PhysGaussian: Physics-Integrated 3D Gaussians for Generative Dynamics

Supplementary Material

1. MPM Algorithm

In MPM, a continuum body is discretized into a set of Lagrangian particles p, and time is discretized into a sequence of time steps $t = 0, t^1, t^2, ...$ Here we take a fixed stepsize Δt, so $t^n = n\Delta t$.

At each time step, masses and momentums on particles are first transferred to grid nodes. Grid velocities are then updated using forward Euler's method and transferred back to particles for subsequent advection. Let m_p, $\boldsymbol{x}_p^n$, $\boldsymbol{v}_p^n$, $\boldsymbol{F}_p^n$, $\boldsymbol{\tau}_p^n$, and $\boldsymbol{C}_p^n$ denote the mass, position, velocity, deformation gradient, Kirchhoff stress, and affine momentum on particle p at time t_n. Here, particle masses are invariant due to mass conservation law. Let m_i^n, $\boldsymbol{x}_i^n$ and $\boldsymbol{v}_i^n$ denote the mass, position, and velocity on grid node i at time t^n. We summarize the explicit MPM algorithm as follows:

1. **Transfer Particles to Grid.** Transfer mass and momentum from particles to grids as

$$
\begin{aligned}
m_i^n &= \sum_p w_{ip}^n m_p, \\
m_i^n \boldsymbol{v}_i^n &= \sum_p w_{ip}^n m_p \left(\boldsymbol{v}_p^n + \boldsymbol{C}_p^n \left(\boldsymbol{x}_i - \boldsymbol{x}_p^n \right) \right).
\end{aligned} \tag{1}
$$

   We adopt the APIC scheme [1] for momentum transfer.

2. **Grid Update.** Update grid velocities based on forces at the next timestep by

$$
\boldsymbol{v}_i^{n+1} = \boldsymbol{v}_i^n - \frac{\Delta t}{m_i} \sum_p \boldsymbol{\tau}_p^n \nabla w_{ip}^n V_p^0 + \Delta t \boldsymbol{g}. \tag{2}
$$

3. **Transfer Grid to Particles.** Transfer velocities back to particles and update particle states.

$$
\begin{aligned}
\boldsymbol{v}_p^{n+1} &= \sum_i \boldsymbol{v}_i^{n+1} w_{ip}^n, \\
\boldsymbol{x}_p^{n+1} &= \boldsymbol{x}_p^n + \Delta t \boldsymbol{v}_p^{n+1}, \\
\boldsymbol{C}_p^{n+1} &= \frac{12}{\Delta x^2 (b+1)} \sum_i w_{ip}^n \boldsymbol{v}_i^{n+1} \left(\boldsymbol{x}_i^n - \boldsymbol{x}_p^n \right)^T, \\
\nabla \boldsymbol{v}_p^{n+1} &= \sum_i \boldsymbol{v}_i^{n+1} \nabla w_{ip}^{n\,T}, \\
\boldsymbol{F}_p^{\text{E, tr}} &= (\boldsymbol{I} + \nabla \boldsymbol{v}_p^{n+1}) \boldsymbol{F}^{E,n}, \\
\boldsymbol{F}_p^{E,n+1} &= \mathcal{Z}(\boldsymbol{F}_p^{\text{E, tr}}), \\
\boldsymbol{\tau}_p^{n+1} &= \boldsymbol{\tau}(\boldsymbol{F}_p^{E,n+1}).
\end{aligned} \tag{3}
$$

   Here b is the B-spline degree, and Δx is the Eulerian grid spacing. The computation of the return map $\mathcal{Z}$ and the Kirchhoff stress $\boldsymbol{\tau}$ is outlined in Sec. 2. We refer the readers to [2] for the detailed derivations from the continuous conservation law to its MPM discretization.

Table 1. **Model Settings.**

Scene	Figure	Constitutive Model
Vasedeck	Fig. 1	Fixed corotated
Ficus	Fig. 2	Fixed corotated
Fox	Fig. 3	Fixed corotated
Plane	Fig. 3	von Mises
Toast	Fig. 3	Fixed corotated
Ruins	Fig. 3	Drucker-Prager
Jam	Fig. 3	Herschel-Bulkley
Sofa Suite	Fig. 3	Fixed corotated
Materials	Fig. 6	Fixed corotated
Microphone	Fig. 7	Neo-Hookean
Bread	Fig. 1 in Supp.	Fixed corotated
Cake	Fig. 1 in Supp.	Herschel-Bulkley
Can	Fig. 1 in Supp.	von Mises
Wolf	Fig. 1 in Supp.	Drucker-Prager

Table 2. **Material Parameters.**

Notation	Meaning	Relation to E, ν
E	Young's modulus	/
ν	Poisson's ratio	/
μ	Shear modulus	$\mu = \frac{E}{2(1+\nu)}$
λ	Lamé modulus	$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$
κ	Bulk modulus	$\kappa = \frac{E}{3(1-2\nu)}$

2. Elasticity and Plasticity Models

We adopt the constitutive models used in [7]. We list the models used for each scene in Tab. 1 and summarize all the parameters needed in discussing the constitutive models in Tab. 2.

In all plasticity models used in our work, the deformation gradient is multiplicatively decomposed into $\boldsymbol{F} = \boldsymbol{F}^E \boldsymbol{F}^P$ following some yield stress condition. A hyperelastic constitutive model is applied to $\boldsymbol{F}^E$ to compute the Kirchhoff stress $\boldsymbol{\tau}$. For a pure elastic continuum, we simply take $\boldsymbol{F}^E = \boldsymbol{F}$.

2.1. Fixed Corotated Elasticity

The Kirchhoff stress $\boldsymbol{\tau}$ is defined as

$$
\boldsymbol{\tau} = 2\mu(\boldsymbol{F}^E - \boldsymbol{R})\boldsymbol{F}^{E^T} + \lambda(J-1)J, \tag{4}
$$

where $\boldsymbol{R} = \boldsymbol{U}\boldsymbol{V}^T$ and $\boldsymbol{F}^E = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$ is the singular value decomposition of elastic deformation gradient. J is the de-

Figure 1. **Additional Evaluation.** Examples from top to bottom are: *vasedeck* (elastic entity), *bread* (fracture), *cake* (viscoplastic material), *can* (plastic metal) and *wolf* (granular material).

terminant of $\boldsymbol{F}^E$ [1].

2.2. StVK Elasticity

The Kirchhoff stress $\boldsymbol{\tau}$ is defined as

$$\boldsymbol{\tau} = \boldsymbol{U}\left(2\mu\boldsymbol{\epsilon} + \lambda\,\mathrm{sum}(\boldsymbol{\epsilon})\mathbf{1}\right)\boldsymbol{V}^T, \tag{5}$$

where $\boldsymbol{\epsilon} = \log(\boldsymbol{\Sigma})$ and $\boldsymbol{F}^E = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$ [3].

2.3. Neo-Hookean Elasticity

The Kirchhoff stress $\boldsymbol{\tau}$ is defined as

$$\boldsymbol{\tau} = \mu(\boldsymbol{F}^E\boldsymbol{F}^{E^T} - \boldsymbol{I}) + \log(J)\boldsymbol{I}, \tag{6}$$

where J is the determinant of $\boldsymbol{F}^E$ [1].

2.4. Drucker-Prager Plasticity

The return mapping of Drucker-Prager plasticity for sand [3] is, given $\boldsymbol{F} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$ and $\boldsymbol{\epsilon} = \log(\boldsymbol{\Sigma})$,

$$\boldsymbol{F}^E = \boldsymbol{U}\mathcal{Z}(\boldsymbol{\Sigma})\boldsymbol{V}^T, \tag{7}$$

$$\mathcal{Z}(\boldsymbol{\Sigma}) = \begin{cases} \mathbf{1}, & \mathrm{sum}(\boldsymbol{\epsilon}) > 0, \\ \boldsymbol{\Sigma}, & \delta\gamma \leq 0, \text{ and } \mathrm{sum}(\boldsymbol{\epsilon}) \leq 0, \\ \exp\left(\boldsymbol{\epsilon} - \delta\gamma\frac{\hat{\epsilon}}{\|\hat{\epsilon}\|}\right), & \text{otherwise.} \end{cases} \tag{8}$$

Here $\delta\gamma = \|\hat{\boldsymbol{\epsilon}}\| + \alpha\frac{(d\lambda+2\mu)\,\mathrm{sum}(\boldsymbol{\epsilon})}{2\mu}$, $\alpha = \sqrt{\frac{2}{3}}\frac{2\sin\phi_f}{3-\sin\phi_f}$, and ϕ_f is the friction angle. $\hat{\epsilon} = \mathrm{dev}(\epsilon)$.

2.5. Von-Mises Plasticity

Similar to Drucker-Prager plasticity, given $\boldsymbol{F} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$ and $\boldsymbol{\epsilon} = \log(\boldsymbol{\Sigma})$,

$$\boldsymbol{F}^E = \boldsymbol{U}\mathcal{Z}(\boldsymbol{\Sigma})\boldsymbol{V}^T,$$

where

$$\mathcal{Z}(\boldsymbol{\Sigma}) = \begin{cases} \boldsymbol{\Sigma}, & \delta\gamma \leq 0, \\ \exp\left(\boldsymbol{\epsilon} - \delta\gamma\frac{\hat{\epsilon}}{\|\boldsymbol{\epsilon}\|}\right), & \text{otherwise}, \end{cases} \tag{9}$$

and $\delta\gamma = \|\hat{\boldsymbol{\epsilon}}\|_F - \frac{\tau_Y}{2\mu}$. Here τ_Y is the yield stress.

2.6. Herschel-Bulkley Plasticity

We follow Yue et al. [6] and take the simple case where $h = 1$. Denote $\boldsymbol{s}^{\text{trial}} = \text{dev}(\boldsymbol{\tau}^{\text{trial}})$, and $s^{\text{trial}} = ||\boldsymbol{s}^{\text{trial}}||$. The yield condition is $\Phi(s) = s - \sqrt{\frac{2}{3}}\sigma_Y \leq 0$. If it is violated, we modify s^{trial} by

$$s = s^{\text{trial}} - \left(s^{\text{trial}} - \sqrt{\frac{2}{3}}\sigma_Y\right) / \left(1 + \frac{\eta}{2\mu\Delta t}\right).$$

$\boldsymbol{s}$ can then be recovered as $\boldsymbol{s} = s \cdot \frac{\boldsymbol{s}^{\text{trial}}}{||\boldsymbol{s}^{\text{trial}}||}$. Define $\boldsymbol{b}^E = \boldsymbol{F}^E \boldsymbol{F}^{E^T}$. The Kirchhoff stress $\boldsymbol{\tau}$ is computed as

$$\boldsymbol{\tau} = \frac{\kappa}{2}\left(J^2 - 1\right)\boldsymbol{I} + \mu\,\text{dev}\left[\det(\boldsymbol{b}^E)^{-\frac{1}{3}}\boldsymbol{b}^E\right].$$

3. Additional Evaluations

We present additional evaluations of our method in Fig. 1. The *vasedeck* data is from the Nerf dataset [4] and the others are synthetic data, generated using BlenderNeRF [5].

References

[1] Chenfanfu Jiang, Craig Schroeder, Andrew Selle, Joseph Teran, and Alexey Stomakhin. The affine particle-in-cell method. *ACM Transactions on Graphics (TOG)*, 34(4):1–10, 2015. 1, 2

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[3] Gergely Klár, Theodore Gast, Andre Pradhana, Chuyuan Fu, Craig Schroeder, Chenfanfu Jiang, and Joseph Teran. Drucker-prager elastoplasticity for sand animation. *ACM Transactions on Graphics (TOG)*, 35(4):1–12, 2016. 2

[4] Ben Mildenhall, Pratul P Srinivasan, Matthew Tancik, Jonathan T Barron, Ravi Ramamoorthi, and Ren Ng. Nerf: Representing scenes as neural radiance fields for view synthesis. *Communications of the ACM*, 65(1):99–106, 2021. 3

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[6] Yonghao Yue, Breannan Smith, Christopher Batty, Changxi Zheng, and Eitan Grinspun. Continuum foam: A material point method for shear-dependent flows. *ACM Transactions on Graphics (TOG)*, 34(5):1–20, 2015. 3

[7] Zeshun Zong, Xuan Li, Minchen Li, Maurizio M Chiaramonte, Wojciech Matusik, Eitan Grinspun, Kevin Carlberg, Chenfanfu Jiang, and Peter Yichen Chen. Neural stress fields for reduced-order elastoplasticity and fracture. *arXiv preprint arXiv:2310.17790*, 2023. 1