Supplementary Material
BiPer: Binary Neural Networks using a Periodic Function

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Introduction

This supplementary document provides details on the mathematical development of the quantization error analysis from section 4.1; the derivation of the quantization error equation for BiPer in Eq. (18); and additional results for the frequency effect on the VGG-small architecture with CIFAR-10 dataset.

1. Convergence of summation in Eq. (14) to Eq. (15).

Recalling Eq. (14), the distribution of the weights for an arbitrary frequency ω_0 is given by

$$f_{\hat{\mathcal{W}}}(\hat{w}) = \sum_{k=-\infty}^{\infty} \frac{1}{2b\omega_0} \frac{1}{\sqrt{1-\hat{w}^2}} \exp\left(\frac{-|(-1)^k \arcsin(\hat{w}) + \pi k|}{b\omega_0}\right). \tag{1}$$

Considering that $-\pi/2 \leq \arcsin(\hat{w}) \leq \pi/2$, if $k < 1$ the absolute value in the exponential term is negative. Therefore,

$$|(-1)^k \arcsin(\hat{w}) + \pi k| = -\left((-1)^k \arcsin(\hat{w}) + \pi k\right). \tag{2}$$

Conversely, if $k > 1$

$$|(-1)^k \arcsin(\hat{w}) + \pi k| = (-1)^k \arcsin(\hat{w}) + \pi k. \tag{3}$$

Using Eqs. (2) and (3), we can rewrite the summation in Eq. (1) as

$$\begin{aligned} f_{\hat{\mathcal{W}}}(\hat{w}) &= \alpha \exp\left(-\frac{|\arcsin(y)|}{b\omega_0}\right) \\ &+ \alpha \sum_{k=-\infty}^{-1} \exp\left(\frac{(-1)^k \arcsin(\hat{w}) + \pi k}{b\omega_0}\right) \\ &+ \alpha \sum_{k=1}^{\infty} \exp\left(\frac{-(-1)^k \arcsin(\hat{w}) - \pi k}{b\omega_0}\right), \end{aligned} \tag{4}$$

where $\alpha = \frac{1}{2b\omega_0} \frac{1}{\sqrt{1-\hat{w}^2}}$. The first summation can be rewritten as

$$\begin{aligned} &\sum_{k=-\infty}^{-1} \exp\left(\frac{(-1)^k \arcsin(\hat{w}) + \pi k}{b\omega_0}\right) \\ &= \sum_{k=\infty}^{1} \exp\left(\frac{(-1)^k \arcsin(\hat{w}) - \pi k}{b\omega_0}\right). \end{aligned} \tag{5}$$

Replacing Eq. (5) in Eq. (4) we obtain

$$\begin{aligned} f_{\hat{\mathcal{W}}}(\hat{w}) &= \alpha \exp\left(-\frac{|\arcsin(y)|}{b\omega_0}\right) \\ &+ \alpha \sum_{k=1}^{\infty} \exp\left(\frac{(-1)^k \arcsin(\hat{w})}{b\omega_0}\right) \exp\left(\frac{-\pi k}{b\omega_0}\right) \\ &+ \alpha \sum_{k=1}^{\infty} \exp\left(\frac{-(-1)^k \arcsin(\hat{w})}{b\omega_0}\right) \exp\left(\frac{-\pi k}{b\omega_0}\right) \end{aligned} \tag{6}$$

Using the fact that $\cosh(x) = \frac{\exp(x)+\exp(-x)}{2}$ is an even function, and the convergence of the harmonic series, we finally obtain the pdf of BiPer weights

$$\begin{aligned} f_{\hat{\mathcal{W}}}(\hat{w}) &= \frac{1}{2b\omega_0} \frac{1}{\sqrt{1-\hat{w}^2}} \exp\left(\frac{-|\arcsin(\hat{w})|}{b\omega_0}\right) \\ &+ \frac{1}{2b\omega_0} \frac{1}{\sqrt{1-\hat{w}^2}} \cosh\left(\frac{\arcsin(\hat{w})}{b\omega_0}\right) \frac{1}{e^{\pi/b\omega_0} - 1}. \end{aligned} \tag{7}$$

2. Derivation of Quantization Error from Eq.(18)

To obtain the QE expression for BiPer in Eq. (19), we start with its general formulation in Eq. (18) given by

$$\text{QE} = \int_0^{+\infty} \frac{1}{b} \exp\left(\frac{-w}{b}\right) \left(|\sin(\omega_0 w)| - \gamma\right)^2 \, \mathrm{d}w. \tag{8}$$

Expanding the square term and splitting it into three integrals yields

$$\mathrm{QE} = \int_0^{+\infty} \frac{1}{b} \exp\left(\frac{-w}{b}\right) \sin(\omega_0 w)^2 \, \mathrm{d}w + \gamma \int_0^{+\infty} \frac{1}{b} \exp\left(\frac{-w}{b}\right) |\sin(\omega_0 w)| \, \mathrm{d}w + \gamma^2 \int_0^{+\infty} \frac{1}{b} \exp\left(\frac{-w}{b}\right) \, \mathrm{d}w \tag{9}$$

For the first integral we use the trigonometric identity $\sin^2(x) = 0.5 - 0.5\cos(2x)$, and integrating by parts leading to

$$\frac{2(\omega_0 b)^2}{4(\omega_0 b)^2 + 1}. \tag{10}$$

Similarly, for the second integral, we used the fact that $|\sin(x)| = \sqrt{\sin^2(x)}$, and integrated by parts so that the solution can be written as

$$-\frac{2\gamma\omega_0 b \left(e^{\pi/\omega_0 b} + 1\right)}{\left((\omega_0 b)^2 + 1\right)\left(e^{\pi/\omega_0 b} - 1\right)}. \tag{11}$$

For the third integral, note that it can be seen as the distribution of a probability density function, which is equal to one. Therefore, its solution reduces to γ^2. Summarizing, the QE for BiPer can be written as

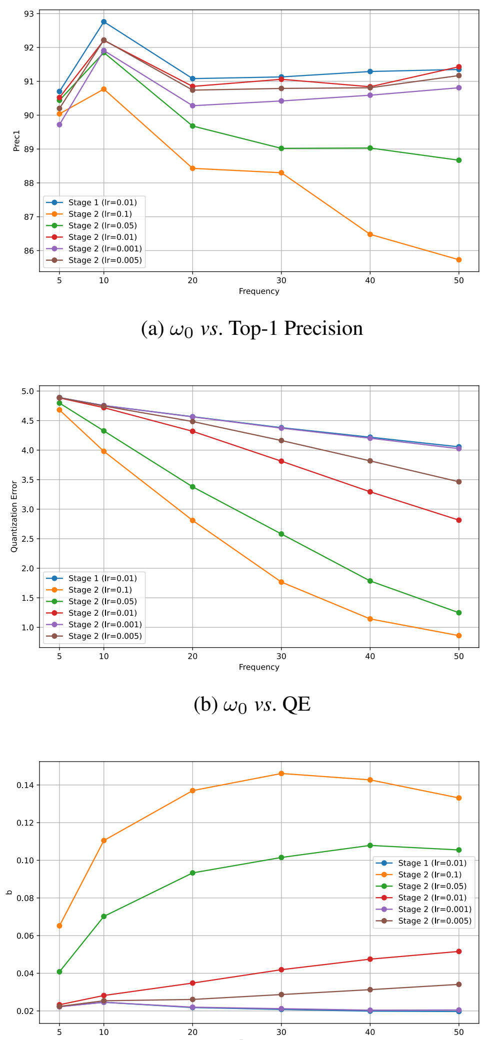
$$\mathrm{QE} = \frac{2(\omega_0 b)^2}{4(\omega_0 b)^2 + 1} - \frac{2\gamma\omega_0 b \left(e^{\pi/\omega_0 b} + 1\right)}{\left((\omega_0 b)^2 + 1\right)\left(e^{\pi/\omega_0 b} - 1\right)} + \gamma^2. \tag{12}$$

3. Additional results for VGG-small architecture on CIFAR 10

This section presents image classification results on the CIFAR-10 dataset using the VGG-small architecture.

Figure 1 illustrates the obtained stage 1 results for (a) Top-1 validation precision, (b) QE, and (c) maximum likelihood estimated parameter b. This figure complements Fig. 4 in the manuscript for VGG small architecture.

These results consistently resemble those obtained for the Resnet-18 architecture, where ensuring the lowest QE in Stage 1 does not result in the best network performance. For this case, we found that an intermediate frequency $\omega_0 = 10$ balances the initial QE and precision of the full binary model.



(a) ω_0 *vs.* Top-1 Precision

(b) ω_0 *vs.* QE

(c) ω_0 *vs.* b

Figure 1. Impact of the frequency of the periodic function ω_0 on BiPer for VGG-small architecture (a) Top-1 classification precision, (b) Quantization error and, (c) Weight distribution parameter b, for the CIFAR-10 dataset.