## Boosting Flow-based Generative Super-Resolution Models via Learned Prior Supplementary Materials

#### 1. Overview

This supplementary material first provides an expanded notation table used in this work. Following this, more detailed information is presented for enhanced clarity, including additional experimental results and implementation details. Lastly, more qualitative comparisons are shown to demonstrate the capabilities of the proposed framework.

#### 2. Notation Table

Table 1 presents the notations used in the main text and their corresponding definitions.

#### 3. Detailed Analyses

This section analyzes the design flexibility of our latent module in arbitrary-scale super-resolution (SR) tasks and demonstrates the visual effects of employing different objective functions.

**Design flexibility in Arbitrary-scale SR tasks.** In the main paper, we present the arbitrary-scale SR results of our proposed framework, which employs UNet [12] as the latent generator. To further validate the design flexibility of our latent module, we provide results in arbitrary-scale SR tasks using EDSR-baseline [8] and Swin Transformer [9] (Swin-T) as alternative latent generators. This analysis includes two variants of our model: EDSR-baseline-LINF-LP and RRDB-LINF-LP, which facilitates a comprehensive comparison. For clarity, we denote them as EDSR-baseline-Ours and RRDB-Ours, respectively. The results in Table 2 demonstrate that our framework achieves promising results with all these backbones as the latent generator, proving the flexibility of our framework in arbitrary-scale SR tasks.

**Visual Effects with Different Objective Functions.** Figure 1 demonstrates the qualitative results obtained by training our latent module using different objective functions. As shown in this figure, our framework generates sharper content when employing the perceptual loss, in contrast to the smooth but blurry results with the latent space loss. In



LINF-LP (Ours, Latent) LINF-LP (Ours, VGG)

Figure 1. A qualitative comparison between the effects of employing different objective functions. "Latent" represents the exclusive use of latent space loss, while "VGG" denotes the sole employment of VGG perceptual loss [5].

addition, our framework effectively mitigates the grid artifacts presented in image produced by an LINF [14] model.

### 4. Implementation Details

In this section, we illustrate the derivation of the "best temperature map" and additional details of SRFlow-LP.

**Best Temperature Map.** We adapt the derivation of the "Optimal Objective Estimation" in SROOE [13] to generate our "best temperature map". Specifically, for each image, we generate a total of 21 outcomes using an LINF model, with sampling temperatures ranging from 0 to 1 at intervals of 0.05. Then, for every pixel in each image, we compute the LPIPS [16] values and select the temperature that yields the optimal LPIPS from these 21 images. Once the optimal temperature for every pixel is determined, we assemble these selections to form a "best temperature map". Note that in [13], they search a *t* value as a conditional input to

Notation	Definition
N	The size of a dataset.
x	Low-resolution image.
$x^{up}$	Bilinear upsampled low-resolution image.
y	High-resolution image.
$\hat{y}$	High-resolution image generated by the flow model.
m	The residual map between $y$ and $x_{up}$ .
z	A latent code in a standard normal distribution (i.e., $z \in \mathcal{N}(0, I)$ ).
$f_{ heta}$	Pre-trained flow-based super-resolution model [10, 14], which is parameterized by $\theta$ .
au	The sampling temperature (i.e., the standard deviation of a Gaussian distribution).
s	The scaling factor.
c	The coordinate of a patch [14].
n	The size of a patch [14].
G	The proposed latent module.
$\hat{z}$	The learned prior generated by $G$ .
$z^*$	The latent code corresponding to the ground truth image $y$ , transformed by $f_{\theta}$ .
$\mathcal{L}_{latent}$	The L1 loss calculated between $z^*$ and $\hat{z}$ .
$\mathcal{L}_{percep}$	The perceptual loss [5] calculated between $y$ and $\hat{y}$ .
$\mathcal{L}_{total}$	The final objective function.
$\Psi_{per}$	A pre-trained VGG19 [6] network.

Table 1. The notations used in this study and their corresponding definitions.

Table 2. The arbitrary-scale SR results on SR benchmark datasets. The names in the parentheses (*e.g.*, UNet) refer to the architecture of our latent generator. "*In-scales*" and "*OOD-scales*" refer to in- and out-of-training-distribution scales. LPIPS [16] scores are reported (lower is better), with the best and second-best highlighted in **red** and blue, respectively.

	Set5 [1]					Set14 [15]					B100 [11]					Urban100 [4]				
Method	In-scales		OOD-scales		In-scales			OOD-scales		In-scales			OOD-scales		In-scales			OOD-scales		
	×2	$\times 3$	$\times 4$	$\times 6$	$\times 8$	$\times 2$	$\times 3$	$\times 4$	×6	$\times 8$	$\times 2$	$\times 3$	$\times 4$	$\times 6$	$\times 8$	$\times 2$	$\times 3$	$\times 4$	$\times 6$	$\times 8$
EDSR-baseline-MetaSR [3]	0.057	0.125	0.175	0.253	0.326	0.094	0.207	0.286	0.395	0.460	0.147	0.285	0.376	0.492	0.565	0.065	0.157	0.233	0.352	0.446
EDSR-baseline-LIIF [2]	0.056	0.124	0.173	0.248	0.307	0.093	0.205	0.284	0.390	0.449	0.147	0.282	0.372	0.486	0.556	0.064	0.155	0.228	0.338	0.422
EDSR-baseline-LTE [7]	0.056	0.123	0.174	0.257	0.326	0.092	0.203	0.283	0.396	0.463	0.146	0.280	0.371	0.495	0.570	0.063	0.152	0.224	0.345	0.436
EDSR-baseline-LINF [14] ( $\tau = \tau_0$ )	0.035	0.067	0.088	0.158	0.249	0.064	0.115	0.163	0.275	0.375	0.108	0.172	0.207	0.319	0.451	0.050	0.110	0.158	0.273	0.386
EDSR-baseline-Ours (EDSR-baseline)	0.026	0.051	0.072	0.157	0.276	0.052	0.097	0.147	0.269	0.390	0.083	0.132	0.177	0.304	0.440	0.043	0.097	0.144	0.260	0.388
EDSR-baseline-Ours (UNet)	0.026	0.047	0.074	0.145	0.243	0.054	0.094	0.144	0.253	0.364	0.084	0.127	0.177	0.289	0.425	0.440	0.098	0.146	0.253	0.377
EDSR-baseline-Ours (Swin-T)	0.029	0.053	0.073	0.149	0.277	0.055	0.100	0.141	0.252	0.379	0.090	0.135	0.176	0.287	0.422	0.043	0.097	0.141	0.251	0.382
RRDB-LINF [14] ( $\tau = \tau_0$ )	0.034	0.064	0.084	0.147	0.247	0.059	0.110	0.146	0.252	0.359	0.097	0.152	0.194	0.306	0.444	0.040	0.093	0.137	0.239	0.354
RRDB-Ours (EDSR-baseline)	0.025	0.044	0.066	0.136	0.249	0.046	0.087	0.129	0.230	0.329	0.069	0.118	0.165	0.272	0.385	0.035	0.082	0.126	0.230	0.352
RRDB-Ours (Unet)	0.023	0.042	0.066	0.131	0.234	0.043	0.087	0.124	0.221	0.322	0.061	0.113	0.163	0.264	0.378	0.033	0.081	0.126	0.219	0.331
RRDB-Ours (Swin-T)	0.025	0.045	0.063	0.129	0.247	0.046	0.086	0.123	0.223	0.328	0.071	0.115	0.162	0.267	0.383	0.034	0.080	0.123	0.221	0.343

their GAN model, while in our approach, the temperature  $\tau$  represents the standard deviation of a Gaussian distribution.

**SRFlow-LP Implementation Details.** We found that employing the latent space loss as a regularization term for training SRFlow-LP effectively prevents exploding inverses. This effectiveness stems from the observation that models trained with the latent space loss tend to produce latent codes that fall within the training distribution, therefore avoiding subsequent exploding inverses. To further stabilize the inference process of SRFlow-LP, the initial prior is normalized before being processed by the latent module. In addition, we skip the iteration which encounters an exploding inverse during training. These techniques allow SRFlow-LP to be more stable during both training and inference without modifying the architecture or inference pipeline of the proposed framework.

#### 5. Additional Qualitative Results

#### 5.1. Qualitative Results of SRFlow-LP

Figs. 2 and 3 demonstrate our SRFlow-LP generates images with sharper details than the original SRFlow [10]. Fig. 4 also presents that SRFlow-LP effectively prevents SRFlow from encountering exploding inverses that display noisy patches within images.

#### 5.2. Qualitative Results of LINF-LP

Fig. 5 illustrates that our LINF-LP mitigates the grid artifacts in images generated by LINF [14], especially in areas with thin, repetitive linear structures.

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LR Image

Ground Truth



SRFlow ( $\tau = 0.9$ )

SRFlow-LP (Ours)

Figure 2. A qualitative comparison between the  $4 \times$  SR results of SRFlow [10] and our SRFlow-LP.



LR Image

Ground Truth



SRFlow ( $\tau = 0.9$ )

# **SRFlow-LP (Ours)**

Figure 3. A qualitative comparison between the  $4 \times$  SR results of SRFlow [10] and our SRFlow-LP.



Figure 4. Our framework SRFlow-LP effectively prevents SRFlow [10] from encountering exploding inverses.



Figure 5. A qualitative comparison between images generated by LINF [14] and LINF-LP.