UltraAugment: Fan-shape and Artifact-based Data Augmentation for 2D Ultrasound Images

Supplementary Material

5. Warping Operation Φ

Warping is performed by sampling all scanlines through a polar sweep across the fan angle α and arranging them next to each other. To find the origin O for sweeping, the four corners of the ultrasound fan P_1, P_2, P_3 and P_4 are identified. Origin O can then be found by calculating the intersection of vectors $\overrightarrow{P_3P_1}$ and $\overrightarrow{P_4P_2}$. For each angle θ of the sweeping line, the distance $r_{min}(\theta)$ from the beginning of the scanline to the origin O needs to be calculated to know where the scanline starts. To achieve this, the arc from P_1 to P_2 is assumed to be part of an ellipse with semi-major axis a, semi-minor axis b and origin Q. An ellipsoidal approximation was chosen over a circular one since the fan shape does not always correspond to a circle sector. To find parameters a and b, point U is identified, which is assumed to be the closest point to O that is still inside the fan shape. To find semi-major axis a, a new ellipse is created that passes through U and O with semi-major axis a and semi-minor axis b'. Origin Q' of this ellipse can be calculated according to

$$Q' = (U_x, \frac{U_y - O_y}{2}). \tag{12}$$

For this ellipse two foci P_1' and P_2' can be defined by setting the y-coordinate of P_1 and P_2 to Q_y'. To find a, we rely on the rule that the sum of the distances from the foci to a point on the ellipse is constant such that

$$\|\overrightarrow{P_1'U}\| + \|\overrightarrow{P_2'U}\| = 2a. \tag{13}$$

Notice that this holds since the summed distances from the foci to a boundary point on the major-axis is equal to the length of the major-axis, namely $2a$. Given that $b = U_y - Q_y$ and $Q_x = U_x$ we can use the ellipse equation to find Q_y:

$$\frac{(x - U_x)^2}{a^2} + \frac{(y - Q_y)^2}{(U_y - Q_y)^2} = 1. \tag{14}$$

Finding Q_y is then possible by filling in either P_1 or P_2 and solving for Q_y. To find $r_{min}(\theta)$ the distance from the origin O to the intersection point between the ellipse and the sweeping line is calculated as

$$r_{min}(\theta) = \sqrt{(Q_x + a\cos(\theta\frac{\eta}{\alpha}) - O_x)^2 + (Q_y + b\sin(\theta\frac{\eta}{\alpha}) - O_y)^2}. \tag{15}$$

The warping is achieved by letting θ range over the entire fan angle α, calculating $r_{min}(\theta)$ and sampling along the sweeping line for a length l. We assume l to be constant and calculate it by measuring the distance between the the closest point U and the furthest point on the same scanline. All math symbols are visualized in Fig. 3.

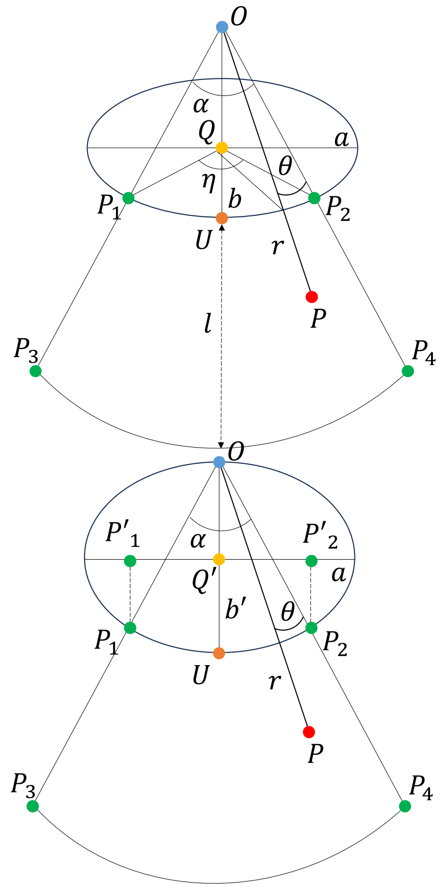

Figure 3. *Top*: Schematic overview of the math symbols involved in the warping operation. *Bottom*: Schematic overview of the second ellipse that is used to find the semi-major axis a.

6. Unwarping Operation Φ^{-1}

Unwarping is performed by calculating for each position (x, y) in the original image space the corresponding position (θ, r) in the warped image space. This can be achieved by doing a conversion from Cartesian coordinates to polar coordinates using origin O. Using polar coordinates it becomes possible to sample the warped image. For coordinates that fall outside the fan shape, and are therefore not valid in the warped image space, a zero value is used to get the original fan shape padding back.