \mathbf{q}_n}\right)^\top \hat{\mathbf{q}}_n + \left(\frac{\partial \phi}{\partial \mathbf{q}_n}\right)^\top \tag{1}$$

The XPBD simulation frameworks uses the following update scheme.

$$\begin{aligned} \mathbf{x}_{n+1} &= \mathbf{x}_n + \Delta\mathbf{x}(\mathbf{x}_{n+1}) + \Delta t\left(\mathbf{v}_n + \Delta t \mathbf{M}^{-1}\mathbf{f}_{\text{ext}}\right) \\ \mathbf{v}_{n+1} &= \frac{1}{\Delta t}\left(\mathbf{x}_{n+1} - \mathbf{x}_n\right) \end{aligned} \tag{2}$$

We find the adjoint evolution for the XPBD integration scheme by combining this with (1) as

$$\begin{aligned} \hat{\mathbf{x}}_n &= \hat{\mathbf{x}}_{n+1} + \left(\frac{\partial \Delta\mathbf{x}}{\partial \mathbf{x}} + \Delta t^2 \mathbf{M}^{-1}\frac{\partial \mathbf{f}_{\text{ext}}}{\partial \mathbf{x}}\right)^\top \hat{\mathbf{x}}_n \\ &\quad + \frac{\hat{\mathbf{v}}_n}{\Delta t} - \frac{\hat{\mathbf{v}}_{n+1}}{\Delta t} + \frac{\partial \phi}{\partial \mathbf{x}}^\top \\ \hat{\mathbf{v}}_n &= \left(\frac{\partial \Delta\mathbf{x}}{\partial \mathbf{v}} + \Delta t^2 \mathbf{M}^{-1}\frac{\partial \mathbf{f}_{\text{ext}}}{\partial \mathbf{v}}\right)^\top \hat{\mathbf{x}}_n \\ &\quad + \Delta t \hat{\mathbf{x}}_{n+1} + \frac{\partial \phi}{\partial \mathbf{v}}^\top \end{aligned} \tag{3}$$

After re-arranging and by substituting $\hat{\mathbf{v}}_n$ we find the adjoint states as

$$\begin{aligned} &\left(I - \frac{\partial \Delta\mathbf{x}}{\partial \mathbf{x}} - \Delta t^2 \mathbf{M}^{-1}\frac{\partial \mathbf{f}_{\text{ext}}}{\partial \mathbf{x}} - \frac{1}{\Delta t}\frac{\partial \Delta\mathbf{x}}{\partial \mathbf{v}} - \Delta t \mathbf{M}^{-1}\frac{\partial \mathbf{f}_{\text{ext}}}{\partial \mathbf{v}}\right)^\top \hat{\mathbf{x}}_n \\ &= 2\hat{\mathbf{x}}_{n+1} - \frac{\hat{\mathbf{v}}_{n+1}}{\Delta t} + \frac{\partial \phi}{\partial \mathbf{x}}^\top + \frac{1}{\Delta t}\frac{\partial \phi}{\partial \mathbf{v}}^\top \end{aligned} \tag{4}$$

1.1. Material Model

We use the orthotropic StVK model for modeling stretching and shearing and a hinge-based bending energy as detailed in [1]. The material parameters are recovered as part of the optimization process. Different material models can also be used.

2. Gradient of 3D Cloth Positions to 2D Patterns

To compute the gradient of the position with respect to the 2D patterns, we need to compute $\frac{\partial \Delta\mathbf{x}}{\partial \bar{\mathbf{x}}_i}$, for each of the 2D cloth vertex i $\in [0, 1, \ldots, n]$. We use the same set of constraints as in DiffXPBD, where $\mathbf{C} = [\boldsymbol{\epsilon}_{00}, \boldsymbol{\epsilon}_{11}, \boldsymbol{\epsilon}_{01}]$, and $\boldsymbol{\epsilon}$ is the Green strain.

$$\frac{\partial \Delta\mathbf{x}}{\partial \bar{\mathbf{x}}_i} = \mathbf{M}^{-1}\left(\frac{\partial \nabla \mathbf{C}}{\partial \bar{\mathbf{x}}_i}\Delta\boldsymbol{\lambda} + \nabla\mathbf{C}\frac{\partial \Delta\boldsymbol{\lambda}}{\partial \bar{\mathbf{x}}_i}\right) \tag{5}$$

$$\begin{aligned}\frac{\partial \Delta \boldsymbol{\lambda}}{\partial \bar{\mathbf{x}}_i} &= -\mathbf{J}^{-1}\frac{\partial \mathbf{J}}{\partial \bar{\mathbf{x}}_i}\mathbf{J}^{-1}\mathbf{b} + \mathbf{J}^{-1}\frac{\partial \mathbf{b}}{\partial \bar{\mathbf{x}}_i} \\ &= -\mathbf{J}^{-1}\left(\frac{\partial \mathbf{J}}{\partial \bar{\mathbf{x}}_i}\Delta \boldsymbol{\lambda} - \frac{\partial \mathbf{b}}{\partial \bar{\mathbf{x}}_i}\right)\end{aligned} \tag{6}$$

where $\frac{\partial \mathbf{b}}{\partial \bar{\mathbf{x}}_i}$ and $\frac{\partial \mathbf{J}}{\partial \bar{\mathbf{x}}_i}\Delta \boldsymbol{\lambda}$ are computed as

$$\begin{aligned}\frac{\partial \mathbf{b}}{\partial \bar{\mathbf{x}}_i} &= -\frac{\partial \mathbf{C}}{\partial \bar{\mathbf{x}}_i} - \frac{\partial \tilde{\boldsymbol{\alpha}}}{\partial \bar{\mathbf{x}}_i}\boldsymbol{\lambda} - \tilde{\boldsymbol{\alpha}}\frac{\partial \boldsymbol{\lambda}}{\partial \bar{\mathbf{x}}_i} \\ &= -\nabla \mathbf{C} - \tilde{\boldsymbol{\alpha}}\sum \frac{\partial \Delta \boldsymbol{\lambda}}{\partial \bar{\mathbf{x}}_i}\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{\partial \mathbf{J}}{\partial \bar{\mathbf{x}}_i}\Delta \boldsymbol{\lambda} &= \frac{\partial \nabla \mathbf{C}^T}{\partial \bar{\mathbf{x}}_i}\mathbf{M}^{-1}\nabla \mathbf{C}\Delta \boldsymbol{\lambda} + \nabla \mathbf{C}^T\mathbf{M}^{-1}\frac{\partial \nabla \mathbf{C}}{\partial \bar{\mathbf{x}}_i}\Delta \boldsymbol{\lambda} \\ &= \frac{\partial \nabla \mathbf{C}^T}{\partial \bar{\mathbf{x}}_i}\Delta \mathbf{x} + \nabla \mathbf{C}^T\mathbf{M}^{-1}\frac{\partial \nabla \mathbf{C}}{\partial \bar{\mathbf{x}}_i}\Delta \boldsymbol{\lambda}\end{aligned} \tag{8}$$

Given that $\boldsymbol{\epsilon}$ is a function of the deformation gradient $\mathbf{F}$, we provide the gradient of $\mathbf{F}$ with respect to the rest positions, and the rest should just follow from chain rule. Note that $\mathbf{F} = \mathbf{D}\bar{\mathbf{D}}^{-1}$, where the columns of $\mathbf{D}$ and $\bar{\mathbf{D}}$ are the edge vectors, such that

$$\begin{aligned}\mathbf{D} &= \begin{bmatrix}\mathbf{x}_0 - \mathbf{x}_2 & \mathbf{x}_1 - \mathbf{x}_2\end{bmatrix} \\ \bar{\mathbf{D}} &= \begin{bmatrix}\bar{\mathbf{x}}_0 - \bar{\mathbf{x}}_2 & \bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2\end{bmatrix}\end{aligned} \tag{9}$$

The dimensions of the matrix $\mathbf{D}$ is 3x2, $\bar{\mathbf{D}}$ is 2x2, and $\mathbf{F}$ is 3x2.

We compute the derivative of the deformation gradient using the Einstein notation for $\bar{\mathbf{x}}_0$ and $\bar{\mathbf{x}}_1$

$$\begin{aligned}\frac{\partial \mathbf{F}_{ij}}{\partial \bar{\mathbf{x}}_{mn}} &= \mathbf{D}_{ik}\frac{\partial \bar{\mathbf{D}}^{-1}_{kj}}{\bar{\mathbf{x}}_{mn}} \\ &= -\mathbf{D}_{ik}\bar{\mathbf{D}}^{-1}_{k\alpha}\frac{\partial \bar{\mathbf{D}}_{\alpha\beta}}{\partial \bar{\mathbf{x}}_{mn}}\bar{\mathbf{D}}^{-1}_{\beta j} \\ &= -\mathbf{D}_{ik}\bar{\mathbf{D}}^{-1}_{km}\bar{\mathbf{D}}^{-1}_{nj},\end{aligned} \tag{10}$$

where $\bar{\mathbf{x}}_{mn}$ is the nth component of $\bar{\mathbf{x}}_m$.

$$\frac{\partial \mathbf{F}_{ij}}{\partial \bar{\mathbf{x}}_2} = -(\frac{\partial \mathbf{F}_{ij}}{\partial \bar{\mathbf{x}}_0} + \frac{\partial \mathbf{F}_{ij}}{\partial \bar{\mathbf{x}}_1}) \tag{11}$$

3. Novel Animations

Figure 1 shows select frames from a novel simulated sequence with the recovered body shapes and garment patterns and materials.

References

[1] Tuur Stuyck and Hsiao-yu Chen. Diffxpbd: Differentiable position-based simulation of compliant constraint dynamics. *Proceedings of the ACM on Computer Graphics and Interactive Techniques*, 6(3):1–14, 2023. 1